Effect of Similarity Measures and Underlying PCA Subspace on Linear Discriminant Analysis

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ABSTRACT

This paper addresses the use of Linear Discriminant Analysis for recognition of human faces. It presents the effect of various similarity measures and the dimensionality of underlying PCA subspace on the recognition rate of the system. Commonly used 4 similarity measures such as City block, Euclidean, Cosine and Mahalanobis are tested. In order to test the performance of the face recognition using Linear Discriminant Analysis, various experiments are carried out on AT&T face database. AT&T is publicly available database which has 400 images of 40 different persons. It was observed that changing similarity measure caused significant change in the performance of the system. The performance improved with the dimensionality of the final subspace. We achieved the best recognition performance using Cosine distance measure. It is observed that recognition rate depends on the dimensionality of underlying PCA subspace as well as on the similarity measure used.

General Terms

Image Processing, Pattern Recognition, Face Recognition

Keywords

Biometrics, Face Recognition, Principal Component Analysis, Linear Discriminant Analysis

1. INTRODUCTION

Biometrics establishes the identity of an individual based on the unique physical or behavioral characteristics (i.e. traits) of the person. As these biometric traits are inherent to an individual, it is more difficult to manipulate, share, or forget them [1]. Biometrics is increasingly employed in several government and civilian identity management applications either to replace or enhance security offered by traditional knowledge-based and token-based schemes.

Face images are the most commonly used biometric characteristic by humans to recognize one another. Face recognition methods are categorized into two types: feature-based and appearance-based. In the appearance-based approaches, whole face image is considered rather than just local features. Feature-based approaches consider the location and shape of facial attributes, such as the eyes, eyebrows, nose, lips, and chin, and their spatial relationships. This paper presents Linear Discriminant Analysis (LDA) [2, 3, 4] which is one of the well known appearance-based techniques for performing face recognition.

The organization of the paper is as follows: Section 2 describes use of LDA/fisherfaces in recognizing faces in detail. Section 3 presents experimental work and results. Section 4 offers the conclusion.

2. METHODOLOGY

2.1 Linear Discriminant Analysis

Belhumeur et. al. [2,3] presented fisherface method which maximizes the ratio of between-class scatter to within-class scatter so as to make accurate classification.

Steps to develop face recognition system using LDA:

- Let the training database of M face images be $\Gamma_1, \Gamma_2, \ldots, \Gamma_M$, each of size N x N. Consider that training database has total C number of persons.
- The mean image is calculated as:

$$\psi = \frac{1}{M}\sum_{i=1}^{M}\Gamma_i \qquad (1)$$

- The deviation of each image from the mean image is given as:

$$\phi_i = \Gamma_i - \Psi\,. \qquad (2)$$

- Calculate M x M matrix L as:

$$L = A^T A \ where\ A = [\phi_1, \phi_2, \ldots \phi_M] \qquad (3)$$

  This gives M eigenvectors (i.e. v) corresponding to M eigenvalues. Using formula $u = A{*}v$, get the most significant M eigenvectors of covariance matrix AA^T.
- Centered image vectors are projected onto subspace formed using the most significant eigenvectors of covariance matrix as done in eigenface method [5, 6].
- Calculate the mean of each class and also the total mean in eigenspace.
- Within class scatter matrix (S_w) and between class scatter matrix (S_b) are calculated as:

$$S_w = \sum_{j=1}^{C}\sum_{i=1}^{N_j}(\Gamma_i^j - \mu_j)(\Gamma_i^j - \mu_j)^T \qquad (4)$$

  where Γ_i^j is the i^{th} sample of class j, C is the number of classes, μ_j is the mean of class j, N_j is the number of samples in class j.

$$S_b = \sum_{j=1}^{C}(\mu_j - \mu)(\mu_j - \mu)^T, \qquad (5)$$

  where μ is the mean of all classes.
- Goal is to minimize S_w while maximizing S_b. This can be achieved by maximizing the ratio det$|S_b|$/det$|S_w|$. This ratio gets maximized when

eigenvectors of $S_b\, S_w$ form the column vectors of the projection matrix *i.e.*

$$W = [w_1, w_2, \ldots, w_{C-1}], \quad (6)$$

where $\{w_i \mid i = 1, 2, \ldots, C-1\}$ are the eigenvectors of S_b and S_w corresponding to the set of decreasing eigenvalues $\{\lambda_i \mid i = 1, 2, \ldots, C-1\}$.

- To prevent S_w from becoming singular, PCA is used to reduce the dimension of the feature space to $M–C$ and then, Fisher Linear Discriminant is applied to further reduce the dimension to $C–1$.

2.2 Similarity Measures

Consider two feature vectors x and y of dimensions n each. The distances between these feature vectors can be calculated as [6, 7, 8]:

1. City block distance (or Manhattan distance):

$$d(x, y) = \sum_{i=1}^{n} |x_i - y_i| \quad (7)$$

2. Euclidean distance:

$$d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \quad (8)$$

3. Cosine distance:

$$d(x, y) = -\frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}} \quad (9)$$

4. Mahalanobis distance:

$$d(x, y) = \sqrt{(x - y) S^{-1} (x - y)^t} \quad (10)$$

where S is the covariance matrix of the distribution.

3. EXPERIMENTAL WORK AND RESULTS

3.1 Data

To perform face recognition, the AT&T face database [9] is used. It has face images taken between April 1992 and April 1994 at the AT&T Laboratories. The database has 400 grayscale face images with 10 images each from 40 persons. All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position with tolerance for some side movement. None of the 10 images of a person is identical to other. They vary in pose, expression, rotation and scale. Each person has changed facial expression in each of the 10 samples (i.e. smiling/non-smiling, open/close eye). For some persons, images are taken at different times, by varying facial details like wearing glasses or no glasses. Size of each image is 112 x 92 pixels, with 256 gray levels per pixel. The files are in PGM format.

3.2 Preprocessing and Training

All pre-processing and implementation is done using MATLAB® R2013a. In our experiments, each face image is resized to 50 x 42 pixels. First 5 images per person are used for training, and remaining five are used for testing. This forms gallery set of 200 images and probe set of 200 images. Figure 1 shows all sample ten images of some of the persons from AT&T face database.

Fig.1: Sample faces from ORL face database

3.3 Results

Let g be the dimensionality of underlying PCA subspace and f be the final dimensionality of LDA. Value of g cannot exceed $M - C$ [10], where M is the number of training images and C is the number of persons. As there are 200 training images and 40 persons, g cannot go beyond 160. Value of g is varied from 2 to maximum possible value of 160.

Table 1 shows recognition rate of LDA corresponding to the dimensionality of final subspace for various similarity measures. It shows recognition rate for each value of f based on that value of g that gave best recognition rate.

Figure 2 shows plot of recognition rate corresponding to the dimensionality of final subspace for various similarity measures. Final dimensionality of LDA can be at the most $C-1$, therefore LDA curve cannot go beyond $f = 39$.

Table 1. Recognition rates of the system corresponding to the dimensionality of final subspace using various similarity measures

Dimensionality of final subspace f	Recognition rate of LDA (%)			
	City Block	Eucl.	Cos	Mahalanobis
2	53	54	27.5	53
3	67	68.5	57.5	69.5
4	79	80.5	76	79.5
5	81	84	83.5	82
6	84	86	87.5	84.5
7	86.5	87	91	85.5
8	87	87.5	91.5	86.5
9	88	89	91.5	86.5
10	88.5	88.5	91.5	85.5
12	88.5	88.5	92.5	88
13	88.5	88	92.5	87.5
14	89	88.5	93	89
15	89.5	89	93	88
16	90	89.5	93.5	88

Dimensionality of final subspace *f*	Recognition rate of LDA (%)			
	City Block	Eucl.	Cos	Mahalanobis
17	90	90.5	94.5	87.5
18	91.5	90	94	88
19	91	90.5	94.5	89
20	93.5	91	95	90
21	91.5	92	95	90
22	91.5	92.5	95	90
23	91.5	92.5	95	91
24	92	93	95	91
25	92	92.5	95.5	91.5
26	90.5	91	95.5	88.5
27	90.5	91	96	89
28	90.5	91.5	96	89.5
29	92	94	95.5	88.5
31	92.5	93	95	89
32	91	94	95.5	89
33	92.5	93	95	89.5
34	92	93	95.5	88.5
35	91	91.5	95	89
36	91	91.5	95	88
37	91	91.5	95.5	88
38	91.5	91.5	95.5	89
39	91.5	92	95.5	88

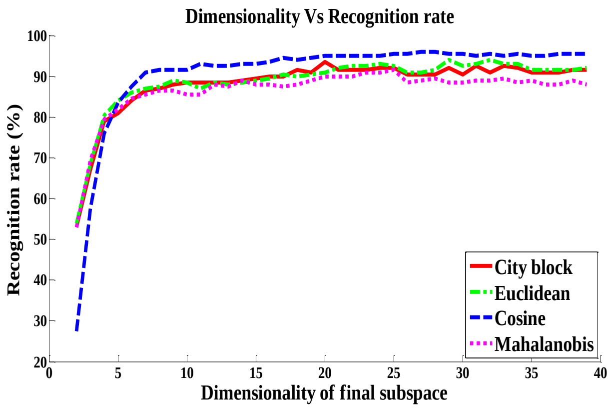

Fig.2: Recognition rate vs. dimensionality of final subspace for face recognition using LDA for various similarity measures

As seen in Figure 2, recognition rate using LDA increases with increase in dimensionality of final subspace. Table 1 shows that maximum recognition rates achieved are 93.5% at f = 20 using City block, 94% at f = 29/32 using Euclidean, 96% at f = 27/28 using Cosine and 91.5% at f = 25 using Mahalanobis distance measure.

4. CONCLUSIONS

We implemented face recognition using Linear Discriminant Analysis. Experimentation work is carried out on AT&T database of faces which has 400 images of 40 persons with 10 images per person. Recognition rates are found using most commonly used similarity measures like City block, Euclidean, Cosine and Mahalanobis. Recognition rate using LDA increases with the increase in the dimensionality of the final subspace. Recognition rates achieved are 93.5%, 94%, 96% and 91.5% using City block, Euclidean, Cosine and Mahalanobis distance measures respectively. It is observed that recognition rate depends on the dimensionality of the underlying PCA subspace as well on the similarity measure used.

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