Modelling and Design Aspects of PI Controller for Coupled Tank Process

Elizabeth Rani T[1],
PG Student,
Karunya University,
Coimbatore, India.

Samson Isaac J [2]
Assistant Professor,
Karunya University,
Coimbatore, India

ABSTRACT
In industry, liquids such as water and chemicals are used in various processes. The amount of such liquids stored can be found by measuring level of the liquid in a container or tank. The level affects not only the quantity delivered but also pressure and rate of flow in and out of the container. Often the tanks are so coupled together that the levels interact and exhibits a nonlinear behaviour. Control of coupled tank system using conventional PI controller, result in poor performance. In this paper the modelling and controller design for coupled tank liquid level process using characteristic ratio assignment (CRA) method is implemented. CRA method is an approach to directly control the transient response of linear time invariant systems. It is very convenient for fast adjustment of speed as well as damping ratio of the response. Finally, the simulation results can be done by MATLAB simulink and LabVIEW. The results can be compared with other tuning techniques also.

Keywords
Characteristic Ratio Assignment (CRA), SISO Process, PI Controller, transient response.

1. INTRODUCTION
Several techniques have been developed for designing LTI control system that is optimum and robust. However despite the theoretical successes of modern control approaches have some significant drawbacks to effective utilization in practice. The drawbacks are related to the fact that the robust optimal controller has higher order [1].

Recently, several robust controllers are developed for uncertain or non linear system. Another robust controller known as Characteristic ratio assignment method is developed [2]. The structure of this controller is based on certain relations between characteristic polynomial coefficient.

Proportional Integral Derivative controllers are widely used in the process control industries. The main reason for that is, its relatively simple structure, which is easily understood and implemented in practice. Owing to their popularity in the industries, many approaches have been developed to determine the parameters of PID controller for single input single output (SISO) systems [6]. Among the well-known formulas are the Cohen–Coon method, Ziegler–Nichols rule, the ITEA, IAE and internal model control. But many PID controllers are poorly tuned in practice. One of the reasons is that most of the tuning methods are derived for particular processes and conditions, and therefore that are apply to their own area [6].

All the control system design for LTI dynamic system depends on proper selection of the characteristic polynomials and proper selection of numerator polynomials for concerned input output relations. When these polynomials are properly selected, the design of controller transfer function requires only simple mathematics. The proper selection of characteristic is not a big deal but it becomes complicated only when robustness issue is present [9].

This paper presents the mathematical modelling and transient response control of the coupled-tank process, the interactive process is controlled by PI controller using characteristic ratio assignment method. The combination of proportional and integral terms is important to increase the speed of the response and also to eliminate the steady state error. By independently adjusting α and τ in all-pole systems, small or no overshoot as well as fast speed of response can be achieved. Explained about how to speed up the response without altering the overshoot of the response. The controller used here is a state feedback plus feed forward. The response of Ziegler-Nichols and characteristic Ratio Assignment method is compared.

2. EXPERIMENTAL SETUP
Two tanks are connected in interactive manner. Both tanks are identical in cross section. The cross sectional area of the tank is represented as $A(cm^2)$. The inlet flow Q_{in} is given to the tank 1 and the outlet flow Q_{out} is from tank 2. A manual valve is available between the tank1 and tank2 can be used to change the interaction between the tanks. The change in water level $h_1(cm)$ in tank1 is affects the water level $h_2(cm)$ in tank 2. The water level variation in tank1 and tank2 depends on the inlet and outlet flows. The control variable is level in tank2 that is $h_2(cm)$. Manipulated variable is input flow rate Q_{in}.

The water level of the tank 2 is measured by capacitive level transmitter. The output of the level transmitter is 4-20mA which is converted into voltage in the range of 0-5v by current to voltage converter. This voltage signal is given to the CRA-PI controller(pc) through data acquisition card. The DAQ card is used as a interface between hardware and software. The output of the controller is 0-5v which is converted into current signal by voltage to current converter. This current signal is given to the current to pressure converter which converts this current into pressure in the range of 3-15psi. This pressure is given to the pneumatic control valve.

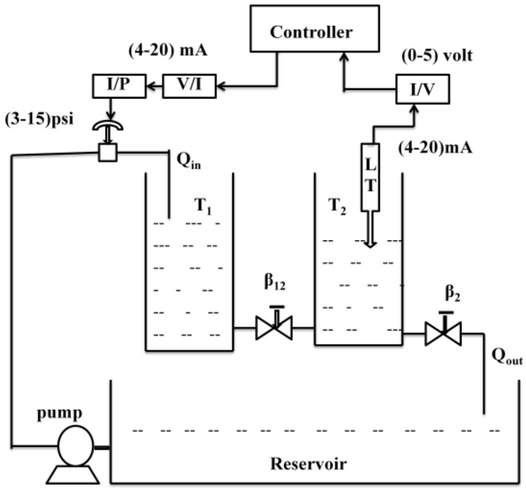

Fig.1 Block diagram of coupled tank system

According to the pressure given to the pneumatic control valve, the position of the valve will change. So, the inlet flow rate F_{in} will vary.

3. THE COUPLED-TANK PROCESS

According to fig.1, the input $u(t)$ is the input pressure which is taken to the pump, and the output $h_2(t)$ is the water level in tank1. The nonlinear equation can be obtained by mass balance equation and Bernoullis law is given by

$$\frac{dh_1(t)}{dt} = \frac{-\beta_{12}a_{12}}{A_1}\sqrt{2g(h_1(t)-h_2(t))} + \frac{k}{A_1}u(t) \qquad (1)$$

$$\frac{dh_2(t)}{dt} = \frac{-\beta_2 a_2}{A_2}\sqrt{2gh_2(t)} + \frac{\beta_{12}a_{12}}{A_2}\sqrt{2g(h_1(t)-h_2(t))} \qquad (2)$$

Where,

A_1, A_2 - cross sectional area of tank 1 & 2 (cm^2)
a_2 - cross sectional area of output pipe in tank 2 (cm^2)
a_{12} - cross sectional area of the interaction pipe between tank 1 & tank 2 (cm^2)
h_1, h_2 - level of water in tank 1 and tank 2(cm)
Q_{in} - inflow (LPH)
Q_{out} - outflow (LPH)
β_{12} -valve ratio of interaction pipe between tank 1and tank 2
β_2 - valve ratio at the outlet of tank 2
k - gain of the pump
g - gravity (m/s^2)

The equation (1) & (2) can be linearized by Taylor's series expansion

$$\frac{dH_1(t)}{dt} = \frac{1}{B_{12}}(-H_1(t)+H_2(t)) + \frac{k}{A_1}U(t) \qquad (3)$$

$$\frac{dH_2(t)}{dt} = \frac{1}{B_2}(-H_2(t)) + \frac{1}{B_{12}}(H_1(t)-H_2(t)) \qquad (4)$$

Where,

$$B_2 = \frac{A_2}{\beta_2 a_2}\sqrt{\frac{2h_2(t)}{g}}\ \text{sec}$$

$$B_{12} = \frac{A_{12}}{\beta_{12}a_{12}}\sqrt{\frac{2(h_1(t)-h_2(t))}{g}}\ \text{sec}$$

Take laplace transform and simplify,
The transfer function of a coupled tank process is obtained from the linear equations (3) & (4) as

$$\frac{H_2(s)}{U(s)} = G(s) = \frac{K}{B_{12}B_2 s^2 + (B_{12}+2B_2)s + 1} \qquad (5)$$

Where, $K = \frac{kB_2}{A_2}$ cm

4. CONTROL SYSTEM STRUCTURE

The two-degree of freedom control system structure, that its parameters are determined by CRA method.

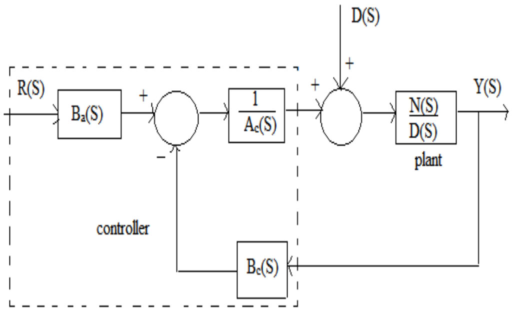

Fig 2: Block diagram of CRA control system

The general transfer function of CRA control system is,

$$\frac{Y(s)}{R(s)} = \frac{B_a(s)N(s)}{A_c(s)D(s)+B_c(s)N(s)}$$

Where $N(s)$ and $D(s)$ is the polynomial equation.
$N(s) = K$, $D(s) = B_{12}s^2 + (B_{12}+2B_2)s + 1$, $A_c(s) = s$
$B_c(s) = K_p s + K_i$, $B_a(s) = K_i$
The characteristic equation of CRA transfer function is,
$P(s) = A_C(s)\,D(s) + B_C(s)\,N(S)$
$P(s) = B_{12}B_2s^3+(B_{12}+2B_2)s^2+(1+KK_p)s+KK_i \qquad (7)$
This Equation is the polynomial characteristic equation to design the PI controller by CRA method.

5. THE CRA METHOD

The general relation of characteristic equation can be given as,

$$Q(s) = b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0 \qquad (8)$$

Where b_n, b_{n-1}, ...b_1, b_0 are coefficients of characteristic equation.
The characteristic ratios of the CRA method is given by

$$\alpha_1 = \frac{b_1^2}{b_0 b_2}, \alpha_2 = \frac{b_2^2}{b_1 b_3}, \ldots \alpha_{n-1} = \frac{b_{n-1}^2}{b_{n-2}b_n} \qquad (9)$$

And the inverse of characteristic ratio is given as,

$$a_0 = \frac{b_0}{b_1}, a_1 = \frac{b_1}{b_2}, \ldots, a_{n-1} = \frac{b_{n-1}}{b_n} \quad (10)$$

Another form of characteristic ratio is given as

$$\alpha_1 = \frac{a_1}{a_0}, \alpha_2 = \frac{a_2}{a_1}, \ldots, \alpha_{n-1} = \frac{a_{n-1}}{a_{n-2}} \quad (11)$$

Thus the time constant is given by

$$\tau = \frac{b_1}{b_0} \quad (12)$$

Given equation (2) – (4), it is able to describe in another form by the coefficient of characteristic equation

$$B = [\, b_n \quad b_{n-1} \ \ldots \ b_1 \quad b_0] \quad (13)$$

$$C = [c_{n-1} \quad c_{n-2} \ \ldots \ c_1 \quad c_0] \quad (14)$$

$$D = [d_{n-2} \quad d_{n-3} \ \ldots \ d_1 \quad d_0] \quad (15)$$

Where,

$$c_i = \frac{b_i}{b_{i+1}}, i = 0,1,2,\ldots,n-1 \quad (16)$$

$$d_i = \frac{c_{i+1}}{c_i}, i = 0,1,2,\ldots,n-2 \quad (17)$$

5.1 Adjust the speed of response

To adjust the speed of response, the CRA method can be tuned by changing a value of time constant. Then the transfer function is

$$G(s) = \frac{b_0}{b_n s^n + b_{n-1} s + \ldots + b_1 s + b_0} \quad (18)$$

Then it is arranged in new format

$$G(s) = \frac{b_0 / b_n}{s^n + \frac{b_{n-1}}{b_n} s + \ldots + \frac{b_1}{b_n} s + \frac{b_0}{b_n}} \quad (19)$$

From equation (12) is able to construct the coefficient as inverse form of characteristic equation

$$\mathrm{B} = \left[\ 1 \quad \prod_{i=n-1}^{n-1} c_i \ \ldots \ \prod_{i=2}^{n-1} c_i \quad \prod_{i=1}^{n-1} c_i \ \right] \quad (20)$$

when increase the gain with equivalent ratio by k, it is obtained by

$$\mathrm{B} = \left[\ 1 \quad k \prod_{i=n-1}^{n-1} c_i \ \ldots \ k^{n-1} \prod_{i=2}^{n-1} c_i \quad k^n \prod_{i=1}^{n-1} c_i \ \right] \quad (21)$$

Coefficient ratio is unchanged but the time constant is able to change as shown in equation (20) and (21).

$$G_K(S) = \frac{k^n b_o}{b_n s^n + k b_{n-1} s^{n-1} + \ldots + k^{n-1} b_1 s + k^n b_0} \quad (22)$$

$$\tau = \frac{1}{k}\left(\frac{b_1}{b_0}\right) \quad (23)$$

5.2 Adjust damping ratio of control system

To adjust the damping ratio, CRA method is designed for tuning only one parameter (α_i) characteristic ratio. By adjusting the characteristic ratio the damping ratio will increase, at the same time overshoot is decreased and the response will be faster.

$$G(s) = \frac{1}{\frac{b_n}{b_0} s^n + \frac{b_{n-1}}{b_0} s^{n-1} + \ldots + \frac{b_1}{b_0} s + 1} \quad (24)$$

$$\boldsymbol{B} = \left[\frac{1}{\prod_{i=0}^{n-1} c_i} \ \frac{1}{\prod_{i=0}^{n-2} c_i} \cdots \frac{1}{\prod_{i=0}^{1} c_i} \ \frac{1}{\prod_{i=0}^{0} c_i} \right] \quad (25)$$

$$A = \left[\frac{1}{[\prod_{i=0}^{n-2} d_i \prod_{i=0}^{n-3} d_i \ldots \prod_{i=0}^{0} d_i] c_0^n} \ \frac{1}{[\prod_{i=0}^{n-3} d_i \prod_{i=0}^{n-4} d_i \ldots \prod_{i=0}^{0} d_i] c_0^{n-1}} \ldots \frac{1}{c_0} 1 \right] \quad (26)$$

Then the coefficient ratio is increased by k, the new characteristic equation is given by

$$A = \left[\frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2}} [\prod_{i=0}^{n-2} d_i \prod_{i=0}^{n-3} d_i \ldots \prod_{i=0}^{0} d_i] c_0^n} \ \frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2} - 1} [\prod_{i=0}^{n-3} d_i \prod_{i=0}^{n-4} d_i \ldots \prod_{i=0}^{0} d_i] c_0^{n-1} \ldots \frac{1}{c_0} 1]} \right]$$

Then the equation (27) is formed from above equations

$$G_k(s) = \frac{k^{\frac{1}{2}n^2 - \frac{1}{2}n} b_0}{b_n s^n + k^{n-1} b_{n-1} s^{n-1} + \ldots + k^{\frac{1}{2}n^2 - \frac{1}{2}n - 1} b_1 s + k^{\frac{1}{2}n^2 - \frac{1}{2}n} b_0} \quad (27)$$

The parameters of the coupled tank as SISO process are tabulated as below.

Table 1. Parameters of Coupled Tank SISO Process

A_1, A_2 (cm^2)	a_2, a_{12} (cm^2)	β_2	β_{12}
367.14	5.75	0.1	0.02

Table 2. Operating Point of Coupled Tank SISO Process

h_1 (cm)	h_2 (cm)	u (v)	k (cm^3/v.s)
29.5	28	3	10.1

6. THE SIMULATION RESULTS

In this paper, the simulation results of the coupled tank SISO process is given by MATLAB simulink. The experiment of PI controller based on CRA shows the adjustment of speed response and damping ratio. Using the parameters on the Table 1 and Table 2, the transfer function for coupled tank SISO process as

$$G(s) = \frac{20.9}{26932.6 s^2 + 1560.847 s + 1} \quad (28)$$

6.1 The Adjustment of Speed of Response

Using Characteristic Ratio Assignment method, the adjustment of speed response can be produced by adjusting the value of k according to equation (20).

$$G_K(S) = \frac{k^3 0.807}{26932.6s^3 + k1614s^2 + k^2 48.42s + k^3 0.807}$$

Table3. Parameters of PI controller

	K_p	K_i
K = 1	2.268	0.0386
K = 1.1	2.755	0.051
K = 1.2	3.288	0.0667

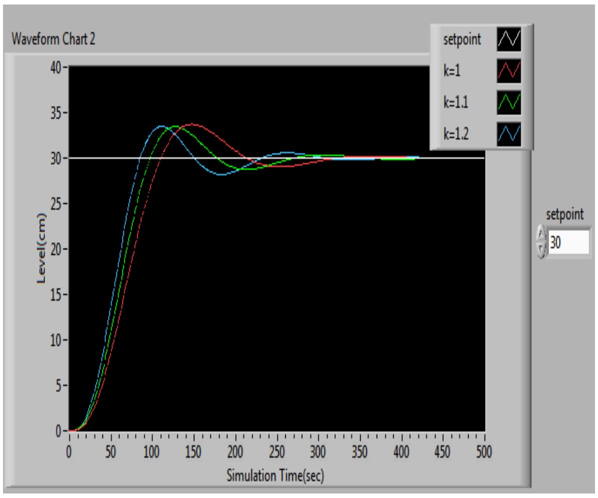

Fig 3: The response when k is changed

When k is adjusted from 1.1 to 1.2, settling time is decreased from 373.2 sec to 319.46 sec but the overshoot is not change. The system response is faster.

6.2 The adjustment of damping ratio

The CRA method can be used to adjust the damping ratio by formulating feedback transfer function.

$$G_K(S) = \frac{k^3 0.807}{26932.6s^3 + k1614s^2 + k^3 48.42s + k^3 0.807}$$

Table 4. Parameters of PI controller

	K_p	K_i
K = 1	2.268	0.0386
K = 1.1	2.867	0.048
K = 1.2	3.737	0.063

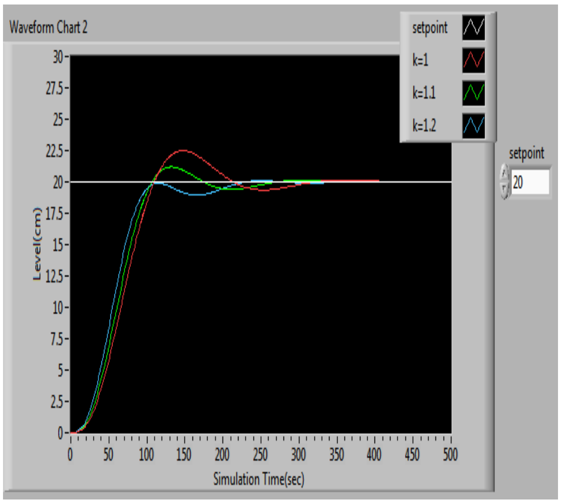

Fig 4: The response when damping ratio adjusted

When k is adjusted from 1.1 to 1.2, setting time is decreased from 371 sec to 291 sec at the same time overshoot is decreased from 0.8 to 0. Therefore the system response is faster.

6.3 Comparison between Ziegler Nichols and CRA PI Controller

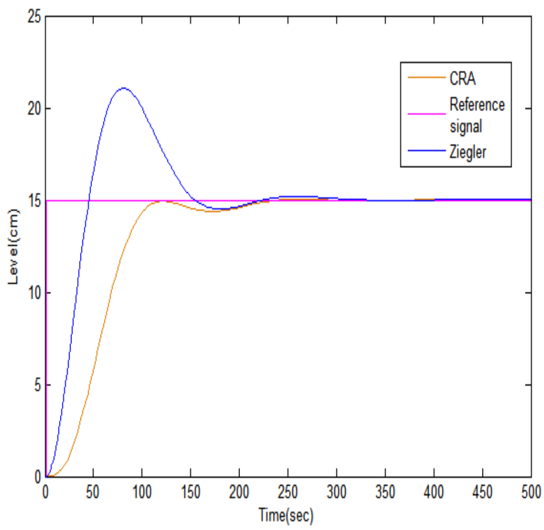

Fig 5: closed loop response

The parameters of the PI controller using Ziegler Nichols method are k_p=2.99, k_i=0.0405.
The overshoot of the response is 40.1%and the settling time is 332 sec.

Table5. Performance values of the Time Response

	Settling Time	%Overshoot
CRA	291s	0
Ziegler-Nichols	332s	40.1

7. CONCLUSION

In this paper, the mathematical model of a coupled tank system was developed and the design of PI controller using CRA method is presented. The MATLAB simulation results shows how effectively faster response is obtained. It also shows that by adjusting only one parameter gives the better result. When compared to the Ziegler Nichols tuning method, the proposed CRA method gives better result with reduced settling time and overshoot. Thus, the proposed method is convenient and suitable for designing and tuning the controller.

8. REFERENCES

[1]Daekwan Kim, Hansil Kim and Wonho Choi, 2003. "A New Design Method for Transient Response Control " , SICE Annual Conference in Fuhi, Fukui University, Japan.

[2] P. Naslin , 1969. Essentials of optimal control London Boston Technical Publishers Inc, Cambridge, MA pp. 30-51

[3]Quing-Guo Wang, Tong-hengLee, Ho - Wang Fung, QiangBi and Yuzhang, 1999. " PID Tuning For Improved Performance ", IEEE Transactions on Control System Technology, Vol.7, No.4.

[4]S.Manabe,1998."Coefficient Diagram Method", Proceedings of 14th IFAC Symposium on Automatic Control in Aerospace, Seoul.

[5]Y.C.Kim, L.H.Keel and S.P.Bhattacharya, 2003. "Transient Response Control via characteristic Ratio Assignment", IEEE Transactions on Automatic control. Vol.48, No. 12 : pp 2238-2244.

[6]S.Jayasuriya and J.W.Song,1988. "On the Synthesis of Compensators for Non Overshooting Step Response", in Proceedings American Control Conference. pp 683-684

[7]Numsomran, A.Wongkhum, T.Suskri, T.Nilas, P.Chaoraingern, 2007. "Design of decoupled controller for TITO system using characteristic ratio assignment", International Conference on Control, Automation and systems. pp 957-962.

[8]YoungcholKim, KeunsiKim, ShunjiManabe, 2006. "Sensitivity of Time Response to Characteristic Ratio", IEICE Trans, Fundamentals.Vol.E89 - A.

[9]M.Zhuang and D.P.Athetron, 1983."PID Controller Design for a TITO System", Proceeding of the American Control Conference, San Francisco, California, pp 3176 - 3177.

[10]Tianchai Suksri U - thai Sritheeravirojana, Arji Numsomran, Viriya Kongrattana, and Thongchai Werataweemart, 2005. "T - DOF PID Controller Design using Characteristic Ratio Assignment Method for Quadruple Tank Process" , World Academy of Science, Engineering and Technology.

[11]J.Chaoraingern,T.Trisuwannawat and A.Numsomran 2009. "PID Controller Design for Following Control of Hard Disk Drive by Characteristic Ratio Assignment Method" World Academy of Science, Engineering and Technology.

[12] A.V. Lipatov and N.I. Sokolov, 1978. "Some Sufficient Conditions for Stability and Instability of Continuous Linear Stationary Systems, "Translated from Automatika, No.9, pp. 30-37