ANFIS Gain Scheduled Johnson’s Algorithm based State Feedback Control of CSTR

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ABSTRACT

CSTR plays a vital role in almost all the chemical industries. Continuous Stirred Tank Reactor (CSTR) is a highly nonlinear process. The CSTR process is analysed for stability conditions by investigating the eigen values. Also Phase plane trajectory is constructed to analyse the CSTR. CSTR exhibits stable and unstable steady state at different regions. Control of CSTR in the complete range is a mind boggling problem. In this paper, a ANFIS scheduled Johnson’s Algorithm based state feedback control of CSTR is proposed. The entire range is divided into three regions; low, middle and the high region. The low and the high region have stable steady state and the middle region have unstable steady state. Initially, the gain matrix of the state feedback controller is computed for each region using Johnson’s Algorithm to have a minimum performance index. Then using ANFIS gain scheduler, the complete controller is formed. The result shows the feasibility of using the proposed controller for the control of CSTR for the tracking and regulatory problems.

Keywords

State feedback, Phase Plane Trajectory, Johnson’s Algorithm, ANFIS Gain Scheduler, Continuous Stirred Tank Reactor.

1 INTRODUCTION

CSTR is part and parcel of any chemical industry. CSTR is a highly complicated nonlinear process. Stability analysis of CSTR is tedious. CSTR is a single process which exhibits stable and unstable linear portions. As a first step the complete nonlinear CSTR has to be split into multiple linear regions. The conventional method of Taylor’s series forms the linear model from the nonlinear model. Each linear region has to be checked for the stability. The stability of CSTR is investigated by checking the eigen values.

The graphical method of analysing the stability by phase plane is investigated in this work.

Due to its strong nonlinear behavior, the problem of controlling of CSTR is always an attracting task for control system engineers. Conventional control techniques are suited for completely known and well understood Linear Time-Invariant systems. When the system exhibits parameter inaccuracy and time varying dynamics, conventional control methods encounters problem. Despite of the difficulty in achieving high control performance, the fine-tuning of controller parameters is a tedious task that requires expert knowledge both in control theory and process information. Therefore, the control of systems with nonlinear, complex, unknown and uncertain dynamics is a serious challenge to control community [6]. Optimal control of systems are used to provide a desired control of the process [3,4]. In order to accommodate the nonlinearity, a gain scheduled control scheme is investigated. A gain scheduled control system consists of a family of controllers (Local Controllers) and a scheduler. The scheduler selects the controller depending on the operating region. Plethora of advanced control schemes such as neural adaptive controller [5,7,8], Nonlinear Internal Model Control Scheme [9] and Adaptive Control Scheme [1,10] and Robust control [2] have been attempted on the CSTR process considered for the simulation study in this paper. Even with the all the nonlinear control strategies, gain scheduling is the simplest and has low computational complexity.

Gain-scheduling is a well-known technique of industrial control and is used when a plant is subject to large changes in its operating state, a situation that is typical in industry. Large changes in the operating state lead to corresponding variations in the parameters of the linearized models of the plant about these operating states, it is well known that it is not possible therefore to design a controller to operate satisfactorily at one operating state and expect it to perform equally well elsewhere without re-tuning it. Closed system performance is degraded since the controller cannot track the changes in the operating states. Considerable effort has gone into developing controllers that can track the variations in plant parameters with a view to achieving invariant operation throughout the domain of operation of the plant. Adaptive controllers are one such approach, yet even these controllers do not always demonstrate satisfactory performance throughout the domain of operation of the plant and may, on occasion, lose control altogether. Robust controllers, another approach, also have their limitations since they must deal with system dynamics that vary over a wide range though using constant parameters only. Clearly this class of controllers can only operate satisfactorily over a limited domain. ANFIS Gain-Scheduling assures smooth transitions in the control law, yet maintains essentially invariant closed system characteristics.

In this paper, the single nonlinear CSTR is split into three regions. Each region is subjected to stability analysis by verifying the eigen values and by investigating the phase plane trajectory. After analysing, control of CSTR using state feedback gain using Johnson’s Algorithm is proposed. The state feedback gain parameters are gain scheduled using ANFIS Logic Control to provide the appropriate values for the different regions.

2. PROCESS DESCRIPTION

Chemical reactions in a reactor are either exothermic (release energy) or endothermic (require energy input) and therefore require that energy either be removed or added to the reactor for a constant temperature to be maintained.

Figure 1 shows the schematic of the CSTR process. In the CSTR process model under discussion, an irreversible exothermic reaction takes place. The heat of the reaction is removed by a coolant medium that flows through a jacket around the reactor. A fluid stream A is fed to the reactor. A catalyst is placed inside the reactor. The fluid inside the reactor is perfectly mixed and sent out through the exit valve. The jacket surrounding the reactor also has feed and exit streams. The jacket is assumed to be perfectly mixed and at a lower temperature than the reactor.

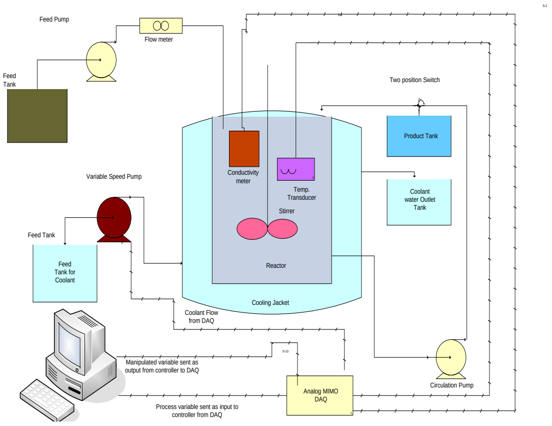

Figure 1. CSTR Process

The reactor is modeled using its material and energy balance equations.

$$\frac{dT}{dt}=\frac{F_i}{V}(T_i-T)+\frac{UA}{V\rho C_p}(T_c-T)-\frac{k_o\Delta H}{\rho C_p}e^{-\Delta E/RT}C \tag{1}$$

$$\frac{dC}{dt}=\frac{F_i}{V}(C_i-C)-k_oe^{-\Delta E/RT}C \tag{2}$$

$$\frac{dT_c}{dt}=F_c(T_{ci}-T_c)+\frac{UA}{\rho C_{pc}}(T_c-T) \tag{3}$$

where T and C are the temperature and concentration in the reactor respectively. F_i and T_i are the Input flow and temperature of the reactant, V the volume flowing rate of the inlet reactants of the CSTR process and F_c & T_{ci} are the coolant flow and temperature. The important process output are C, the concentration, T the temperature of the product and T_c coolant temperature. Table 1 gives the parameters of the CSTR.

Table 1. Parameters of CSTR

Parameter	Data
F/V	$1hr^{-1}$
k_0	$9{,}703*3600\ hr^{-1}$
(-ΔH)	5960kcal/kgmol
E	11,920 kcal/kgmol
ρCp	$500\ kcal/(m^3\ {}^oC)$
T_f	$25oC/298.5^oK$
CA_f	$10\ kgmol/m^3$
UA/V	$150\ kcal/(m^3\ {}^0K\ hr)$
T_c	$25oC/298.5^oK$

3. STEADY STATE CHARACTERISTICS

By simulating the differential equations, the steady state behaviour of the CSTR is obtained. The process outputs Effluent Temperature, Effluent Concentration and Coolant Temperature are plotted against the coolant flow.

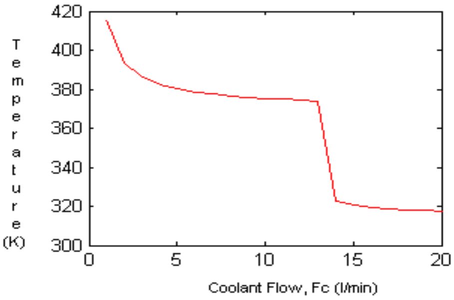

Figure 2. Coolant Flow versus Temperature

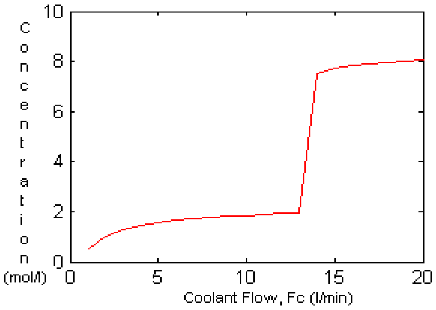

Figure 3. Coolant Flow versus Concentration

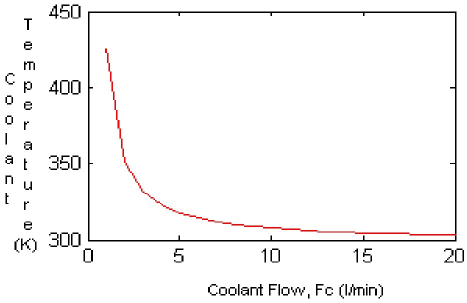

Figure 4. Coolant Flow versus Coolant Temperature

On analyzing the responses in the figures 2, 3 & 4, the curve signifies nonlinear characteristics and operates in three regions: low, middle and high region. The middle region has the property of output multiplicity. It is likely to operate the CSTR at the middle unstable steady state; as at very low temperature, steady states cause low yields as the temperature is very low and at high temperatures, steady state may be very high causing unsafe conditions, destroying the catalyst for a catalytic rector degrading the product.

4. LINEARISATION

On analyzing the equations, it is nonlinear due to the presence of the nonlinear term $e^{-E/RT}C$ in equation 1 & 2 and F_cT_c & F_cT_{ci} in equation 3. The nonlinear term has to be linearized around the operating point (C_0, T_0).

Applying Taylor’s series and linearising the nonlinear term $e^{-E/RT}C$

$$e^{-\Delta E/RT}C = e^{-\Delta E/RT_0}C_0 + \frac{E}{RT_0^{\ 2}}e^{-\Delta E/RT_0}C_0(T - T_0) + e^{-\Delta E/RT_0}(C - C_0) \quad (4)$$

Linearising the nonlinear terms F_cT_c & F_cT_{ci}

$$F_cT_c = F_cT_{c0} + F_{c0}T_c - F_{c0}T_{c0} \quad (5)$$

$$F_cT_{ci} = F_cT_{ci0} + F_{c0}T_{ci} - F_{c0}T_{ci0} \quad (6)$$

Applying the linearised equation, converting the equations 1, 2 and 3 into the deviation variable form,

$$\frac{dC'}{dt} = -\left[\frac{F_i}{V} + k_0e^{-\Delta E/RT_0}\right]C' - k_0\frac{E}{RT_0^{\ 2}}e^{-\Delta E/RT_0}C_0T' + \frac{F_i}{V}C_i' \quad (7)$$

$$\frac{dT'}{dt} = Jk_oe^{-\Delta E/RT_0}C' + \left[Jk_o\frac{E}{RT_0^{\ 2}}e^{-\Delta E/RT_0}C_0 - \frac{U_A}{V\rho C_p} - \frac{F_i}{V}\right]T' + \frac{U_A}{V\rho C_p}T_c' + \frac{F_i}{V}T_i' \quad (8)$$

$$\frac{dT_c'}{dt} = F_c'\left(T_{ci} - T_{co}\right) + \left[\frac{U_A}{\rho C_{pc}} - F_{co}\right]T_c' - \frac{U_A}{\rho C_{pc}}T' \quad (9)$$

where $J = \dfrac{-\Delta H}{\rho C_p}$

$C' = C - C_0$, $C_i' = C_i - C_{i0}$, $T' = T - T_0$, $T_i' = T_i - T_{i0}$,

$T_c' = T_c - T_{c0}$, $F_c' = F_c - F_{c0}$

where

C_0, C_{i0}, T_0, T_{i0}, T_{c0}, F_{c0} are the operating conditions

C', T', T_c' are the deviation state variables

C_i', T_i', F_c' are the deviation input variables

The normal feed concentration of A is 10 kgmol/m^3 and the concentration of A in the reactor will be in the range $0 < CA < 10$. The lower bound for temperature is 298 K, which occurs if there is no reaction at all, since the feed and jacket temperatures are 298K. There is a correlation between concentration and temperature. If the concentration of A is high, little reaction occurs, so little energy is released by reaction and therefore the temperature will not be much different than the feed and jacket temperatures.

CSTR is operated in the entire range and from the I/O characteristic, piecewise linearization of the nonlinearity is done.

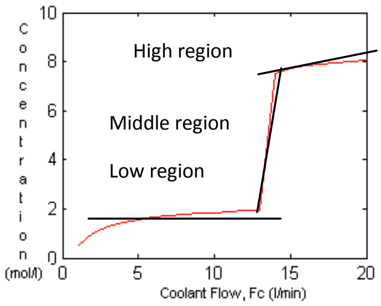

Figure 5. Input Output Characteristics –Three Linear regions

In order to apply linear control techniques, the nonlinear plant is divided into three linear regions. The State space matrices are found for each region

Table 2 shows the operating regions with operating points for the states and the input. The operating points are found for each linear region.

Table 2 Operating regions with operating points

Operating regions	Fc0	Ca0	T0	Tc0
Low Region	5	1.5635	380.25	317.55
Middle Region	14.25	4.8446	345.81	304.7
High Region	17	7.919	318.24	303.9

5. STABILITY ANALYSIS BY INVESTIGATION OF THE EIGEN VALUES

The stability characteristics are determined by the eigen values of **A**, which are obtained by solving det **(λI- A)** = 0.

$$|\lambda I - A| = \begin{bmatrix} \lambda - a_{11} & -a_{12} & -a_{13} \\ -a_{21} & \lambda - a_{22} & -a_{23} \\ -a_{31} & -a_{32} & \lambda - a_{33} \end{bmatrix} \quad (10)$$

The eigen values are tabulated in Table 3.

Table 3. Eigen values for the different regions of the CSTR

Operating Regions	Eigen Values
Low Region	$\begin{bmatrix} -1.7727+j1.0773 \\ -1.7727-j1.0773 \\ -4.7161 \end{bmatrix}$ -
Middle Region	$\begin{bmatrix} 0.6091 \\ -0.7836 \\ -13.9445 \end{bmatrix}$

High Region	$\begin{bmatrix}-0.8976\\-0.2142\\-16.6947\end{bmatrix}$

On investigation of the eigen values of the low operating region, all the three eigen values are negative, indicating that the point is stable. One of the eigen value is positive in the middle region, indicating that the point is unstable. The real part of the high region has negative eigen values, indicating that the point is stable.

6. PHASE PLANE TRAJECTORY OF THE CSTR

A phase-plane plot is constructed by performing simulations for a large number of initial conditions. The phase-plane plot generated for different coolant flow is shown in Figure 6. Three steady-state values are clearly shown; 2 are stable, that is the high and low temperature steady-states, while the middle one is unstable. The initial conditions of low concentration (0.4884 kgmol/m3) and relatively low-to-intermediate temperatures (315 to 365 K) all converge to the low temperature steady-state. When the initial temperature is increased above 365 K, convergence to the high temperature steady-state is achieved.

With a high concentration (8 kgmol/m3) and low temperature (315 to 325 K) initial conditions, the phase plane converges to the low temperature steady-state. When the initial temperature is increased above 325 K, phase portrait converges to the high temperature steady-state. When the initial temperature is increased around 340 K, a very high overshoot to above 425 K occurs, before the system settles down to the high temperature steady-state. No initial conditions converge to the intermediate temperature steady-state, since it is unstable.

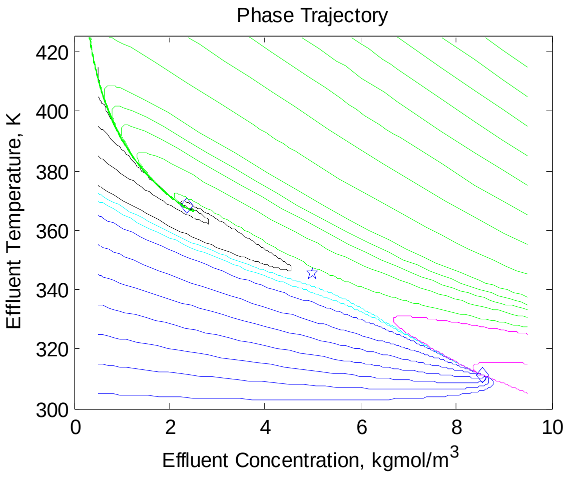

Figure 6. Phase Trajectory of the CSTR

On investigation of the Phase trajectory , the blue and the magenta trajectories reveals that the process output the converges to Low Temperature for the initial conditions as Low Concentration and Low to intermediate T as well as for High Concetration and Low Temperature. The black and the green trajectories shows that the convergence to high temperature is obtained for an initial condition of Low Concentration & High Temperature and High Concentration and Intermediate to High Temperature.

The CSTR process exhibits stable Node, Saddle point and stable Focus for the Low, Intermediate and High Temperature regions respectively.

7. STATE FEEDBACK CONTROL USING JOHNSON'S ALGORITHM

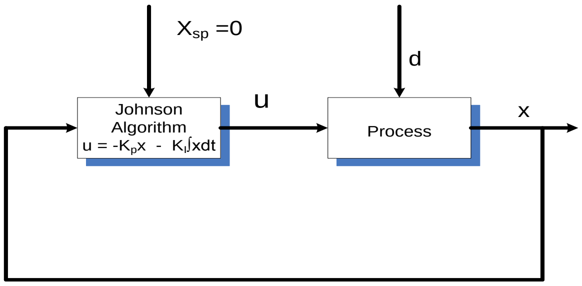

Figure 7 Block diagram of the Johnson Controller

The system in the presence of the unmeasured disturbances

$$dx = Ax + Bu + Wd \qquad \text{-----------(11)}$$

Where d is the unmeasured step disturbances.

Output y = Bu +Wd ----------- (12)

Then for piecewise constant disturbances,

$$y_e = Bu_e \qquad \text{----------- (13)}$$

With the definition of the vector, y, the original process is expressed as

$$dx = Ax + y \qquad \text{----------- (14)}$$

Combining equations 13 and 14

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{bmatrix} A & I \\ 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u_e \qquad \text{----------- (15)}$$

The control input generated is

$$u = -K_2 x - [K_1 - K_2 A] \int xdt \qquad \text{----------- (16)}$$

The optimal regulator for the linear system with unmeasurable constant disturbance is a state feedback proportional-integral controller. Figure 7 shows the block diagram of the Johnson's Controller. Figure 8 shows the Feedback Gain matrix convergence for the low region of the CSTR. Figure 9 shows the Feedback Gain Matrix convergence for the middle unstable region of the CSTR and figure 10 shows the Feedback gain matrix convergence for the high region of the CSTR.

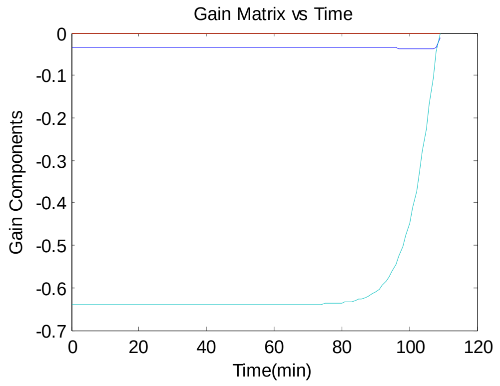

Figure 8 Feedback Gain Matrix for Low region

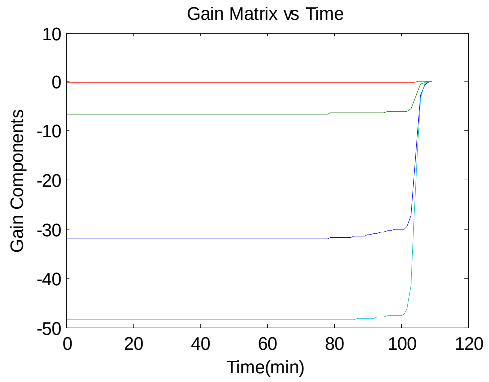

Figure 9 Feedback Gain Matrix for unstable middle region

Table 4 Gain Matrices by State Feedback Controller using Johnson Algorithm

	K_{ss}		
I Region	-0.6397	-0.079	-0.0050
II Region	-48.573	-10.10	-0.2077
III Region	-0.5710	-0.561	-0.0101

Individually for the state models of the low, middle and high region, the Gain matrix is computed. Table 4 shows the Gain matrices for the individual regions. Finally using ANFIS gain scheduler, overall process is controlled.

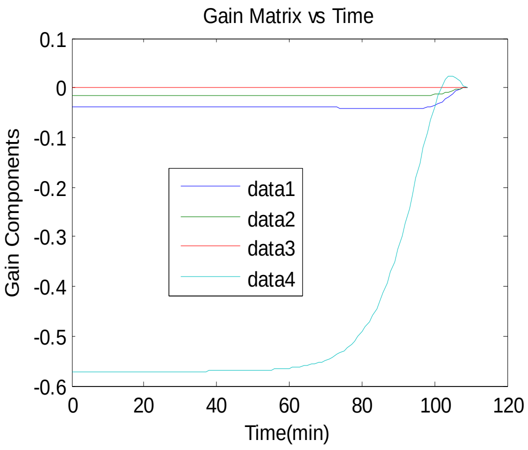

Figure 10 Feedback Gain Matrix for High region

8. ANFIS GAIN SCHEDULER FOR STATE FEEDBACK CONTROLLER USING POLE PLACEMENT TECHNIQUE FOR CSTR

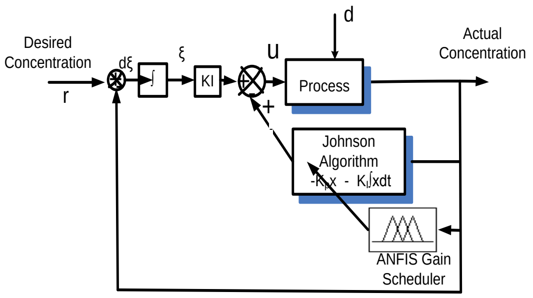

Figure 11. Block diagram for ANFIS gain scheduled CSTR with State Feedback using Johnson Algorithm

In gain-scheduling controllers, the parameters (i.e. gains) of the controller are varied usually as a function of the exogenous variable, here the effluent temperature in an attempt to compensate for the changes in the operating state of the plant through *stepwise* changes in the controller parameters. The gain scheduled controller output is based on multiple local linear controllers. To combine multiple local linear PID controller outputs a method is be devised to partition the operating state space. Figure 11 shows the block diagram of the ANFIS gain scheduled CSTR with State Feedback using Johnson Algorithm.

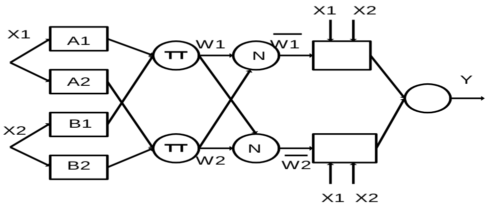

Figure 12. Architecture of ANFIS

An ANFIS model is used to provide the gain scheduling of the Gain generated by the State Feedback. Considering the Effluent Temperature as the input for the ANFIS model and k_I, k_1, k_2 & k_3 as the output, the ANFIS model is developed. Figure 12 shows the architecture of ANFIS.

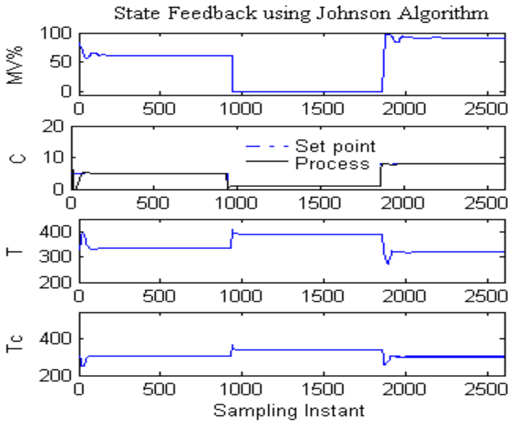

Figure 13 Servo control of the state feedback using Johnson Algorithm

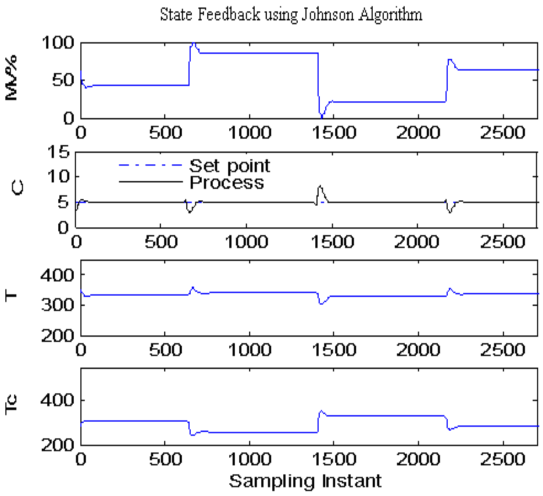

Figure 14. Regulatory control for varying feed of the state feedback controller using Johnson Algorithm

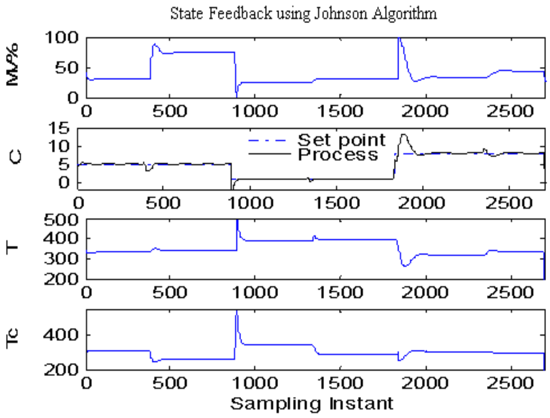

Figure 15 Servo-Regulatory control for varying feed of the state feedback controller using Johnson Algorithm

Figure 13 shows the servo control of the state feedback controller using Johnson Algorithm and figure 14 shows the regulatory control for varying feed of the state feedback controller using Johnson Algorithm. Figure 15 shows the servo-regulatory response of the state feedback controller using Johnson Algorithm. All the above controllers are augumented with the Integral action to assist in the servo control.

9. CONCLUSION

In this paper, the CSTR process is divided into three linear regions. Each linear region is analysed for stability by Eigen values and Phase Trajectory. The eigen values of the Low and High region have negative real parts specifying that the said region possess stable steady state whereas the Middle region has one positive eigen values specifying it as an unstable steady state. On analysing the phase portrait of the CSTR, it is evident that the CSTR process exhibits stable Node, Saddle point and stable Focus for the Low, Intermediate and High Temperature regions respectively. Here State Feedback controller is designed using Johnson’s Algorithm for each region. The ANFIS gain scheduler combines the multiple local linear State feedback gains. The proposed controller shows best performance for both set point tracking and regulatory conditions.

10. REFERENCE

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