Design of Fractional Order Controllers for First Order Plus Time Delay Systems

Rakhi Gopinath
Assistant Professor, EEE
Sree Narayana Guru Institute of science and Technology,
North Paravur
rg.rakhigopinath@gmail.com

ABSTRACT

In this paper, a fractional order proportional integral controller (FOPI) and Fractional order[Proportional Integral] (FO[PI]) controller is designed for controlling the level of a spherical tank which is modeled as a first order plus dead time system These controllers are designed based on the same set of design specifications which will satisfy the desired given gain cross over frequency and phase margin. The performance of the designed FOPI and FO [PI] controller is compared with the Conventional integer order Proportional Integral (IOPI) controller. The simulations are done in MATLAB simulink.

General Terms

Controller Design , Comparison of the Controllers , Fractional Calculus.

Keywords

Fractional order controller, Integer order controller , Spherical tank, First order plus delay time systems

1. INTRODUCTION

The control engineering deals with understanding the plant under operation, and obtaining a desired output response in presence of system constraints, So the development of better and simpler control algorithms is a continuing objective.PID (proportional integral derivative) controllers are the most popular controllers used in industry because of their simplicity, and the availability of many effective and simple tuning methods based on minimum plant model knowledge.

Clearly, for closed-loop control systems, there are four cases: (1) integer order (IO) plant with IO controller; (2) IO plant with fractional-order (FO) controller; (3) FO plant with IO controller, and (4) FO plant with FO controller. In control practice, the fractional-order controller is more common, because the plant model can be obtained as an integer-order model in the classical sense [3].From an engineering point of view, improving or optimizing performance is the major concern. Hence, our objective is to apply the fractional-order control (FOC) to enhance the (integer order) dynamic system control performance.

The primary concern for the controller design and tuning is to maintain the stability of the control system. Stability is the minimal requirement for the controller design. Where after, some specific controller need to be determined to meet the desired robustness and performance criteria by searching over the stabilizing controller set. Recently, several schemes have been proposed to analyze the stability region for the traditional integer order PID controllers, and also for the fractional order PID controllers [3]. Within the complete stabilizing set, it is important to design the proper controllers. to guarantee the robust requirement, and satisfy performance specifications, e.g., phase margin, gain margin, gain crossover frequency, etc.

For all first order plus time delay (FOPTD) systems, a controller can be designed to satisfy the given crossover frequency, phase margin and a flat phase criteria. This flat phase means that the system open loop phase is a constant around the given gain crossover frequency, which can show the iso-damping property for the system response. This scheme has been discussed in a previous work [4].In recent years there is remarkable increase in the number of studies related with the application of fractional controllers in many areas of science and engineering. This is due to a better understanding of the fractional calculus potentialities.

2. FRACTIONAL CALCULUS

Calculus is a branch of mathematics focused on limits, derivatives, integration etc. Conventional calculus deals with differentiation and integration of integer order. The generalization of conventional calculus to any order gave birth to fractional calculus. Since its birth the fractional calculus investigation was mostly mathematical for almost 300 years. There were notable contributions in the field of fractional calculus from Liebnitz (1695), Euler(1730), Lagrange(1772), Laplace(1812), Fourier(1822), Abel(1823), Liouville(1832), Rieman (1876), Oldham and Spanier (1974) etc.

A commonly used definition of fractional differo-integral is the Riemann-Liouville definition which is given by

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t)}{(t-\tau)^{\alpha-n+1}} d\tau $$

${}_aD_t^{\alpha} f(t)$ is a differintegral operator,which represents differeintegration operation of order α at time t. The function f(t) has history from time a to t. n is integer and α is not necessarily an integer.

Another commonly used definition is Grunwald Letnikov definition given of the form,

$$ {}_aD_t^{q} f(t) = \lim_{h\to 0} \frac{1}{h^q} \sum_{j=0}^{[(t-q)/h]} (-1)^j \begin{bmatrix} q \\ j \end{bmatrix} f(t-jh) $$

3. LEVEL SYSTEM

The control parameter which we have chosen is the level. Capacity sensor and level transmitter arrangement will sense the level from the process and converts into electrical signal. Then this electrical signal is fed to a current to voltage converter which in turn provides corresponding voltage signal and will be given to the computer (controller). The control system maintains water level in a storage tank. The control system performs this task by continuously monitoring the level in the tank and adjusting a supply valve to add more or less water to the tank. The desired level is maintained by an operator.

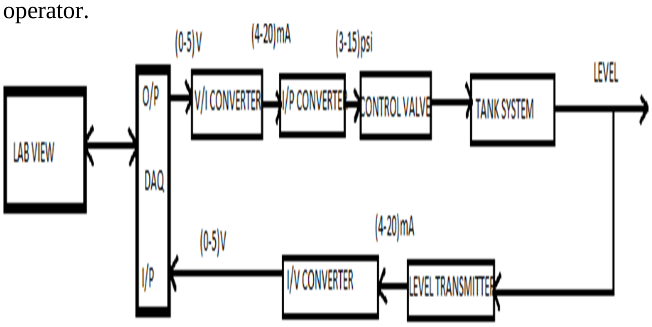


Figure 3.1 Closed loop level control system

The actual storage tank water level, sensed by the level transmitter, is then compared with a desired level to produce the required control action that will position the level control as needed to maintain the desired level. Now the controller decides the control action and it is given to the V/I converter and then to I/P converter. The final control element (pneumatic control valve) is now controlled by the resulting air pressure. This in turn control the inflow to the spherical tank and the level is maintained.

3.1 System Identification

The empirical method of identifying the system is the most modern method. Empirical models use data gathered from experiments to define the mathematical model of a system. A step change in the input to a process produces a response, which is called process reaction curve .A variety of empirical modelling methods exists. One method for developing models uses system identification methods. System identification methods use the input and output data to create difference equation which are used for representations that model the data.

In general terms, the time constant (τ), describes how fast the process variable (PV) moves in response to a change in the controlled output (CO). the time constant must be positive and it must have units of time. Most often it has units of minutes or seconds. Step test data implies that the process is in manual mode (open loop) and initially at steady state.

The transfer function models are required only for the simulation studies of the controller design. Here we are controlling the level (H) of the tank by manipulating the flow rate (Q). The most commonly used model to describe the dynamics of the industrial level process is general First Order plus Time Delay Process (FOPTD). And the FOPTD model structure is given in equation (3.1)

$$G(s) = \frac{K}{\tau s + 1} e^{-\theta ds}$$

Ɵd – Time delay
Kp – Process gain
τ - Time constant

Here the process of interest is approximated by a First Order plus Time Delay Process. The dead time approximation can be done in several methods; some methods are discussed below:

1. The simplest approximation method is taking the first two terms of Taylor series expansion of the Laplace transfer function of the dead time element.

$$e^{-\tau_d s} \cong 1 - \tau_d s$$

2. The pade approximation

$$e^{-\tau_d s} = \frac{e^{\frac{-\tau_d s}{2}}}{e^{\frac{\tau_d s}{2}}} \cong \frac{1 - \frac{\tau_d s}{2}}{1 + \frac{\tau_d s}{2}}$$

3. The crude approximation

$$e^{-\tau_d s} \cong \frac{1}{1 + \tau_d s}$$

Thus by approximating the dead time using any mentioned methods, we can conclude that a First Order plus Time Delay Process can be approximated as a higher order process. The transfer function parameters of the process are obtained by doing the step test. That is at a fixed inlet flow rate the system is allowed to reach the steady state. Various readings are noted by giving a step increment in the input flow rate. Steps to find transfer function are:

1. Note down the Initial Steady State (I.S.S) value of the process variable (PV).
2. Give a noticeable change in the input at time (t1) and the time delay (L).
3. Observe the change in the process variable and note down the New Steady State (N.S.S) value.
4. Find out the total change in PV,ΔPV=N.S.S – I.S.S
5. Compute the value (ΔPV*.632) + I.S.S
6. Note down the time (t2) corresponding to (ΔPV*0.632) + I.S.S value.
7. Then the time constant (τ) can be calculated as τ=t2-t1
8 The process gin (Kp) is ΔPV/ΔV, where ΔV is the change in input in Volts.

The transfer function parameter for the region (0 - 15)cm is experimentally obtained as

$$G(S) = \frac{0.93}{89.59s + 1} e^{-15s} \quad .$$

Table 1 Piecewise Transfer Function Models

Operating Range (cm)	Transfer Functions models
(0-15)cm	$G(S) = \frac{0.93}{89.59s + 1} e^{-15s}$
(15-25)cm	$G(S) = \frac{0.36}{206.54s + 1} e^{-20s}$

The spherical tank is modeled into piecewise transfer functions and the controllers (IOPI, FOPI AND FO[PI] are designed for each transfer function. The performance of the controllers can be compared.

4. CONTROLLER DESIGN

4.1 Design Specifications

According to the desired gain margin *Am* and phase margin *φm*, the designed fractional order PID controller should meet the stability robustness of the feedback control loop[3]. There are three interesting specifications to be met by the fractional order controller. From the basic definition of gain cross over frequency and phase margin we get the following:

I. Phase margin constraint,
Arg[G(jωc)]=Arg[C(jωc)P(jωc)]=-∏+φm

II. Gain crossover frequency constraint
|G(jωc)|db=Gc(jωc)P(jωc)|db=0

III. Robustness to loop gain variation constraint.

$$\frac{d(Arg[C(jwc)P(jwc)])}{dw}=0$$

With the condition that the phase derivative with respect to the frequency is zero i.e the phase Bode Plot is flat, at the gain cross over frequency, it means that the system is more robust to gain changes and the overshoots of the response are almost the same.

4.2 FOPI Controller Design

The tuning of the FOPI controller (λ, kp, ki) for the desired phase margin and gain cross over frequency can be found out. The open-loop transfer function of the system can be taken as

$$G(s)=C(s)P(s) \tag{4.1}$$

The closed-loop transfer function of the overall system can be expressed as

$$G(s)=\frac{\mathrm{C(s)P(s)}}{1+\mathrm{C(s)P(s)}} \tag{4.2}$$

The FOPI controller is of the form ,

$$\mathrm{C(s) = Kp+Ki/s^{\lambda}} \tag{4.3}$$

From [i] specification we get,

$$Arg[G2(jwc)]=-\arctan\frac{Kiwc^{-\lambda}\sin(\lambda\pi/2)}{1+Kiwc^{-\lambda}\cos(\lambda\pi/2)}-\arctan(wcT)-Lwc=-\pi+\phi m$$

Then, the equation between Ki and λ can be obtained as follows,

$$Ki=\frac{-\tan[\arctan(\omega_c T)+\phi_m+L\omega_c]}{M} \tag{4.4}$$

Where ,

$$M=\omega_c^{-\lambda}\sin(\lambda\prod/2)+\omega_c^{-\lambda}\cos(\lambda\prod/2)\tan[\arctan(\omega_c T)+\phi_m+L\omega_c]$$

From [iii] specification we get,

$$\frac{d(Arg[C(jwc)P(jwc)])}{dw}=0 \tag{4.5}$$

$$[-\arctan[\frac{Ki\omega^{-\lambda}\sin(\lambda\prod/2)}{1+Ki\omega^{-\lambda}\cos(\lambda\prod/2)}\tan[\arctan(\omega T)-L\omega]'_{\omega=\omega_c}=0$$

$$A_2Ki^2+[2w_c^{\lambda}\cos(\lambda\pi/2)A_2-\lambda w_c^{\lambda-1}\cos(\lambda\pi/2)]Ki+A_2w_c^{2\lambda}=0$$

$$B_2=2A\omega_c^{\lambda}\cos(\lambda\prod/2)-\lambda\omega_c^{-\lambda-1}\sin(\lambda\prod/2)$$

According to Specification (ii), we can establish an equation about Kp

$$Kp=\sqrt{\frac{1+(\omega_c T)^2}{(1+Ki\omega_c^{-\lambda}\cos(\lambda\prod/2))^2+(Ki\omega_c^{-\lambda}\sin(\lambda\prod/2))^2}}$$

On solving the equations we get the value of Ki for each λ . Using the values of Ki and λ, we will get the value of Kp which will satisfy the desired specifications.
So for the given λ, the ki value and kp value can be obtained.
Design procedure

- Given ωc, the gain crossover frequency;
- Given ϕm, the desired phase margin;
- With the pre-specified ϕm = 50 ωc=0.01rad/sec ,and fixed λ = 0.7,Kp = 1.3354, Ki = 0.1379, λ =0.7

4.3 FO[PI] Controller Design

The transfer function *G*(*s*) of the FO[PI] controller for the system is

$$G(s)=C(s)P(s) \tag{4.6}$$

According to the fractional order [PI] controller transfer function, we can get its frequency response as follows,

$$C(s)=(Kp+\frac{Ki}{jw})^{\lambda} \tag{4.7}$$

According to Specification (i), the phase of *G*3(*j*ω) can be expressed as,

$$\mathrm{Arg[G(j\omega c)]} = -\pi+\varphi\mathrm{m} \tag{4.8}$$

i.e $-\lambda\arctan\frac{Ki}{Kp\omega c}-\arctan(\omega cT)-L\omega c=-\pi+\varphi m$

According to Specification (iii) about the robustness to gain variations in the plant

$$\frac{d(\mathrm{Arg}(G3(j\omega)))}{d\omega}=0 \tag{4.9}$$

$$\frac{\lambda KiKp}{(Kp\omega_c)^2+Ki}=\frac{T}{1+(T\omega_c)^2}+L=\frac{T}{A_3}+L$$

$$A_3=1+(\omega_c T)^2$$

According to Specification (ii), we can establish an equation about *Kp*,

$$Ki=\omega_c\sqrt{\frac{(T+LA_3)\omega_c}{\lambda}}A_3^{(1/\lambda-1)}B_3 \tag{4.10}$$

$$Kp=\frac{Ki}{\omega_c B_3}$$

Where,

$$\mathrm{B3}=\tan[(\pi-\arctan(\omega_c\mathrm{T})-\phi\mathrm{m}-\mathrm{L}\omega_c)/\lambda].$$

On solving the equations we get the value of λ and Ki. Using the values of Ki and λ, we will get the value of Kp which will satisfy the desired specifications.
So for the given λ, the ki value and kp value can be obtained.
Design procedure

- Given ωc, the gain crossover frequency;
- Given ϕm, the desired phase margin;

- With the pre-specified value of ϕm = 50, ωc=0.01rad/sec ,and fixed λ = 0.7, the values can be obtained as Kp = 3.54, Ki = 0.29, λ =0.7

5. SIMULATION RESULTS

Ziegler-Nichols open loop Tuning is performed in order to get the controller parameters. A step signal is applied to get the open loop response . The Ziegler Nichols tuning was done for the Integer order controller parameters. The fractional order controllers were designed based upon the minimization of ISE , integral square error.

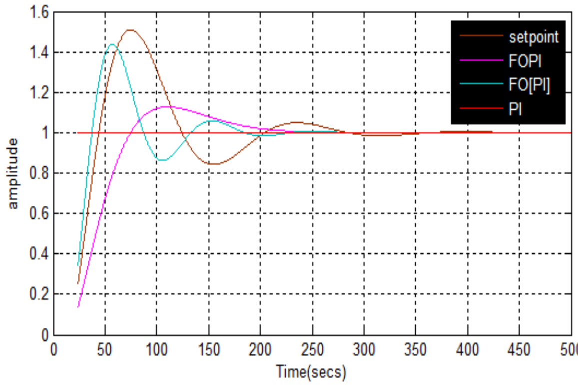
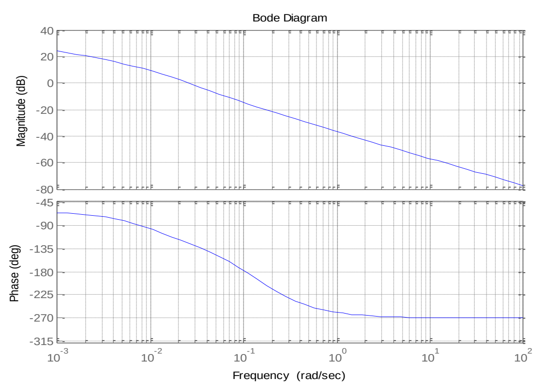


Figure 5.1 Response for the PI and FOPI controller for the region (0 -15)cm

Table 5.1. Performance values of the controllers for the range (0 – 15)cm

Controllers	Settling Time(secs)	Rise time(secs)	ISE
IOPI Controller	450	199	53.3
FO[PI] Controller	250	78	35.5
FOPI Controller	200	75	27

Figure 5.2 Bode Plot of C(s)P(s)



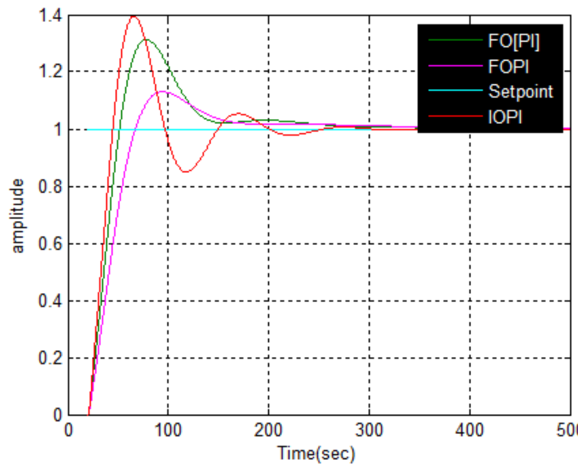


Figure 5.1 Response for the PI and FOPI controller for the region (15-30)cm

Table 5.2. Performance values of the controllers for the range (0 – 15)cm

Controllers	Settling Time(secs)	Rise time(secs)	ISE
IOPI Controller	425	75	35
FO[PI] Controller	300	65	33
FOPI Controller	220	40	32

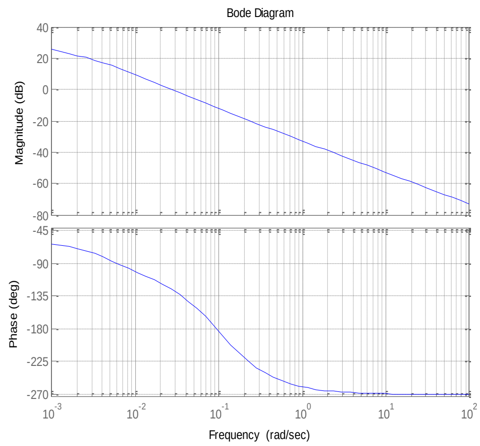


Figure 5.2 Bode Plot of C(s)P(s)

Here from the response it is inferred that , PID control has the faster response and the rise timet is less when compared to PI controller. But when Fractional order PI controller is used it has lesser overshoot and the response obtained was better compared to the ZN PI and ZN-PID controllers. The table 5.1 shows the performance values of the controllers.

6. CONCLUSION

The system identification of the spherical tank using emperical method was done and the piecewise transfer function was obtained in real time and hence the system was studied. The controllers parameters for an integer order system was obtained using Ziegler Nichols method and the simulation was done using Matlab Simulink The FOPI controller was implemented and the response obtained is compared with the ZN PI controller and FO[PI] controller.We can see that the overshoot of the unit step response using the FOPI controller is even much shorter than those using the FO[PI] and the IOPID controllers with the designed parameters. Thus, we can conclude that the FOPI controller outperforms the FO[PI] and the IOPID controllers.

7. ACKNOWELDGEMENT

I wish to express my sincere thanks to my Department Head, Prof. Rajasekaran, M.Tech, (Ph.D), and all other teaching faculty members and supporting Staff of our department for co-operating and arranging the necessary facilities.

I take immense pleasure in conveying my thanks and deep sense of gratitude to my guide Mr. A.Sanjeevi Gandhi,ME,(PhD), Assistant Professor, timely suggestions during all phases of this work

I would like to convey gratitude to my Parents whose prayers and blessing where always there with me throughout this work. Last but not the least I would like to thank my Friends and Others who directly or indirectly helped me in successful completion of this work.

8. REFERENCES

[1] J. G. Ziegler and N. B. Nichols, "Optimum setting for automatic controller," *Trans. ASME*,vol. 64, pp.759–768, 1942.

[2] K. Bettou., A. Charef, "Improvement of control performances using fractional PIλDμ controllers"

[3] Ying Luo, and YangQuan Chen, "Stabilizing and Robust FOPI Controller Synthesis for First Order Plus Time Delay Systems" IEEE Conference on Decision and Control and European Control Conference,2011

[4] Y.Q.Chen, D,Xue, and H,Dou, "Fractional Calculus and Biomimetic Control '', *IEEE Int. Conf. on robotics and biomimetics* (ROBIO 04), Shengang, China, Aug 2004.

[5] Chunna Zhao and Dingy¨u Xue, YangQuan Chen,"A Fractional Order PID Tuning Algorithm for A Class of Fractional Order Plants" Proceedings of the IEEE International Conference on Mechatronics & Automation Niagara Falls, Canada • July 2005.

[6] K Vaithiyanathan ,P.K. Bhaba," Design and Real Time Implementation of Fractional Order Proportional-Integral Controller (PI λ) in A Liquid Level System", Modern Applied Science Vol. 5, No. 6; December 2011.

[7] Pankaj Rai, Vineet Shekher "Determination of Stabilizing Parameter of Fractional Order PID Controller Using Genetic Algorithm" IJCEM International Journal of Computational Engineering & Management, Vol. 15 Issue 1, January 2012

[8]S. Nithya, N Sivakumaran," Model based controller design for a spherical tank process in real time", IJSSST Vol. 9 No. 4, November 2008

[9]Radek Matusu "Application of fractional order calculus to control theory",International journal of Maths and Modern applied sciences, Issue 7, Volume 5, 2011

[10]Ying Luo, Yang Quan Chen , Chun Yang Wang,"Tuning Fractional order Proportional Integral controllers for Fractional order systems".