Modelling of Geared DC Motor and Position Control using Sliding Mode Controller and Fuzzy Sliding Mode Controller

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ABSTRACT

Geared Dc motors can be controlled by many controllers which may include the conventional PID controllers and various other techniques. The sliding mode control is one of the good controllers which can be used for conditions like disturbances and parameter variations. The system responses are compared when PID controller is applied to the system. SMC methods yield nonlinear controllers which are robust against unmodeled dynamics and, internal and external perturbations. Also, due to system uncertainties that can lead to chattering phenomena in control law which can excite non-modeled dynamics and may damage the process, different approaches, like intelligent techniques, are used to minimize these effects. In this paper, the FL will be considered in the design of SMC. Also, a PID will be used in the outer loop in the control law then the gains of the sliding term and PID term are tuned on-line by a fuzzy system, so the chattering is eliminated and response of system is improved against external load here. Presented simulation results make sure that the above proposal and demonstrate the performance improvement to the example of DC motor. The results obtained with SMC with changed gains are compared with the traditional PID controller and fuzzy sliding mode controller. The advantages and limitations of each method are discussed.

General Terms
Position control

Keywords
PID controller, Sliding Mode Controller, Fuzzy Sliding Mode Controller, Geared DC Motor.

1. INTRODUCTION

DC geared motors are essentially a DC shunt motor which has been specially designed for low inertia, symmetrical rotation and smooth low-speed characteristics. Geared motor is a motor with a closed feedback system in which the position of the motor will be communicated back to the control circuit in the motors.

Geared motors are formed from four different elements: a DC motor, a position-sensing device (a potentiometer), a gear reducing part and a control unit. All of these components work together to make the motor to accept control signals that represent the desired output of the motor shaft and power the DC motor until its shaft is turned to the right position. The shaft in geared motors doesn't rotate as freely as those in regular DC motors; it is only able to rotate around 200 degrees in both directions. The position-sensing device in a geared motor determines the rotation of the shaft and thus the way the motor needs to turn in order to arrive at the desired position.

The sliding mode control is robust to plant uncertainties and insensitive to external disturbances. It is commonly used to get good dynamic performance of controllable systems. Even then, the chattering phenomena due to the finite speed of the switching devices can affect the system behavior significantly. Besides, the sliding control needs the knowledge of mathematical model of the system with bounded uncertainties [4].

Reduced chattering may be achieved without sacrificing robust performance by combining the attractive features of fuzzy control with SMC. FL is a potent tool for controlling ill-defined or parameter-variant plants. By generalizing fuzzy rules, a FL controller can cope well with severe modelling mistakes. Fuzzy logic with excellent expressions for tuning can avoid the heavy computational burden [5].

DC motors are generally controlled by conventional PID controllers, since they can be designed and implemented easily, have minimal cost, low cost maintenance and effectiveness. It is important to know system's mathematical model or to make some experiments for tuning PID parameters. However, it has been well known that conventional PID controllers generally do not work well for non-linear applications, and especially complex systems that have no precise mathematical models [6].

To overcome these difficulties, various types of modified conventional PID controllers such as auto-tuning and adaptive PID controllers were developed lately. Also FLC can be used for this type of problems. When matching with the conventional controller, the main advantage of FL is that no mathematical modeling is required [7].

In this paper the combined solution we have proposed and designed a robust and efficient controller. Here we have used a PID outer loop in the control law then the gains of the sliding term and PID term are tuned on-line by a fuzzy system.

Computer simulations are performed to show the validity of the proposed system. The results obtained with SMC with changed gains are compared with the traditional PID controller and fuzzy sliding mode controller. The advantages and limitations of each method are discussed.

2. MODELLING AND SYSTEM DESCRIPTION [1]

A geared DC motor is used in a control system where an appreciable amount of shaft power is needed. The DC motors can be armature-controlled having fixed field, or field-controlled with fixed armature current. DC motors found in instruments employ a fixed permanent-magnet field, and the control signal is given to the armature terminals.

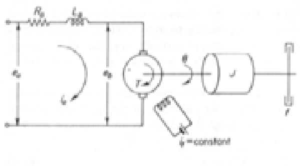

Fig: 1 Schematic diagram of an armature controlled geared DC motor

In order to model the DC motor shown in Fig.1, the parameters and variables are defined as follows.

Ra = armature-winding resistance (ohms)

La = armature-winding inductance (henrys)

ia = armature-winding current(amperes)

if = field current(amperes)

ea = applied armature voltage(volts)

eb = back emf(volts)

θ= angular displacement of the motor shaft (radians)

T = torque delivered by the motor (lb-ft)

J = moment of inertia of the motor and load referred to the motor shaft (slug-ft)

f = viscous-friction coefficient of the motor and load referred to the motor shaft (lb-ft/rad/sec)

The torque T is delivered by the motor is proportional to the product of the armature current ia and the air gap flux ψ, which in turn is directly proportional to the field current

$$\psi = K_f i_f$$

Where K_f is a constant. The torque can be written as

$$T = K_f i_f K_a i_a$$

Where K_a is also constant. Therefore, the torque is directly proportional to the armature current so that with a motor torque constant K,

$$T = Ki_a$$

The back emf is proportional to the angular velocity $\frac{d\theta}{dt}$ .Thus, with a back emf constant K_b, we have

$$e_b = K_b \frac{d\theta}{dt} \tag{2.1}$$

The speed of an armature controlled DC servo motor is controlled by the armature voltage ea, which is supplied by a power supply. The differential equation for the armature circuit is

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \tag{2.2}$$

The armature current creats the torque which is applied to the inertia and friction

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T = Ki_a \tag{2.3}$$

Assuming that the initial condition is zero, and taking the Laplace transforms of the above three differential equations, we get the following equations in the Laplace transform.

$$\begin{aligned} K_b\theta(s) &= E_b(s) \\ (L_a s + R_a)I_a(s) &= E_a(s) \\ (Js^2 + fs)\theta(s) &= T(s) = KI_a(s) \end{aligned} \tag{2.4}$$

Considering Ea(s) as the input, and θ(s) as the output, the block diagram can be constructed as shown in Fig. below from these three equations. The effect of the back emf was seen to be the feedback signal proportional to the speed of the motor. This back emf thus builds up the effective damping of the system.

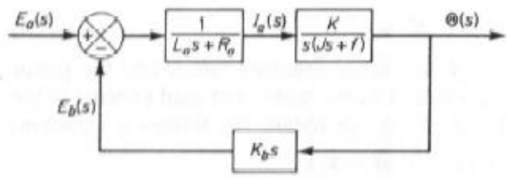

Fig: 2 block diagram of a geared DC motor

The transfer function of the system is obtained as follows

$$\frac{\theta(s)}{E_a(s)} = \frac{K}{s[(L_a f + R_a J)s + R_a f + KK_b]} \tag{2.5}$$

The inductance La in the armature circuit is usually small and maybe neglected. If La is neglected, the transfer function is reduced to,

$$\frac{\theta(s)}{E_a(s)} = \frac{K}{s[(L_a f + R_a J)s + R_a f + KK_b]} = \frac{K_m}{s(T_m s + 1)} \tag{2.6}$$

Where,

$$K_m = \frac{K}{R_a f + KK_b} \text{ = motor gain constant}$$

$$T_m = \frac{R_a J}{R_a f + KK_b} \text{ =motor time constant}$$

The following tests were conducted for obtaining motor parameters.

2.1 No Load Test

The connections are made as per the circuit diagram. Now vary the power supply and note down the armature current reading on the ammeter. Speed is also noted down using a tachometer. Now we can calculate angular velocity, back emf and power using formulas. Using these values, electrical power and P/ω is calculated. The rate of change of Eb with respect to ω is plotted on a graph, the slope of which is used to obtain Kb (back emf constant) Kt (motor torque constant). In no load test, the motor is not loaded.

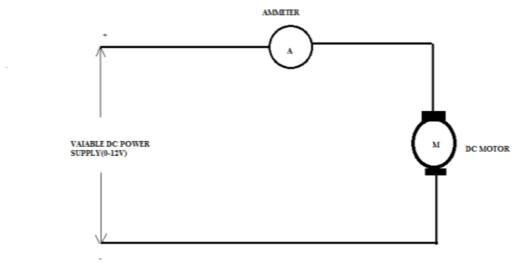

Fig: 3 Test Circuit

2.2 Retardation Test

Same circuit is used for retardation test also. The applied voltage is turned to maximum such that the machine will run at the maximum rated speed. Now, we have to disconnect the armature circuit. As the machine stops to a standstill position, the reading of rpm v/s time will be taken. Retardation test is used to find the change in angular velocity with respect to time. Retardation test readings are shown in table 2.

2.3 Determination of Armature Resistance and Armature Inductance

For determination of armature resistance (Ra), Wheatstone’s bridge can be used and for determination of Armature inductance (La), Hay’s bridge can be used. The use of bridge circuit will yield the accurate values of armature resistance (Ra), and armature inductance (La).

Table1. No Load Test

Sl No	V_a (V)	I_a (A)	N (rpm)	ω (rad/sec)	E_b (V)	E_bI_a(P) (VA)	P/ω(VA/rad/sec)
1	9	0.07	39	4.08	8.36	0.585	0.1434
2	10	.07	43	4.50	9.36	0.655	0.1456
3	11	0.08	63	6.59	10.2	0.821	0.1246
4	12	0.10	63	6.59	11.1	1.108	0.1681

Table 2. Retardation test

Sl No	Speed,N (rpm)	Time,T (sec)	Ang. Speed (rad/sec)
1	43	0	4.50
2	25	1	2.62
3	0	2	0

2.4 Responses Obtained From No Load Test and Retardation Test

Angular velocity ω can be plotted against time (T) to obtain the change in angular velocity with respect to time. The response is shown in figure. The back emf Eb v/s angular velocity ω graph is shown in figure. The rate of change of Eb with respect to ω is used to obtain Kb (back emf constant) and Kt (motor torque constant).

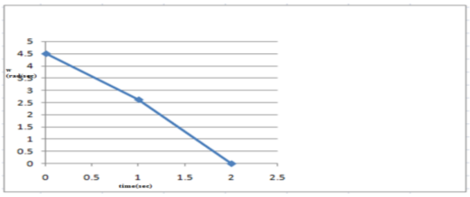

Figure 4 Angular velocity(ω) v/s time(T)

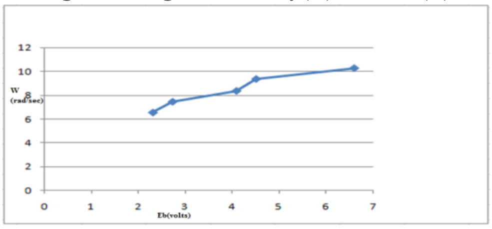

Figure 5angular velocity v/s back emf

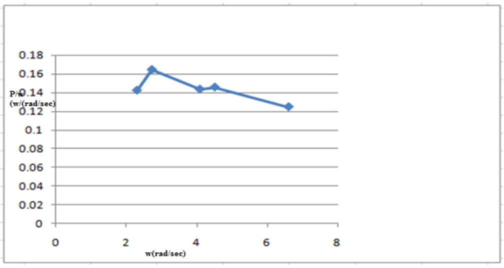

Figure 6 P/ω v/s ω

From the responses the DC motor parameters are calculated as:

Ra = 8.6Ω
La= 6.085mH
Electrical time constant = 0.7076mH/Ω
Back emf constant, Kb = Kt = 0.843V/(rad/sec)
A0 = 0.012 w/(rad/sec)
F0 = 0.01537Ns/m
J0 = A0+ F0ω/|dω/dt|
J0 =0 .03582N/m
Mechanical Time Constant Tm = J0/ F0
= 2.3305 sec

$$\frac{\theta(s)}{E_a(s)} = \frac{K_m}{s(T_m s+1)}$$

=0.9894/s (2.3305s+1)

3. SLIDING MODE CONTROLLER

In control theory, sliding mode control, or SMC, being a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior. It will slide along the boundaries of the controller structures. The movement of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding (hyper) surface.

4. CONTROLLER DESIGN

The design of SMC consists of two main steps. Firstly, one can select a sliding surface that models the desired closed loop performance. Secondly, a control law is designed such that the system state trajectory is forced toward the sliding surface. The system state trajectory in the period of time before reaching the sliding surface is called the reaching phase. The system dynamics in the reaching phase is still influenced by uncertainties. Ideally, the switching of the control should occur at infinitely high frequency to eliminate the deviation from sliding manifold. In practice, the switching frequency is not infinitely high due to the finite switching interval. Thus, undesirable chattering occurs in the control effort. Chattering is not preffered because it excites unmodeled high frequency plant dynamics and this can cause unforeseen instability. Different studies tried to solve this problem by combining fuzzy or neural controller with the sliding mode. For obtaining the actual controller design modeling of the plant is done along with the controller.

The plant consists of a DC motor with an inertial load. The DC motor is separately excited and armature controlled, which schematic diagram is shown in Figure 1. In this part a controller is used to control the motor-load angle speed. The system parameters are shown in appendix.

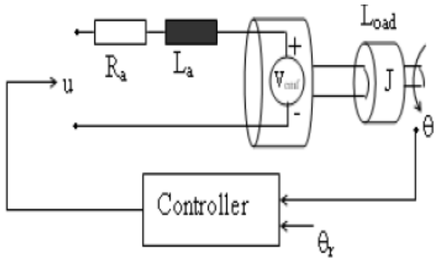

Figure 7 DC motor circuit diagram

The state equations that describe the DC geared motor behavior [2] are:

$$\frac{d^2\theta}{dt^2} = \frac{K_m}{J} i_a - \frac{K_f}{J}\frac{d\theta}{dt} - T_L \quad (1)$$

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a}\frac{d\theta}{dt} + \frac{V_a}{L_a} \quad (2)$$

Let

$$Va=u \quad (3)$$

Substituting (1) into (2) and using (3) one gets:

$$u\frac{K_m}{R_a J_m} = \frac{d^2\theta}{dt^2} + \left[\frac{BR_a + K^2{}_m}{R_a J_m}\right]\frac{d\theta}{dt} + \left[\frac{1}{J_m}\right]T_L$$

In state space matrix form, one gets:

$$\frac{dx}{dt} = Ax + Bu + Dv$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{BR_a + K^2 m}{R_a J_m} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{K_m}{R_a J_m} \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ -\frac{1}{J_m} \end{bmatrix}$$ with

$$x = [x_1 \quad x_2]^t, x_1 = \theta, x_2 = \frac{d\theta}{dt} = \omega, v = T_L$$

And

X1	rotor position
X2	rotor speed
u	control input
v	disturbance
t	transpose

In this section we will show first the design of the SM controller and later, based on the analysis and interpretation of the controller block diagram we compare it with the conventional PID controller.[3].

The sliding surface σ is defined as:

$$\sigma = e^{\cdot} + C.e = (\theta_r - \theta).C + \theta^{\cdot}_r - \omega$$

Where

The reference position

C A positive constant

From the second theorem of Lyapunov, the stability condition can be written as:

$$\frac{1}{2}\frac{d\sigma^2}{dt} = \sigma.\sigma^{\cdot} \leq -K|\sigma|$$

Where,

$$\frac{1}{2}\sigma^2 > 0$$

(Positive definite) is a Lyapunov function and K a positive constant. The control voltage command is calculated by substituting and in equation (8) as follows [3]:

$$u = \frac{R_a J_m}{K_m}\left[\theta^{\cdot\cdot}_r + C.\theta^{\cdot}_r + \left(\frac{BR_a + K^2{}_m}{R_a J_m} - C\right)\theta^{\cdot} + \frac{1}{J_m}T_L + K.sign(\sigma)\right]$$

A block diagram of this conventional SM controller is shown in Fig.8

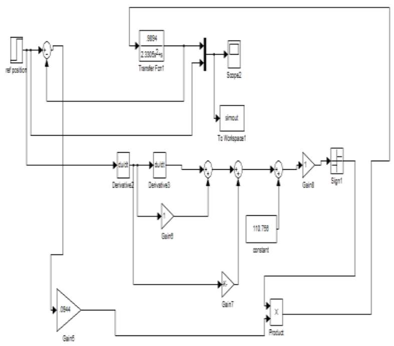

Figure 8 Block diagram of sliding mode controller

The problem with this conventional controller is that it has large chattering in the control output and the drive has large amount of noise. Furthermore, due to chattering, it is difficult to achieve small enough positioning error in steady state [3]. To reduce chattering the sign function (infinite gain) of the conventional SM is substituted with a finite gain K within a small boundary. This also affects controller's robustness, but then also the controller will remain robust enough if gain K is chosen large enough.

5. FUZZY SLIDING MODE CONTROLLER (FSMC) [6]

In recent years the fuzzy logic control technique has been used in many areas, using which the controller can be performed significantly as compared to conventional methods in presence of model uncertainties. As its name tells, the theory of fuzzy sets is basically a theory of graded concepts- a theory in which everything is a matter of degree. From last two decades since its inception, the theory matured in to a wide-ranging collection of concepts and techniques for dealing with complex phenomena that do not lend themselves to analysis by classical methods based on probability theory and bivalent logic. Sliding mode control being a discontinuous control, state trajectories move back and forth around a certain average surface in the state space and ripples come into being, which is called chattering. In order to eliminate the chattering phenomenon fuzzy logic is being introduced also; fuzzy logic systems do not need accurate mathematical models of the controlled system and hence, have been applied to many unknown nonlinear control problems.

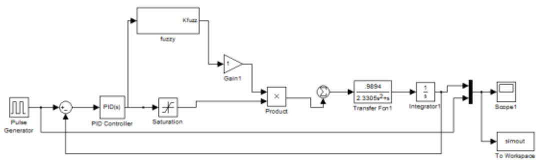

Figure 9 Simulink block diagram of FSMC for position control of geared DC motor

The inputs to fuzzy are error (e) and derivative of error ($e^{\cdot}$).The output is the fuzzy gain (k fuzz).The fuzzy controller consists of three stages: fuzzyfication, inference engine and defuzzyfication. A 5x5 rule base was defined - Table (3) - to develop the inference system. Both fuzzyfication and inference systems were tuned experimentally There are 25 rules used for the fuzzy inference which are listed in the below given table3.

Table 3. Rule Base for Fuzzy Sliding Mode Control

		change in error é				
		NB	NS	ZO	PS	PB
error,e	NB	uNB	uNB	uNS	uNS	uZO
	NS	uNB	uNS	uNS	uZO	uPS
	ZO	uNS	uNS	uZO	uPS	uPS
	PS	uNS	uZO	uPS	uPS	uPB
	PB	uZO	uPS	uPS	uPB	uPB

6. PID CONTROLLER [4]

PID controllers are dominant and popular and, have been widely used because one can obtain the desired system responses and it can control a wide class of systems. This may lead to the thought that the PID controllers give solutions to all requirements, but not to all [4]. Alternative tuning methods have been recently presented including disturbance reduction, magnitude optimum [5,6], pole placement and optimization methods [6,7].

In this work, the PID optimal tuning method used is found in [7]. In this method, the parameters of PID controller satisfying the constraints correspond to a given domain in a plane. The optimal controller lies on the curve. The design plot enables identification of the PID controller for desired robust conditions, and in particular, gives the PID controller for lowest sensitivity. By applying this method, trade-off among high frequency sensor disturbances, low frequency sensitivity, gain and phase margin constraints are also directly available.

The PID controller block diagram used for the closed loop tuning is shown in figure5.1

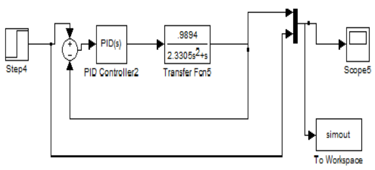

Figure 10 Block diagram of conventional PID controller

The transfer function of a PID controller is obtained as:

$$K(s) = K_p(1 + \tfrac{1}{T_I s} + T_D s)$$

Where K_p, $\frac{K_p}{T_I}$ and $K_P T_D$ represent the proportional, integral and derivative gains used for the controller respectively. Define $\omega_n = \frac{1}{\sqrt{T_I T_D}}$ and $\zeta = \frac{1}{2}\sqrt{\frac{T_I}{T_D}}$ as the controller's natural frequency and the damping coefficient, respectively. Then the PID transfer function can be written as:

$$K(s) = K_P \frac{\omega^2{}_n + 2\xi\omega_n s + s^2}{2\xi\omega_n s}$$

7. RESULTS AND DISCUSSION

The purpose of this part showing the validity and robustness of the proposed SMC approaches as applied to a DC geared motor system. SMC scheme is applied to control the position of the DC geared motor. Also, digital simulation is used to evaluate the model.

7.1Digital Simulation Results

After proper tuning, the output response of PID controller is obtained as below which shows the reduced rise time and minimized steady state error.

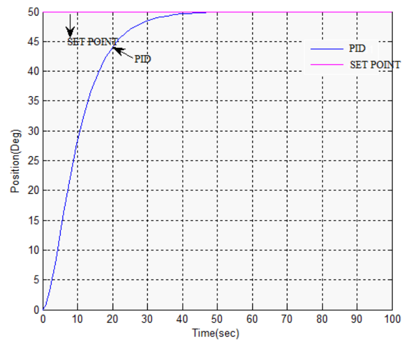

Figure11. Closed loop response of PID controller

As the response of the DC geared motor with PID controller did not respond well to the significant changes in the operating points, a SMC was designed in Simulink in MATLAB. The simulink model of the DC geared motor with SMC control is shown in Fig.5. The switching function is selected depending upon the order of the system. If N is the order then the order of switching function must me N-1. Since the DC motor's model is of second order, a first order switching function S is designed where,

S = iL - iref

Switching law or control law is selected to be as ,

u= 1/2(1-sign(S))

The response of the SMC is (shown in Figure 6.2) was compared to that of the PID controller. We can see that the settling time and rise time of SMC is less than that of the PID controller.

The conventional SMC can be simulated with the sliding surfaces and is shown in the below given figure 11.

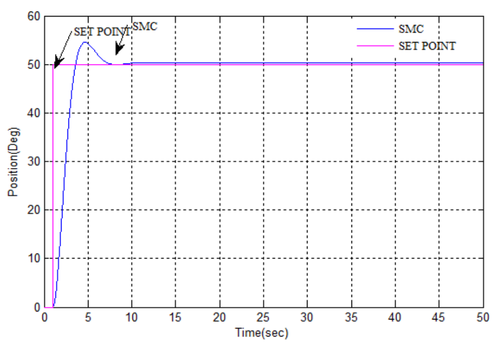

Figure12. Response for conventional SMC of DC geared motor

The response of the SMC is (shown in Figure 5) was compared to that of the PID controller response. We can see that the settling time and rise time of SMC is less than that of the PID controller.

Response of the DC geared motor for position control based on FSMC is shown below in figure (13).

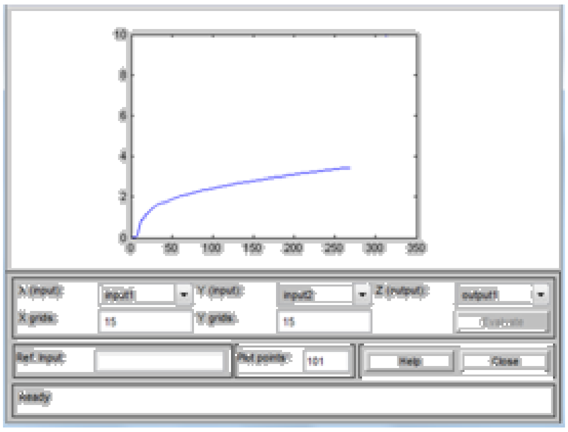

Figure 13 Response of geared DC motor for FSMC

From the response it is clear that the rise time and settling time is much lowered when compared to the PID and SMC controllers**.**

The responses of PID, SMC and FSMC are compared with respect to rise time and settling time and it is tabulated in table (3)below. It is observed that FSMC gives better performance when compared to PID and SMC controllers**.**

Table3Comparison of rise time and settling time for different controllers

Controller	Settling Time (sec)	Rise Time (sec)
PID	31.67	1.74
SMC	8.2	1.65
FSMC	2.2	0.5

From table 3 it is clear that the settling time and rise time of SMC is better than that of the PID controller**.**

8. CONCLUSION

In this paper, SMC with switched gains, fuzzy sliding mode controller and PID controllers have been considered for controlling the position of DC motor in geared system. A comparison method has been studied to show the relative advantages and limitations of each method. PID controllers are suitable if there is no disturbance in the system. However, the settling time is longer than when SMC is applied to the system. FSMC gives better responses when compared to PID and SMC in terms of rise time and settling time.

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APPENDIX

Table 4. Parameter of the DC motor

parameters	values
R_a	8.6Ω
L_a	6.085mH
K_m	0.9894Nm/W
K_v	0.843V/(rad/sec)
V_a	12 V
J_m	0.03582kg-m^2/rad