

# A Study on $(i,j)g^{*s}I$ Closed and Open Sets in Bitopological Spaces

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## ABSTRACT

An Ideal on a set  $X$  is a non empty collection of subsets of  $X$  with heredity property which is also closed under finite unions. In this paper,  $(i,j)g^{*s}$  closed and open sets are introduced with respect to an ideal in a bitopological space and their properties are investigated. Additionally, we compare them with other sets to show their relationships and characterize many other results.

### Keywords

$(i,j)g^{*s}$  closed sets,  $(i,j)g^{*s}I$  closed sets and  $(i,j)g^{*s}I$  open sets .

## 1. INTRODUCTION

J.C.Kelly[8] introduced the notion of bitopological spaces. Such spaces are equipped with two arbitrary topologies. Furthermore, Kelly extended some of the standard results of separation axioms in a topological space to a bitopological space. Fukutake [31] introduced the concepts of  $g$ -closed set in bitopological spaces and after that Sheik John and P.Sundaram[17] introduced  $g^*$ -closed sets in bitopological spaces.

Recently G.B.Navalgi[4] introduced a new class of  $g^{*s}$  closed sets and after that several authors turned towards generalization of various concepts of topology by considering bitopological spaces.

In this paper, the concept of  $(i,j)g^{*s}$  closed and open sets with respect to ideal bitopological spaces are introduced and their properties are discussed.

## 2. PRELIMINARIES

Throughout this paper  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  represents bitopological spaces on which separation axioms are assumed unless otherwise mentioned. If  $A$  is a subset of  $X$  with a topology  $\tau$ , then closure of  $A$  is denoted by  $\tau\text{-cl}(A)$  or  $\text{cl}(A)$  and the interior of  $A$  is denoted by  $\tau\text{-int}(A)$  or  $\text{int}(A)$ .

We recall the following definitions which are useful in the sequel.

**Definition 2.1.** An ideal  $I$  on a topological space[12]  $(X, \tau)$  is a non-empty collection of subsets of  $X$  which satisfies the following properties. (1)  $A \in I$  and  $B \subseteq A$  implies  $B \in I$ , (2)  $A \in I$  and  $B \in I$  implies  $A \cup B \in I$ .

An ideal topological space is a topological space  $(X, \tau)$  with an ideal  $I$  on  $X$  and is denoted by  $(X, \tau, I)$ . For a subset  $A \subseteq X$ ,  $A^*(I, \tau) = \{x \in X : A \cap U \notin I \text{ for every } U \in \tau(X, x)\}$  is called the local function of  $A$  with respect to  $I$  and  $\tau$  [12]. We simply write  $A^*$  in case there is no chance for confusion. A kuratowski closure operator  $\text{cl}^*(\cdot)$  for a topology

$\tau^*(I, \tau)$  called the  $\tau^*$ -topology, finer than  $\tau$  is defined by  $\text{cl}^*(A) = A \cup A^*$  [32].

**Definition 2.2:** A subset  $A$  of a topological space  $(X, \tau)$  is called

- 1) Generalized closed [20] (briefly  $g$ -closed) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- 2)  $\alpha$ -open [21] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ ,  $\alpha$ -closed [21] if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .
- 3) Semi-open [14] if  $A \subseteq \text{cl}(\text{int}(A))$ , semi-closed [14] if  $\text{int}(\text{cl}(A)) \subseteq A$
- 4) Semi-pre-open [3] if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ , semi-pre-closed [3] if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ .
- 5) Pre-closed [19] if  $\text{int}(\text{cl}(A)) \subseteq A$ , pre-open set [19]  $A \subseteq \text{int}(\text{cl}(A))$ .

**Definition 2.3:** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called

- 1)  $(i,j)$ - $g^*$ -closed [17] if  $\tau_j\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $\tau_i$
- 2)  $(i,j)$ - $g^s$ -closed [22] if  $\tau_j\text{-scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$
- 3)  $(i,j)$ - $\alpha g$ -closed [22] if  $\tau_j\text{-}\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$
- 4)  $(i,j)$ - $sg$ -closed [13] if  $\tau_j\text{-scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $\tau_i$
- 5)  $(i,j)$ - $rg$ -closed [9] if  $\tau_j\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular-open in  $\tau_i$
- 6)  $(i,j)g^{*s}$  closed [11] if  $\tau_j\text{-scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $rg$ -open in  $\tau_i$
- 7)  $(i,j)$ - $gsp$ -closed [5] if  $\tau_j\text{-spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$
- 8)  $(i,j)$ - $gp$ -closed [29] if  $\tau_j\text{-}\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $\tau_i$
- 9)  $(i,j)$ - $gpr$ -closed [21] if  $\tau_j\text{-pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular-open in  $\tau_i$
- 10)  $(i,j)$ - $\alpha gr$ -closed [20] if  $\tau_j\text{-pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular-open in  $\tau_i$
- 11)  $(i,j)$ -pre-semi-closed [25] if  $\tau_j\text{-spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $\tau_i$
- 12)  $(i,j)$ - $g\alpha$  closed [20] if  $\tau_j\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $\tau_i$
- 13)  $(i,j)g^{*\alpha s}$  closed [11] if  $\tau_j\text{-}\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $rg$ -open in  $\tau_i$

**Definition 2.4.** A subset  $A$  of an ideal topological space  $(X, \tau, I)$  is said to be

- 1)  $\alpha I$ -open [6], if  $A \subseteq \text{int}(\text{Cl}^*(\text{int}(A)))$ .
- 2)  $g\alpha - I$ -closed [16], if  $\alpha I \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $X$

- 3)  $\alpha g$ -I-closed [16], if  $\alpha I cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- 4)  $wg\alpha$ -I-closed [10], if  $\alpha I cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$ .
- 5)  $w\alpha g$ -I-closed [10], if  $\alpha I cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- 6)  $Ig$ -closed [15], if  $A^* \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

The complements of the above mentioned closed sets are called their respective open sets.

### 3. SOME RESULTS ON $(i, j)^1$ - $\mathcal{G}^{*s}$ CLOSED SETS

**Definition 3.1:** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $(i, j)$ - $\mathcal{G}^{*s}$ -closed set [11] if  $\tau_j$ - $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_j$ -rg-open in  $\tau_i$ .

**Proposition 3.2:**

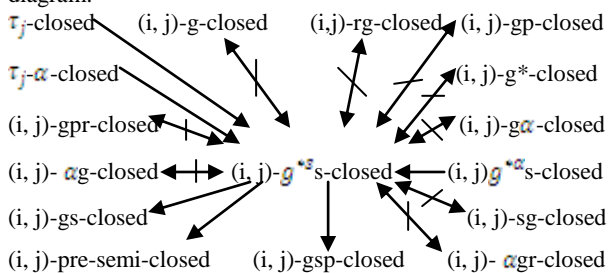
1. Every  $\tau_j$ -closed set is  $(i, j)$ - $\mathcal{G}^{*s}$ -closed.
2. Every  $\tau_j$ - $\alpha$ -closed set is  $(i, j)$ - $\mathcal{G}^{*s}$ -closed.
3. Every  $(i, j)$ - $\mathcal{G}^{*s}$ -closed set is  $(i, j)$ -gs-closed.
4. Every  $(i, j)$ - $\mathcal{G}^{*s}$ -closed set is  $(i, j)$ -gsp-closed.
5. Every  $(i, j)$ - $\mathcal{G}^{*s}$ -closed set is  $(i, j)$ -pre-semi-closed.
6. Every  $(i, j)$ - $\mathcal{G}^{*\alpha}$ -closed set is  $(i, j)$ - $\mathcal{G}^{*s}$ -closed.

The converse of the above theorems need not be true.

**Remark 3.3:** The concept of  $(i, j)$ - $\mathcal{G}^{*s}$ -closed sets is independent of the following classes of sets namely,  $(i, j)$ - $\mathcal{G}^*$ -closed,  $(i, j)$ -gp-closed,  $(i, j)$ -rg-closed,  $(i, j)$ -g-closed,  $(i, j)$ - $\alpha$ -g-closed,  $(i, j)$ -gpr-closed,  $(i, j)$ -sg-closed,  $(i, j)$ - $\alpha$ gr-closed and  $(i, j)$ - $\alpha$ g-closed.

For detailed proof and counter example refer [11].

All the above results can be represented by the following diagram.



**Proposition 3.4:** If  $A$  and  $B$  are  $(i, j)$ - $\mathcal{G}^{*s}$ -closed set then

$A \cup B$  is also  $(i, j)$ - $\mathcal{G}^{*s}$ -closed.

**Proof:** Let  $A$  and  $B$  be  $(i, j)$ - $\mathcal{G}^{*s}$ -closed. Let  $U$  be  $\tau_j$ -rg-open in  $\tau_i$  such that  $A \cup B \subseteq U$ . Then  $A \subseteq U$  and  $B \subseteq U$ . Hence  $\tau_j$ - $scl(A) \subseteq U$  and  $\tau_j$ - $scl(B) \subseteq U$ . Therefore  $\tau_j$ - $scl(A \cup B) \subseteq \tau_j$ - $scl(A) \cup \tau_j$ - $scl(B) \subseteq U$ . Hence  $A \cup B$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -closed.

**Proposition 3.5:** If  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -closed in  $(X, \tau_1, \tau_2)$  then  $\tau_j$ - $scl(A)$ - $A$  contains no non empty  $\tau_j$ -rg closed set.

**Proof:** Let  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -s closed set and  $F$  be a non-empty  $\tau_j$ -rg closed set, such that  $F \subseteq \tau_j$ - $scl(A)$ - $A$ . Then  $F \subseteq \tau_j$ - $scl(A) \cap A^c$ . Then  $F \subseteq \tau_j$ - $scl(A)$  and  $F \subseteq A^c \dots (1)$

Since  $F \subseteq A^c \Rightarrow A \subseteq F^c$ . Also  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -s closed,  $A \subseteq F^c \Rightarrow F^c$  is  $\tau_j$ -rg-open. Then  $\tau_j$ - $scl(A) \subseteq F^c$ . Then

$F \subseteq (\tau_j - scl(A))^c \dots (2)$ . From (1) and (2) we have  $F \subseteq \tau_j$ - $scl(A) \cap (\tau_j - scl(A))^c$ . Therefore  $F \subseteq \phi$ ,

which is a contradiction. Hence  $\tau_j$ - $scl(A)$ - $A$  contains no non-empty  $\tau_j$ -rg-closed set.

### 4. $(i, j)^1$ - $\mathcal{G}^{*s}$ I CLOSED SETS

**Definition 4.1:** Let  $(X, \tau_1, \tau_2)$  be a bitopological space and  $I$  be an Ideal on  $X$ . A subset  $A$  of  $X$  is said to be  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed set with respect to an ideal  $I$  if  $\tau_j$ - $scl(A) \setminus U \in I$ , whenever  $A \subseteq U$  and  $U$  is  $\tau_j$ -rg open in  $X$ , for  $i, j=1, 2$  and  $i \neq j$ .

**Theorem 4.2:** The intersection of two  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed sets is also an  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed set.

**Proof:** Let  $A$  and  $B$  be two  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed sets in  $(X, \tau_1, \tau_2, I)$ . If  $U$  is  $\tau_j$ -rg open in  $X$ , then  $A \subseteq U$  and  $B \subseteq U$ . Therefore  $A \cap B \subseteq U$ . Also  $\tau_j$ - $scl(A) \setminus U \in I$  and  $\tau_j$ - $scl(B) \setminus U \in I$  and hence  $\tau_j$ - $scl(A \cap B) \setminus U = \tau_j$ - $scl(A) \setminus U \cap \tau_j$ - $scl(B) \setminus U \in I$ . Thus  $A \cap B$  is also an  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed set.

**Remark 4.3:** The Union of two  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed sets need not be an  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed set, as shown by the following example.

**Example 4.4:** Let  $X = \{a, b, c\}$  with topologies  $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$  and  $\tau_2 = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and an ideal  $I = \{\phi, \{a\}\}$ . Then  $\{a\}$  and  $\{c\}$  are  $(1, 2)$ - $\mathcal{G}^{*s}$ I Closed but their union  $\{a, c\}$  is not  $(1, 2)$ - $\mathcal{G}^{*s}$ I Closed.

**Theorem 4.5:** Let  $A \subset Y \subset X$  and suppose that  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed in  $(X, \tau_1, \tau_2, I)$ , then  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed relative to the subspace  $Y$  of  $X$  and with respect to the Ideal  $I_Y = \{F \subset Y; F \in I\}$ .

**Proof:** Suppose  $A \subset U \cap Y$  and  $U$  is  $\tau_j$ -rg open in  $X$ , then  $A \subseteq U$ . Since  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -I Closed in  $(X, \tau_1, \tau_2, I)$  we have  $\tau_j$ - $scl(A) \setminus U \in I$ . Then  $(\tau_j$ - $scl(A) \cap Y) \setminus (U \cap Y) = (\tau_j$ - $scl(A) \setminus U) \cap Y \in I_Y$ , whenever  $A \subset U \cap Y$  and  $U$  is  $\tau_j$ -rg open. Hence  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed relative to the subspace  $(Y, \tau_1 \setminus Y, \tau_2 \setminus Y)$ .

**Theorem 4.6:** If  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed and  $A \subset B \subset \tau_j$ - $scl(A)$  in  $(X, \tau_1, \tau_2, I)$ , then  $B$  is  $(i, j)$ - $\mathcal{G}^{*s}$ -I Closed in  $(X, \tau_1, \tau_2, I)$ .

**Proof:** Let  $A$  be  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed and  $A \subset B \subset \tau_j$ - $scl(A)$  in  $(X, \tau_1, \tau_2, I)$ . Suppose  $B \subset U$  and  $U$  is  $\tau_j$ -rg open, then  $A \subset U$ , since  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed. We have  $\tau_j$ - $scl(A) \setminus U \in I$ . Now  $B \subset \tau_j$ - $scl(A)$ . Therefore  $\tau_j$ - $scl(B) \setminus U \subset \tau_j$ - $scl(A) \setminus U \in I$ . Therefore,  $B$  is also  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed in  $(X, \tau_1, \tau_2, I)$ .

**Theorem 4.7:** Let  $Y$  be a subspace of  $X$ . Then a set  $A$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed in  $Y$  if and only if it is equal to the intersection of a  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed set of  $X$  with  $Y$ .

**Proof:** Necessary: Let  $A = C \cap Y$ , where  $C$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I Closed in  $X$ . Then  $X \setminus C$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I open in  $X$ , so that  $(X \setminus C) \cap Y$  is  $(i, j)$ - $\mathcal{G}^{*s}$ I open in  $Y$  (By definition of subspace Topology). But

$(X \setminus C) \cap Y = Y \setminus A$ . Therefore,  $Y \setminus A$  is  $(i,j)g^{*s}I$  open in  $Y$ . Therefore  $A$  is  $(i,j)g^{*s}I$  Closed in  $Y$ .

Sufficient: Conversely assume that  $A$  is  $(i,j)g^{*s}I$  Closed in  $Y$ . Then  $Y \setminus A$  is  $(i,j)g^{*s}I$  open in  $Y$ , so that by definition, it equals the intersection of a  $(i,j)g^{*s}I$  Open set  $U$  of  $X$  with  $Y$ . Therefore,  $(X \setminus U)$  is  $(i,j)g^{*s}I$  Closed in  $X$  and  $A = Y \cap (X \setminus U) =$  intersection of a  $(i,j)g^{*s}I$  Closed set of  $X$  with  $Y$ .

**Theorem 4.8 :** A set  $A$  is  $(i,j)g^{*s}I$  Closed in  $(X, \tau_1, \tau_2, I)$  if and only if  $F \subset \tau_j - scl(A) \setminus A$  and  $F$  is  $\tau_i - rg$  closed in  $X$  implies  $F \in I$ .

Proof: Necessary: Assume that  $A$  is  $(i,j)g^{*s}I$  Closed. Let  $F \subset \tau_j - scl(A) \setminus A$  and suppose  $F$  is  $\tau_i - rg$  closed, then  $A \subset X \setminus F$ . By our assumption,  $\tau_j - scl(A) \setminus (X \setminus F) \in I$ . But  $F \subset \tau_j - scl(A) \setminus (X \setminus F)$  and hence  $F \in I$ .

Sufficient: Conversely, assume that  $F \subset \tau_j - scl(A) \setminus A$  and  $F$  is  $\tau_i - rg$  closed in  $X$ , implies  $F \in I$ .

Suppose  $A \subset U$  and  $U$  is  $\tau_i - rg$  open, then  $\tau_j - scl(A) \setminus U = \tau_j - scl(A) \cap (X \setminus U)$  is a  $\tau_i - rg$  closed set in  $X$  that is contained in  $\tau_j - scl(A) \setminus A$ . By assumption,  $\tau_j - scl(A) \setminus U \in I \Rightarrow A$  is  $(i,j)g^{*s}I$  Closed.

**Theorem 4.9:** Every  $\tau_j$ -closed set is  $(i,j)g^{*s}I$  Closed in  $X$ .

Proof: Let  $A$  be a  $\tau_j$ -closed set. Let  $U$  be  $\tau_i - rg$  open and  $A \subseteq U$ . Since  $A$  is  $\tau_j$ -closed,  $\tau_j - cl(A) = A$ . Then  $\tau_j - cl(A) \subseteq U$ ,  $\tau_j - scl(A) \subseteq \tau_j - cl(A) \subseteq U$ . Therefore  $\tau_j - scl(A) \setminus U = \emptyset \in I$ .  $\therefore A$  is  $(i,j)g^{*s}I$  Closed in  $X$ .

The converse is not true as seen by the following example:

**Example 4.10:** Let  $X = \{a, b, c\}$  with topologies  $\tau_1 = \{\emptyset, \{a\}, \{a, c\}, X\}$  and  $\tau_2 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$  and an ideal  $I = \{\emptyset, \{a\}\}$ . Then  $\{a\}$  and  $\{c\}$  are  $(1,2)g^{*s}I$  Closed but not  $\tau_2$ -closed.

**Theorem 4.11:** Every  $\tau_j$ - $\alpha$ -closed set is  $(i,j)g^{*s}I$  Closed in  $X$ .

Proof: Let  $A$  be a  $\tau_j$ - $\alpha$ -closed set. Let  $U$  be  $\tau_i - rg$  open and  $A \subseteq U$ . Since  $A$  is  $\tau_j$ - $\alpha$ -closed,  $\tau_j - \alpha - cl(A) = A$ . Then  $\tau_j - \alpha - cl(A) \subseteq U$ ,  $\tau_j - scl(A) \subseteq \tau_j - \alpha - cl(A) \subseteq U$ . Therefore  $\tau_j - scl(A) \setminus U \in I$ .  $\therefore A$  is  $(i,j)g^{*s}I$  Closed in  $X$ .

The converse is not true as seen by the following example:

**Example 4.12:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ ,  $\tau_2 = \{\emptyset, \{a\}, \{a, c\}, X\}$  and  $I = \{\emptyset, \{a\}\}$ . Then  $\{a, b\}$  and  $\{c\}$  are  $(1,2)g^{*s}I$  Closed but not  $\tau_2 - \alpha$ -closed.

**Remark 4.13:** Every  $(i,j)g^{*s}I$  Closed set is  $(i,j)g^{*s}I$  Closed but the converse is not true as seen from the following example:

**Example 4.14:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, \{a\}, \{a, c\}, X\}$ ,  $\tau_2 = \{\emptyset, \{a, b\}, X\}$  and  $I = \{\emptyset, \{a\}\}$ . Then  $\{b, c\}$  is  $(1,2)g^{*s}I$  Closed but not  $(1,2)g^{*s}I$  Closed.

**Theorem 4.15:** Let  $A$  be a  $(i,j)g^{*s}I$  Closed in  $(X, \tau_1, \tau_2, I)$  and  $F$  be a  $\tau_j$ -closed set in  $(X, \tau_1, \tau_2)$ , then  $A \cap F$  is a  $(i,j)g^{*s}I$  Closed in  $(X, \tau_1, \tau_2, I)$ .

Proof: Let  $A$  be a  $(i,j)g^{*s}I$  Closed in  $(X, \tau_1, \tau_2, I)$  and  $F$  be a  $\tau_j$ -closed set in  $(X, \tau_1, \tau_2)$ . By Theorem 4.11, every  $\tau_j$ -closed set,  $F$  is  $(i,j)g^{*s}I$  Closed in  $X$ . Also by Theorem 4.2 the intersection of two  $(i,j)g^{*s}I$  Closed sets is also an  $(i,j)g^{*s}I$  Closed set. Then  $A \cap F$  is a  $(i,j)g^{*s}I$  Closed in  $(X, \tau_1, \tau_2, I)$ .

## 5. (i,j)g<sup>\*s</sup>I OPEN SETS

**Definition 5.1:** Let  $(X, \tau_1, \tau_2)$  be a bitopological space and  $I$  be an ideal on  $X$ . A subset  $A \subset X$  is said to be  $(i,j)g^{*s}I$  Open in  $X$ , with respect to the ideal  $I$ , if  $X \setminus A$  is  $(i,j)g^{*s}I$  Closed.

**Theorem 5.2:** A set  $A$  is  $(i,j)g^{*s}I$  Open in  $(X, \tau_1, \tau_2, I)$  if and only if  $F \setminus U \subset \tau_j - sint(A)$ , for some  $U \in I$ , whenever  $F \subset A$  and  $F$  is  $\tau_i - rg$  closed.

Proof: Necessary: Let  $A$  be  $(i,j)g^{*s}I$  Open. Suppose  $F \subset A$  and  $F$  is  $\tau_i - rg$  closed. Then  $F$  is  $\tau_i - rg$  closed. We have,  $X \setminus A \subset X \setminus F$ . By assumption, let  $\tau_j - scl(X \setminus A) \subset (X \setminus F) \cup U$ , for some  $U \in I$ . Then  $(X \setminus [(X \setminus F) \cup U]) \subset X \setminus \tau_j - scl(X \setminus A)$ . Therefore  $F \setminus U \subset \tau_j - sint(A)$ .

Sufficient: Conversely, assume that  $F \subset A$  and  $F$  is  $\tau_i - rg$  closed and  $F \setminus U \subset \tau_j - sint(A)$ , for some  $U \in I$ . Consider a  $\tau_i - rg$  open set  $G$ , such that  $X \setminus A \subset G$ . Then  $X \setminus G \subset A$ . By assumption,  $(X \setminus G) \setminus U \subset \tau_j - sint(A) = X \setminus \tau_j - scl(X \setminus A)$ , for some  $U \in I$ . Therefore  $X \setminus (G \cup U) \subset X \setminus \tau_j - scl(X \setminus A)$ . Then  $\tau_j - scl(X \setminus A) \subset G \cup U$ , for some  $U \in I$ . Therefore  $\tau_j - scl(X \setminus A) \setminus G \in I$ . Hence  $X \setminus A$  is  $(i,j)g^{*s}I$  Closed. Therefore  $A$  is  $(i,j)g^{*s}I$  Open.

**Remark 5.3:** If  $A$  and  $B$  are  $(i,j)g^{*s}I$  open sets in  $(X, \tau_1, \tau_2, I)$ , then  $A \cap B$  need not be an  $(i,j)g^{*s}I$  open set, as shown by the following example.

**Example 5.4:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, \{a\}, \{a, c\}, X\}$ ,  $\tau_2 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$  and  $I = \{\emptyset, \{a\}\}$ . Then  $\{a, b\}$  and  $\{b, c\}$  are  $(1,2)g^{*s}I$  open but  $\{b\}$  is not  $(1,2)g^{*s}I$  open.

**Theorem 5.5:** If  $A \subset B \subset X$ ,  $A$  is  $(i,j)g^{*s}I$  open relative to  $B$  and  $B$  is  $(i,j)g^{*s}I$  open relative to  $X$ , then  $A$  is  $(i,j)g^{*s}I$  open relative to  $X$ .

Proof: Suppose  $A \subset B \subset X$  and let  $A$  be  $(i,j)g^{*s}I$  open relative to  $B$  and  $B$  be  $(i,j)g^{*s}I$  open relative to  $X$ . Suppose let  $F \subset A$  and  $F$  is  $\tau_i - rg$  closed. Since  $A$  is  $(i,j)g^{*s}I$  open relative to  $B$ , by theorem 5.2,  $F \setminus U_1 \subset \tau_j - sint_B(A)$ , for some  $U_1 \in I$ .  $\Rightarrow$  there exists an  $\tau_i - rg$  open set  $G_1$  such that  $F \setminus U_1 \subset G_1 \cap B \subset A_1$ , for some  $U_1 \in I$ . Since  $B$  is  $(i,j)g^{*s}I$  open relative to  $X$  and let  $F \subset B$ , then we have  $F \setminus U_2 \subset \tau_j - sint(B)$ , for some  $U_2 \in I$ .  $\Rightarrow$  there exists an  $\tau_i - rg$  open set  $G_2$  such that  $F \setminus U_2 \subset G_2 \subset B$ , for some  $U_2 \in I$ . Now  $F \setminus (U_1 \cup U_2) \subset (F \setminus U_1) \cap (F \setminus U_2) \subset A \cap B \subset A$ .  $\Rightarrow F \setminus (U_1 \cup U_2) \subset \tau_j - sint(A)$ , for some  $U_1 \cup U_2 \in I$ . Hence  $A$  is  $(i,j)g^{*s}I$  open relative to  $X$ .

**Theorem 5.6:** If  $\tau_j - sint(A) \subset B \subset A$  and if  $A$  is  $(i,j)g^{*s}I$  open in  $(X, \tau_1, \tau_2, I)$ , then  $B$  is  $(i,j)g^{*s}I$  open in  $(X, \tau_1, \tau_2, I)$ .

Proof: Suppose  $\tau_j - sint(A) \subset B \subset A$  and if  $A$  is  $(i,j)g^{*s}I$  open, then  $(X \setminus A) \subset (X \setminus B) \subset \tau_j - scl(X \setminus A)$  and  $(X \setminus A)$  is  $(i,j)g^{*s}I$  closed. Then by theorem 4.6,  $(X \setminus B)$  is  $(i,j)g^{*s}I$  closed and hence  $B$  is  $(i,j)g^{*s}I$  open.

**Theorem 5.7:** A set  $A$  is  $(i,j)g^{*s}I$  closed in  $(X, \tau_1, \tau_2, I)$  if and only if  $\tau_j - scl(A) \setminus A$  is  $(i,j)g^{*s}I$  open.

Proof: Necessary: Suppose  $F \subset \tau_j - scl(A) \setminus A$  and  $F$  is  $\tau_i - rg$  closed then by theorem 4.8,  $F \in I$ .

Therefore  $F \setminus U = \emptyset$  for some  $U \in I$ . Clearly,  $F \setminus U \subset \tau_j - scl(A) \setminus A$ . Then by theorem 5.2,  $\tau_j - scl(A) \setminus A$  is  $(i,j)g^{*s}I$  open.

Sufficient: Conversely, suppose  $A \subset G$  and  $G$  is  $\tau_j$ -closed in  $(X, \tau_1, \tau_2, I)$  then  $\tau_j - scl(A) \cap (X \setminus G) \subset \tau_j - scl(A) \cap (X \setminus A) = \tau_j - scl(A) \setminus A$ . By hypothesis,  $\tau_j - scl(A) \cap (X \setminus G) \setminus U \subset \tau_j - sint(\tau_j - scl(A) \setminus A) = \emptyset$ , for some  $U \in I$ . Then  $\tau_j - scl(A) \cap (X \setminus G) \subset U \in I$  and hence  $\tau_j - scl(A) \setminus G \in I$ . Then  $A$  is  $(i,j)g^{*s}I$  closed.

## 6. CONCLUSION

In this paper  $(i,j)g^{*s}$  closed and open sets with respect to an ideal in a bitopological space are defined and some of their properties are investigated. Additionally, the inclusion relations of these sets are compared with many other existing sets in the literature and they are depicted in the diagram for quick reference. Also we have characterized many other results on it.

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