Supply Chain Inventory Model for Time Dependent Deteriorating Items, Variable Holding Cost and Trade Credit

N.Rajeswari[1]
Department of Mathematics
Sri GVG Visalakshi College for women

K.Indrani[2]
Department of Mathematic
Nirmala College for women

ABSTRACT
The objective of this paper is to formulate and determine the optimal replenishment policy for a retailer's EOQ model with time dependent deterioration and variable holding cost. Here two levels of trade credit policies in which the supplier offers the retailer a permissible delay period and the retailer in turn provides a maximal trade credit period to their customers in a supply chain system are considered. Some results have been developed to determine the optimal ordering policies for the retailer. These results help the retailer to take appropriate inventory decisions. A numerical example is used to highlight the application of the EOQ model. Sensitivity analysis of the optimal solution with respect to major parameters is carried out.

Keywords
Time dependent deteriorating items, EOQ, variable holding cost, Trade credit, Permissible delay in payments, Supply chain.

INTRODUCTION

Supply Chain Management (SCM) is now at the centre stage of manufacturing and service organizations. A simple supply chain system can be assumed to have a supplier, retailer and a customer. Trade credit plays an important role in the supply chain system. In real life situations the supplier allows a certain fixed period to the retailer for settling the amount that the supplier owes to the retailer to repay the amount for the items supplied. The retailer can sell the goods and earn interest on the revenue obtained before the end of the trade credit period. This permissible delay policy given by the supplier to the retailer is to promote the sale of these goods and paying later also indirectly reduces the cost of holding stock.

Goyal[1] introduced the concept of permissible delay in payments. Goyal further assumed that the supplier would offer the retailer delay period whereas the retailer doesn't offer such facility to the customers. This situation defines one level of trade credit period. Chand and Ward [2] proposed a different model based on Goyal's problem. The supply chain consisting of the supplier, retailer and customers is considered as a situation of two levels of trade credit. Yang and Wee [3,4] conducted research on the inventory policy for deteriorating item in the supply chain including a single-vendor and multi-buyers.

Compared to the deteriorating items inventory study in a single enterprise, the inventory studies in the supply chain emphasize how to maintain the stability of the whole system while achieving the minimum of inventory costs in the supply chain. Therefore, researchers focused on supply chain coordination mechanism in the deteriorating items inventory problem study. Haung [5] extended one level trade credit to an EOQ model with two levels of trade credit to develop a retailer's replenishment policy in the supply chain.

Mahata and Mahata [6] developed an EOQ model for deteriorating items with two levels of trade credit financing. Hou and Lin [7] derived an optimal policy for a situation in which deterioration and demand rate are constant. Apart from deterioration the holding cost also plays a significant role in EOQ policies. The main purpose of this paper is to extend Jui-Jung Liao and Kun-Jen Chung [8] incorporating variable holding cost under two levels of trade credit .This study thus develops an inventory model for time dependent deteriorating items, variable holding cost and permissible delay in payments. Three cases are discussed Case I: $T \geq M$. Case II: $N \leq T < M$. Case III: $T \leq N$. The optimal cycle time and order quantity are obtained. Finally numerical example with sensitivity analysis is used to validate the model

2 MODEL FORMULATION

Following notations are used in the paper:

2.1 Notations

A: ordering cost per order
s: unit selling price per item
c: unit purchasing price per item.
D: demand rate per year
Ie: interest earned per $ per year
Ik: interest charged per $ in stock per year by the supplier
M: the retailer's trade credit period offered by the supplier in years
N: the maximal trade credit period for customers offered by retailer in years
$h(t)$: unit stock holding cost per unit per year
T: the cycle time in years
$C(T)$: the annual total relevant cost
T^*: the optimal cycle time

In addition, the following assumptions are used throughout:

2.2 Assumptions

1. Demand rate is known and constant.
2. The shortages are not allowed.
3. Holding cost is a linear function of time $h(t)=h+\alpha t$, $h \geq 0$, $\alpha \geq 0$
4. Time period is infinite and replenishment lead time is zero.
5. The distribution of time to deterioration of the items follows exponential distribution with parameter θt, $0 \leq t \leq 1$.
6. $Ie \leq Ik$, $M \geq N$ and $s \geq c$.
7. The supplier allows a fixed period, M, to settle the account. During this fixed period no interest is changed by the supplier but beyond this period, interest $I_k$is charged by the supplier under the terms and conditions agreed upon. The account is settled completely either at the end of the credit period or at the end of the cycle.

8.The retailer in turn allows a maximal trade credit period N for customers to settle the account. If a customer buys one item from the retailer at time t belonging to $(0,N]$, then the customer will have a trade credit period $N- t$ and makes the payment at time$N$.

Furthermore, the retailer can accumulate revenue and earninterest after the customer pays for the amount of purchasing cost until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

3 MATHEMATICAL FORMULATIONS

The depletion of inventory occurs due to combined effect of the demand and deterioration in the time interval [0,T]. The differential equation governing the instantaneous state of inventory $I(t)$ at some instant of time t is given by

$$\frac{dI(t)}{dt}+\theta t I(t)=-D,\ 0\le t\le T \tag{1}$$

with the boundary condition $I(0) = Q$ and $I(T) = 0$. The solution of (1) is given by

$$I(t)=D\left[\left(1-\frac{\theta t^2}{2}\right)(T-t)+\frac{\theta}{3}\left(T^3-t^3\right)\right],\quad 0\le t\le T \tag{2}$$

and the order quantity is

$$Q=I(0)=D\left(T+\frac{\theta T^3}{6}\right) \tag{3}$$

The annual total relevant cost consists of the following elements:

1. Annual ordering cost $\frac{A}{T}$ (4)
2. Annual inventory holding cost (excluding interest charges) $$\frac{1}{T}\int_0^T (h+\alpha t)I(t)dt=hD\left(\frac{T}{2}+\frac{\theta T^3}{12}\right)+\alpha D\left(\frac{T^2}{6}+\frac{\theta T^4}{40}\right)$$ (5)
3. Annual cost of deteriorated units $\frac{c(Q-DT)}{T}=\frac{cD\theta T^2}{6}$ (6)
4. Regarding interests charged and earned, we have the following three cases for discussion:

Case(I) $T\geq M$

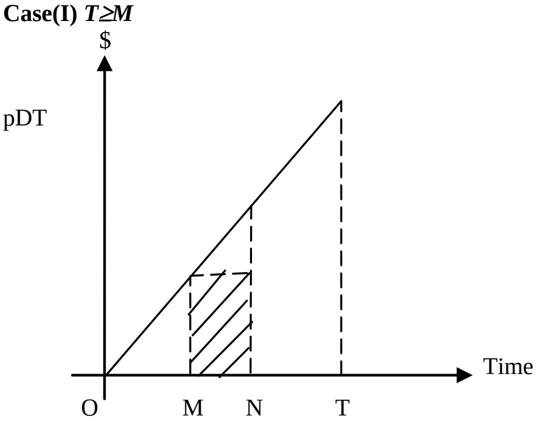

Figure 1: The total accumulation of interest earned when $M \le T$

In this case, the sales revenue is utilized to earn interest $I_e$during the period (N,M). When the account is settled, the item in the inventory has to be financed with annual rate I_k.

Therefore, the annual interest payable is

$$\frac{cI_k\int_M^T I(t)dt}{T}=\frac{cI_kD}{T}\left[\frac{T^2}{2}+\frac{\theta T^4}{12}-TM+\frac{M^2}{2}+\frac{\theta TM^3}{6}-\frac{\theta MT^3}{6}-\frac{\theta T^4}{12}\right] \tag{7}$$

From Figure 1, it implies that the retailer sells products and deposits the revenue into an account during period (0, N], but receives the money at time N. Therefore, sales revenue, sDN, is continuously accumulated in the period (N,M) and the interest earned from this is sI_e, multiplied by the area of $NMYZ$. In addition, the sales revenue from period (N, M) is continuously accumulated, so the interest earned thereby is sI_e multiplied by the area of XYZ. Combining the above argument, the annual interest earned is

$$\frac{sI_e\int_N^M Dtdt}{T}=\frac{sI_eD}{2T}\left[M^2-N^2\right] \tag{8}$$

Case(II) $N \le T < M$

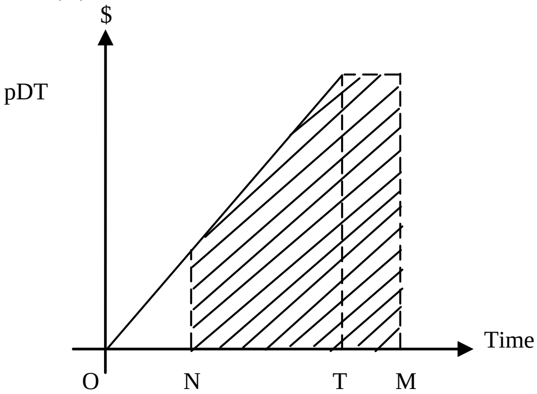

Figure 2: The total accumulation of interest earned when $N\le T \le M$

In this case, all the sales revenue is utilized to earn interest with annual rate $I_e$during the period (N,M) and pays no interest for the items kept in stock. Therefore, the annual interest payable is 0, and the annual interest earned is

$$\frac{sI_e\left(\int_N^T Dtdt+DT(M-T)\right)}{T}=\frac{sI_eD}{2T}\left[2MT-N^2-T^2\right] \tag{9}$$

Case(III) $T \le N$

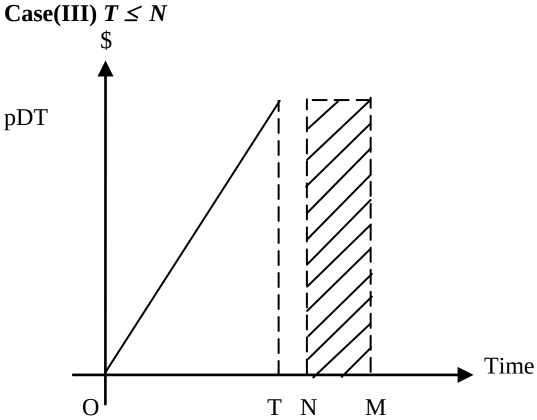

Figure 3: The total accumulation of interest earned when $T \le N$

In this case, all the sales revenue is utilized to earn interest $I_e$during the period (N,M) and pays no interest for the items kept in stock as well. Therefore, the annual interest payable is 0, and the annual interest earned is

$$\frac{sI_e\int_N^M DTdt}{T}=sI_eD\left[M-N\right] \tag{10}$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as,

$C(T)$ = ordering cost + procurement cost + inventory holding cost + interest payable – interest earned.

.

$$C(T)=\begin{cases} C_1(T);\ if\ M\le T \\ C_2(T);\ \ if\ N\le T\le M \\ C_3(T);\ \ if\ T\le N \end{cases}$$

Where

$$C_1(T)=\frac{A}{T}+\frac{cD\theta T^2}{6}+hD\left(\frac{T}{2}+\frac{\theta T^3}{12}\right)+\alpha D\left(\frac{T^2}{6}+\frac{\theta T^4}{40}\right)$$
$$+\frac{cI_kD}{T}\left[\frac{T^2}{2}+\frac{\theta T^4}{12}-TM+\frac{M^2}{2}+\frac{\theta TM^3}{6}-\frac{\theta MT^3}{6}-\frac{\theta M^4}{12}\right]$$
$$-\frac{sI_eD}{2T}\left[M^2-N^2\right]$$

(12)

$$C_2(T)=\frac{A}{T}+\frac{cD\theta T^2}{6}+hD\left(\frac{T}{2}+\frac{\theta T^3}{12}\right)+\alpha D\left(\frac{T^2}{6}+\frac{\theta T^4}{40}\right)-\frac{sI_eD}{2T}\left[2MT-N^2-T^2\right]$$

(13)

and

$$C_3(T)=\frac{A}{T}+\frac{cD\theta T^2}{6}+hD\left(\frac{T}{2}+\frac{\theta T^3}{12}\right)+\alpha D\left(\frac{T^2}{6}+\frac{\theta T^4}{40}\right)-sI_eD\left[M-N\right]$$

(14)

For convenience of discussion, we extend the domain of $C_i(T)$ (i= 1, 2, 3) to (0,∞). Then, equation (12) yields

$$C_1^{'}(T)=\frac{1}{T^2}\left\{-A+\frac{cD\theta T^3}{3}+hD\left(\frac{T^2}{2}+\frac{\theta T^4}{4}\right)+\alpha D\left(\frac{T^3}{3}+\frac{\theta T^5}{10}\right)\right.$$
$$\left.+cI_kD\left[\frac{T^2}{2}-\frac{M^2}{2}+\frac{\theta M^4}{12}-\frac{\theta MT^3}{3}+\frac{\theta T^4}{4}\right]+\frac{sI_eD}{2}\left[M^2-N^2\right]\right\}$$
$$=\frac{1}{T^2}f(T)$$

(15)

Since

$$f(T)=-A+\frac{cD\theta T^3}{3}+hD\left(\frac{T^2}{2}+\frac{\theta T^4}{4}\right)+\alpha D\left(\frac{T^3}{3}+\frac{\theta T^5}{10}\right)$$
$$+cI_kD\left[\frac{T^2}{2}-\frac{M^2}{2}+\frac{\theta M^4}{12}-\frac{\theta MT^3}{3}+\frac{\theta T^4}{4}\right]+\frac{sI_eD}{2}\left[M^2-N^2\right]$$

(16)

Since

$$f^{'}(T)=cD\theta T^2+hD(T+\theta T^3)+\alpha D\left(T^2+\frac{\theta T^4}{2}\right)+cI_kD(T-\theta MT^2+\theta T^3)>0$$

(17)

$f(T)$ is increasing on T >0 . Let T_1^* denote the root of $C_1^{'}(T)$ = 0. Since $\lim_{T\to\infty}f(T)=\infty>0$, the following results are obtained

(i) If $f(M)$ <0, then $T_1^* > M$.

(ii) If $f(M)\ge 0$, then $T_1^* = M$.

(18)

On the other hand, equations (13) and (14) yield

$$C_2^{'}(T)=\frac{-A}{T^2}+\frac{cD\theta T}{3}+hD\left(\frac{1}{2}+\frac{\theta T^2}{4}\right)+\alpha D\left(\frac{T}{3}+\frac{\theta T^3}{10}\right)+\frac{sI_eD}{2T^2}\left[T^2-N^2\right]$$

(19)

$$C_2^{''}(T)=\frac{2A}{T^3}+\frac{cD\theta}{3}+\frac{hD\theta T}{2}+\alpha D\left(\frac{1}{3}+\frac{3\theta T^2}{10}\right)+\frac{sI_eDN^2}{T^3}>0$$

(20)

$$C_3^{'}(T)=\frac{-A}{T^2}+\frac{cD\theta T}{3}+hD\left(\frac{1}{2}+\frac{\theta T^2}{4}\right)+\alpha D\left(\frac{T}{3}+\frac{\theta T^3}{10}\right)$$

(21)

$$C_3^{''}(T)=\frac{2A}{T^3}+\frac{cD\theta}{3}+\frac{hD\theta T}{2}+\alpha D\left(\frac{1}{3}+\frac{3\theta T^2}{10}\right)>0$$

(22)

Since $C'_1(M) = C'_2(M)$ and $C'_2(N) = C'_3(N)$, $C(T)$ is continuous and well defined on $T > 0$.

4 DECISION RULE OF THE OPTIMAL CYCLE TIME T^*

Consider the equations $C'_i(T) = 0$, (i = 1, 2, 3)

(23)

If the root of equation (23) exists, then it is unique. Let T^*_i (i = 1,2,3) denote the root of equation (23). Further, equations (15), (19) and (21) yield that

$C'_1(M) = C'_2(M)$

$$=\frac{-A}{M^2}+\frac{cD\theta M}{3}+hD\left(\frac{1}{2}+\frac{\theta M^2}{4}\right)+\alpha D\left(\frac{M}{3}+\frac{\theta M^3}{10}\right)+\frac{sI_eD}{2M^2}\left[M^2-N^2\right]$$

(24)

and $C'_2(N) = C'_3(N)$

$$=\frac{-A}{N^2}+\frac{cD\theta N}{3}+hD\left(\frac{1}{2}+\frac{\theta N^2}{4}\right)+\alpha D\left(\frac{N}{3}+\frac{\theta N^3}{10}\right)$$

(25)

$C_2(T)$ is convex on T >0 implies that $TV\ C'_2(M) > TV\ C'_2(N)$.

For convenience, let

$$\Delta_1=\frac{-A}{M^2}+\frac{cD\theta M}{3}+hD\left(\frac{1}{2}+\frac{\theta M^2}{4}\right)+\alpha D\left(\frac{M}{3}+\frac{\theta M^3}{10}\right)+\frac{sI_eD}{2T^2}\left[M^2-N^2\right]$$

(26)

and

$$\Delta_2=\frac{-A}{N^2}+\frac{cD\theta N}{3}+hD\left(\frac{1}{2}+\frac{\theta N^2}{4}\right)+\alpha D\left(\frac{N}{3}+\frac{\theta N^3}{10}\right)$$

(27)

Then $\Delta_1>\Delta_2$. Moreover, equations (26) and (27) yield

$\Delta_1<0$ if and only if $C'_1(M)$ <0 if and only if $T_1^* > M$.

$\Delta_1<0$ if and only if $C'_2(M)$ <0 if and only if $T_2^*> M$.

$\Delta_2<0$ if and only if $C'_2(N)$ <0 if and only if $T_2^* > N$.

$\Delta_2<0$ if and only if $C'_3(N)$ <0 if and only if $T_3^* > N$.

Furthermore, if $\Delta_1\ge 0$, then $C_1(T)$ is increasing on [M,∞).The above arguments lead to the following results.

Theorem 1

1. If $\Delta_1<0$, then $C(T^*) = C(T_1^*)$. Hence T^* is T_1^*.

2. If $\Delta_2>0$, then $C(T^*) = C(T_3^*)$. Hence T^* is T_3^*.

3. If $\Delta_1\ge 0$ and $\Delta_2<0$, then $C(T^*) = C(T_2^*)$. Hence T^* is T_2^*.

Proof:

1. If $\Delta_1<0$, then $\Delta_2<0$ implies that $T_1^* > M$, $T_2^* > M$, $T_2^*> N$ and $T_3^*> N$, respectively. Furthermore, $C(T)$ has the minimum value at $T = N$ when $T \le N$, $C(T)$ has the minimum value at $T = M$ when $N\le T \le M$ and $C(T)$ has the minimum value at $T = T_1^*$ when $T \ge M$. Since $C_3(N) = C_2(N) > C_2(M)$ and $C_2(M) = C_1(M) > C_1(T_1^*)$, $C(T)$ has the minimum value at T_1^* for T >0. Hence, we conclude that $C(T^*) = C(T_1^*)$. Consequently, $T^*= T_1^*$.

2. If $\Delta_2>0$, then $\Delta_1>0$ which implies that $T_2^* < M$, $T_2^* < N$, $T_3^* < N$ and $C_1(T)$ is increasing on [$M;1$). Furthermore, $C(T)$ has the minimum value at $T = T_3^*$ when $T \le N$, $C(T)$ has the minimum value at $T = N$ when $N \le T \le M$ and $C(T)$ has the minimum value at $T = M$ when $T \ge M$. Since $C_3(T_3^*) < C_3(N) = C_2(N) < C_2(M)$ and $C_2(M) = C_1(M)$, $C(T)$ has the minimum value at T_3^* for T >0. Hence, we conclude that $C(T^*) = C(T_3^*)$. Consequently, $T^*= T_3^*$.

3. If $\Delta_1\ge 0$ and $\Delta_2<0$ which implies that $T_1^* < M$, $T_2^* < M$, $T_2^* > N$ and $T_3^*> N$. Furthermore, $C(T)$ has the minimum value at $T = N$ when $T\le N$, $C(T)$ has the minimum value at $T = T_2^*$ when $N \le T \le M$ and $C(T)$ has the minimum value at $T = M$ when $T \ge M$.

Since $C_3(N) = C_2(N) > C_2(T_2^*)$ and $C_2(T_2^*) < C_2(M) = C_3(M)$. Hence, we conclude that $C(T^*) = C(T_2^*)$. Consequently, $T^* = T_2^*$.

5 NUMERICAL EXAMPLES

To illustrate the optimal policies of the above model, the following numerical examples are given.

Let $A = \$80$ / order, $M = 0.3$ year, $N = 0.25$ year, $c = \$50$ / unit, $s = \$60$ /unit, h=\$4/unit/year, $\alpha = 0.015$, I_c= \$0.20/\$/year, I_e= 0.15/\$/year and $\theta = 0.1$

Example 1: When $D = 900$ units/year then $\Delta_1>0$ and $\Delta_2> 0$.

Example 2: When $D = 430$ units/year then $\Delta_1>0$ and $\Delta_2< 0$.

Example 3: When $D = 150$ units/year then $\Delta_1<0$ and $\Delta_2< 0$.

For various values of θ(0.1 to 0.4), the optimal value of T and $C(T)$ have been computed. Computed results are displayed in

TABLE I: OPTIMAL SOLUTION OF EXAMPLES

Example1.			Case(I): $T\geq M$		
θ	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.1	> 0	> 0	0.195307	175.888046	385.082428
0.2	> 0	> 0	0.184140	165.913315	412.216553
0.3	> 0	> 0	0.175457	158.154373	436.596985
0.4	> 0	> 0	0.168391	151.838394	458.892090
Example2.			Case(II): $N\leq T\leq M$		
θ	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.1	> 0	< 0	0.230483	99.195435	374.279022
0.2	> 0	< 0	0.224548	96.717926	397.979645
0.3	> 0	< 0	0.219290	94.521423	420.425171
0.4	> 0	< 0	0.214578	92.551765	441.788788
Example3.			Case(III): $T\leq N$		
θ	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.1	< 0	< 0	0.439994	66.212051	298.181488
0.2	< 0	< 0	0.398322	60.064289	298.970520
0.3	< 0	< 0	0.370108	55.896431	304.323944
0.4	< 0	< 0	0.349049	52.782616	311.522644

For various values of α(0.005 to 0.020), the optimal value of T and $C(T)$ have been computed. Computed results are displayed in
Table 2.

TABLE 2 OPTIMAL SOLUTION OF EXAMPLES

Example 1 Case(I): $T\geq M$					
α	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.005	> 0	> 0	0.195332	175.910599	385.024902
0.010	> 0	> 0	0.195320	175.899765	385.053680
0.015	> 0	> 0	0.195307	175.888046	385.082428
0.020	> 0	> 0	0.195294	175.876328	385.111237
Example 2 Case(II): $N\leq T\leq M$					
α	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.005	> 0	< 0	0.230496	99.201042	374.229980
0.010	> 0	< 0	0.230490	99.198456	374.254089
0.015	> 0	< 0	0.230483	99.195435	374.279022
0.020	> 0	< 0	0.230477	99.192856	374.303131
Example 3 Case(III): $T\leq M\leq N$					
α	Δ_1	Δ_2	T^*	Q^*	$C(T^*)$
0.005	< 0	< 0	0.440097	66.227654	298.192444
0.010	< 0	< 0	0.440046	66.219925	298.187256
0.015	< 0	< 0	0.439994	66.212051	298.181488
0.020	< 0	< 0	0.439942	66.204178	298.175720

Following observations can be made from the table:

- Increase in the deterioration parameter decreases the optimal cycle time and optimal order quantity but increases the total annual cost.
- Increase in the parameter α has a marginal effect on the optimal cycle time, optimal order quantity and the total annual cost.

CONCLUSION

In this paper a simple two level supply chain is considered under the conditions of permissible delay in payments. The optimal policies have been discussed using some decision rules. Theorem 1 gives the solution procedure to find the optimal cycle time T*. Numerical examples are given to illustrate the use of the theorem. Sensitivity analysis is carried with reference to the deterioration parameter θ and the holding cost parameter α.

REFERENCES

[1] S.K. Goyal,1985, Economic order quantity under conditions of permissible delay in payment, Journal of the Operational Research Society, 36, 335–338.

[2] Chand.P and ward.J, 1987, A note on economic order quantity under conditions of permissible delay in payment, Journal of operational research society,38, 83–84.

[3] Yang.P.C and wee H.M 2001, A single-vendor and multi-buyers integrated inventory policy, journal of the Chinese institute of industrial engineering, 18, 22– 27.

[4] Yang.P.C and wee H.M 2005, A win-win strategy for an integrated vendor-buyer deteriorating inventory system, mathematical modeling and analysis, 541–546.

[5] Y.F. Huang, 2007, Optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy, European Journal of Operational Research, 176, 1577–1591.

[6] Mahata, G.C. and Mahata, P.2009, Optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain, International Journal of Mathematical, Physical and Engineering Sciences3 (1), 1–7.

[7] Hou, K.L. and Lin, L. C.,2009, A cash flow oriented EOQ model with deteriorating items under permissible delay in payments, Journal of Applied Sciences, 9(9), 1791–1794.

[8] Jui-Jung Liao and Kun-Jen Chung, 2009, An EOQ model for deterioration items under trade Credit policy in a supply chain system, Journal of the Operations Research Society of Japan 52(1), 46–57.

[9] Chandra K. Jaggi, Mona Verma and AmrinaKausar, 2011 Customer Based Two Stage Credit Policies In A Supply-Chain, Proceedings of the 2011 International Conference on Industrial Engineering and Operations Management Kuala Lumpur, Malaysia, January 22–24.