AHP Combined with Fuzzy Topsis for Evaluating a Best Alternative

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Abstract:

Fuzzy TOPSIS is one of the various models of multiple attributes Decision Making with Fuzzy values. In this paper a new method is presented for fuzzy TOPSIS with Triangular fuzzy number. In today's world a consumer while purchasing a product looks after various criteria of a particular product and the place where it is sold. So different products are taken into account along with associated sales counters is identified and formulated. In this paper AHP and Fuzzy TOPSIS methods are proposed to determine the best alternatives using the subjective criteria. The fuzzy distance between each alternative and the ideal solution is found out with the greatest relative closeness to the ideal solution is obtained.

Key Words: Fuzzy number**,** AHP, Fuzzy TOPSIS, Multi Criteria Decision Making

1. Introduction:

The idea of fuzzy set was first proposed by Bellman and Zadeh [3], as a mean of handling uncertainty. Decision making is the process of finding the best alternative from a number of feasible alternatives which is called as Multi Criteria Decision Making. The MCDM problems may be divided into the classical MCDM in which the ratings are measured as crisp numbers and the other is the FMCDM, which is based on the vagueness of the problem and expressed in linguistic terms.In[5],[7],[13]and[15]a fuzzy version of Saaty's AHP method was developed by Triangular Fuzzy Numbers for Linguistic terms**.**

The brand managers of a company wants to sell their products in all the counters of a particular city whereas there has been a lot of ambiguity between various counters and all the products are not selling in a single counter and there are a lot of variations in the sales patterns of different products in different counters. So the specific data is collected and formulated and the results are useful for the brand mangers to take a decision on further sales planning.

Let P_1, P_2, P_3----- ,$P_{n...}$be n alternatives available and C_1, C_2, C_3----- ,$C_{m...}$be criteria involved in the expression of alternative. Let A_{ij} be the performance of alternative P_i with respect to criteria C_j and w_j be the relative importance of criteria. Then the decision making is the selection of the best alternative with respect to criteria

2. Literature Review:

Saaty(1980) [15] introduced Analytical Hierarchy Process which is one of the approaches in Multicriteria Decision Making. This plays an important role in selecting alternatives. This is one of the widely used methods for the practical solution of MCDM problems. Traditional AHP method does not directly use fuzzy numbers or membership functions to express fuzzy information. Instead it uses the fuzziness of a multiple attribute decision making problem. Laurhoven and Pedrycz (1985)[14] proposed a method of fuzzy judgment using triangular fuzzy numbers (TFN). They applied arithmetic operation for TFN and logarithmic least square method to calculate fuzzy utilities. TOPSIS (Techniques for order Performance by similarity to ideal solution) is one of the best grading methods using multi criteria decision making (MCDM) problem was first developed by Hwang and Yoon[12]. Chen and Hwang (1992)[9] first applied fuzzy numbers to establish fuzzy TOPSIS. Later some efforts were done to expand TOPSIS model to TOPSIS fuzzy as given by Abo-Sina M.A, A.H. Amer, 2004. [1].Chen, T.C., 2000[10] made the study on extensions of the TOPSIS for group decision making under fuzzy environment.

3. Preliminaries:

3.1Fuzzy number: [13]

A fuzzy set Ã of the real line R with membership function $\mu_{\breve{A}}(x) : R \to [0,1]$ is called fuzzy number if i) A must be normal and convex fuzzy set

ii) the support of Ã, must be bounded

iii) α_A must be a closed interval for every $\alpha \in [0,1]$;

3.2Triangular Fuzzy number: [9]

A fuzzy number A is defined to be a triangular fuzzy number if its membership functions

$\mu_{\tilde{A}}(x) : R \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \; for\, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} \; for\, a_2 \le x \le a_3 \\ 0, elsewhere \end{cases}$$

3.3Sum of Two Fuzzy numbers: [9]

Let A = (l_1, m_1, n_1) and B= (l_2, m_2, n_2) be two fuzzy numbers. Then their sum A+B is given by

A+B = $(l_1 + l_2, m_1 + m_2, n_1 + n_2)$ Similarly

$\alpha A = (\alpha l_1, \alpha m_1, \alpha n_1)$ where α is a real number.

3.4 Normal: [16]

The fuzzy set Ã is normal if height (A) = 1. In other words there exist at least one $x \in X$ such that $\mu_{\tilde{A}}(x)=1$

3.5 Convex Fuzzy set: [16]

A fuzzy set is convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \forall x_1, x_2 \in X, \lambda \in [0,1]$

3.6 Distance of two Triangular Fuzzy numbers: [2]

If $\tilde{A}=(a_1,a_2,a_3)$, $\tilde{B}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers, then the distance is

$$S(\tilde{B},\tilde{A}) = \frac{(b_1+b_2+b_3)-(a_1+a_2+a_3)}{3}$$

4. Decision Making Using Analytical Hierarchy Process and Evaluation of Alternatives:

In this paper we have used a real life example of a particular product wherein the sales of the products in different sales counters are estimated. Here a company is selling five different variants of a particular product in five different sales counters. The sale is spread unevenly for different products which is very much heterogeneous. So a new formulation is worked out so that the sales of five different products and five sales counters become homogenous. This is done by the analysis of sales data from the company. The data is tabulated and later fuzzy rating is given and priority vector is calculated and weights were obtained to give the result.

4.1 Comparative Scale of relative importance by AHP

Verbal Judgment	**Numerical rating**	**Fuzzy rating**
Unsuitable	1	(1,1,1)
Below Average	3	(1,3,5)
Average	5	(3,5,7) Fair
	7	(3,5,9)
Good	9	(5,7,9)

Algorithm for the proposed method:

Step 1: Construct a comparison decision matrix using AHP Techniques.

A comparison matrix between Product and sales counter is formed and each entry is divided by its

column sum .Now the average of the corresponding row entry is the priority vector w_j

where j=1, 2,3,4,5 with respect to each criterion.

ALT	C_1	C_2	C_3	C_4	C_5
P_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	$\tilde{x}_{13}$	$\tilde{x}_{14}$	$\tilde{x}_{15}$
P_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	$\tilde{x}_{23}$	$\tilde{x}_{24}$	$\tilde{x}_{25}$
P_3	$\tilde{x}_{31}$	$\tilde{x}_{32}$	$\tilde{x}_{33}$	$\tilde{x}_{34}$	$\tilde{x}_{35}$
P_4	$\tilde{x}_{41}$	$\tilde{x}_{42}$	$\tilde{x}_{43}$	$\tilde{x}_{44}$	$\tilde{x}_{45}$
P_5	$\tilde{x}_{51}$	$\tilde{x}_{52}$	$\tilde{x}_{53}$	$\tilde{x}_{54}$	$\tilde{x}_{55}$

Step 2: **C**onstruct the fuzzy decision matrix for ranking.

The fuzzy decision matrix for the alternatives, product $\tilde{P}_i$ and the criteria is constructed as follows

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Step 3: Construct a normalized decision matrix. The normalized value $\tilde{n}_{ij} = (n_{ij}^a, n_{ij}^b, n_{ij}^c)$.

The normalized scores are obtained as follows:

$$n_{ij} = \frac{\hat{x}_{ij}}{\sqrt{\sum_{i=1}^{m} (s(\hat{x}_{ij},0))^2}} \qquad i, j= 1,2,3,4,5\backslash$$

where x_{ij} - the score of the i^{th} option, with respect to the j^{th} criterion

$$s(\tilde{x}_{ij},0) = \frac{x_{ij}^a + x_{ij}^b + x_{ij}^c}{3}$$

Step 4: Construct a normalized weighted decision matrix.

The weighted normalized value $\tilde{v}_{ij}$ = (v_{ij}^{a}, v_{ij}^{b}, v_{ij}^{c},) is calculated as

$\tilde{v}_{ij} = w_j * \tilde{n}_{ij}$, i= 1, 2, 3, 4,5 j=1, 2, 3,4,5

Assuming that there is set of weights for each w_j, criteria, from the priority vector (w_i) from step 1, where w_j is the weight of the i^{th} criterion and $\sum_{j=1}^{n} w_j$ =1.

Step 5: Determine the positive ideal solutions and negative ideal solutions respectively

Positive ideal solution $A^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, \tilde{v}_3^+, \tilde{v}_4^+, \tilde{v}_5^+\}$

Negative ideal solution $A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \tilde{v}_3^-, \tilde{v}_4^-, \tilde{v}_5^-\}$

Step 6: Calculate the distance of each alternative from positive and negative ideal:

Positive ideal alternative $d_i^+ = \sqrt{\sum_{j=1}^{n} (s(\hat{v}_j^{+}, \hat{v}_{ij}))^2}$ i=1, 2, 3,4,5.

Negative ideal alternative $d_i^- = \sqrt{\sum_{j=1}^{n} (s(\hat{v}_j^{-}, \hat{v}_{ij}))^2}$ j= 1, 2, 3,4,5

Step 7: Calculate the relative closeness for each alternative from the ideal solution by using the relation $cl_i^+ = \frac{d_i^+}{d_i^+ + d_i^-}$, i= 1, 2, 3,4,5

Step 8: Grading Alternatives: Rank the preference order with the highest value of closeness coefficient where cl_i^+ indicates a good performance of the alternative p_i. The best alternative is one with the greatest relative closeness to the ideal solution.

5. Mathematical Approach to the study:

Step I: Comparison matrix between Product and Sales counter

Sales counter / Product	C_1	C_2	C_3	C_4	C_5	Priority Vector (w)
$P10_1$	1	3	3	9	9	0.192
P_2	3	5	9	5	9	0.276
P_3	3	5	7	7	9	0.276
P_4	1	3	3	3	7	0.216
P_5	1	1	3	1	9	0.104
Total	9	17	25	25	43	

Table : 1

Step II: Fuzzy decision matrix:

Salescounter / *Product*	C_1	C_2	C_3	C_4	C_5
P_1	(1,1,1)	(1,3,5)	(1,3,5)	(5,7,9)	(5,7,9)
P_2	(1,3,5)	(3,5,7)	(5,7,9)	(3,5,7)	(5,7,9)
P_3	(1,3,5)	(3,5,7)	(3,7,9)	(3,7,9)	(5,7,9)
P_4	(1,1,1)	(1,3,5)	(1,3,5)	(1,3,5)	(3,7,9)
P_5	(1,1,1)	(1,1,1)	(1,3,5)	(1,1,1)	(5,7,9)
Total	(5,9,13)	(9,17,25)	(11,23,33)	(13,23,,31)	(23,35,45)

Table : 2

Step III: Normalized Fuzzy decision matrix:

Salescounter / Product	C_1	C_2	C_3	C_4	C_5
P_1	(0.21,0.21,0.21)	(0.12,0.36,0.6)	(0.09,0.29,0.49)	(0.44,0.62,0.80)	(0.32,0.45,0.58)
P_2	(0.21,0.65,1.09)	(0.36,0.60,0.84)	(0.49,0.69,0.89)	(0.26,0.44,0.62)	(0.32,0.45,0.58)
P_3	(0.21,0.65,1.09)	(0.36,0.60,0.84)	(0.29,0.69,0.89)	(0.27,0.63,0.81)	(0.32,0.45,0.58)
P_4	(0.21,0.21,0.21)	(0.12,0.36,0.6)	(0.09,0.29,0.49)	(0.08,0.26,0.44)	(0.19,0.45,0.58)
P_5	(0.21,0.21,0.21)	(0.12,0.12,0.12)	(0.09,0.29,0.49)	(0.08,0.08,0.08)	(0.32,0.45,0.58)

Table : 3

Step IV: The weighted normalized value $\tilde{v}_{ij}$ = (v_{ij}^a, v_{ij}^b, v_{ij}^c,) is calculated as $\tilde{v}_{ij} = w_j * \tilde{n}_{ij}$, i= 1, 2, 3, j=1, 2, 3 where wj is the weight of the ith criterion and $\sum_{j=1}^{n} w_j$ =1.

W_1=0.192 ,W_2=0.276, W_3= 0.276, W_4=0.216, W_5=0.104

Weighted Decision Making Matrix:

Sales counter / Product	C_1	C_2	C_3	C_4	C_5
P_1	(0.04,0.04,0.04)	(0.03,0.1,0.17)	(0.03,0.08,0.14)	(0.06,0.09,0.11)	(0.02,0.07,0.12)
P_2	(0.04,0.13.0.21)	(0.1,0.17,0.23)	(0.14,0.19,0.25)	(0.04,0.06,0.09)	(0.03,0.05,0.06)
P_3	(0.04,0.13,0.21)	(0.1,0.17,0.23)	(0.08,0.19,0.25)	(0.04,0.09,0.11)	(0.03,0.05,0.06)
P_4	(0.04,0.04,0.04)	((0.03,0.1,0.17)	(0.03,0.08,0.14)	(0.01,0.04,0.06)	(0.02,0.05,0.06)
P_5	(0.04,0.04,0.04	(0.03,0.03,0.03)	(0.03,0.08,0.14)	(0.01,0.01,0.01)	(0.03,0.05,0.06)

Table : 4

Step V: The positive ideal solutions and negative ideal solutions respectively

A^+ ={(0.04,0.13,0.21),(0.1,0.17,0.23),(0.14,0.19,0.25),(0.06,0.09,0.11), (0.02,0.07,0.12)}

A^-={(0.04,0.04,0.04),((0.03,0.03,0.03), (0.03,0.08,0.14),),(0.01,0.01,0.01) (0.02,0.05,0.06)}

Step VI: Distance of each alternative from positive and negative ideal:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (s(\hat{v}_j^{\,+}, \hat{v}_{ij}))^2} \qquad i=1, 2, 3...$$

$$d_i- = \sqrt{\sum_{j=1}^{n} (s(\hat{v}_j^{\,-}, \hat{v}_{ij}))^2} \qquad j= 1, 2, 3$$

PRODUCT	Distance from FPIS	Distance from FNIS
P_1	0.147	0.106
P_2	0.023	0.20
P_3	0.030	0.20
P_4	0.16	0.05
P_5	0.176	0

Table 5

Step VII: Relative closeness of the each alternative from positive ideal cl_1=0.43 , cl_2=0.89, cl_3= 0.86, cl_4 = 0.23, cl_5= 0

Step VIII: Grading Alternatives

Rank the alternatives cl_i in descending order

$$P_2 > P_3 > P_1 > P_4 > P_5$$

Therefore the alternative P_2 is the best alternative.

6. Conclusions and Discussion:

The results on using AHP and Fuzzy TOPSIS were found and the alternative P_2 has the highest score. This score indicates that the above alternative P_2 which is the product has the highest sales in all the counters. This gives a crisp data for the decision maker to come out with a decision and plan for further improvement of sales in other products equally as P_2 .This method makes an easy and understandable approach rather than the traditional statistical method which involves a number of criteria and also is a longer process. In this study we not only find which is the best alternative we also give a structured analysis for the P_1, P_3, P_4 and P_5 where the brand managers are able to understand and improve the various aspects and parameters for selling their products.

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