Quantitative Feedback Theory based Control of Light-Weight Flexible Manipulators

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ABSTRACT
The control of lightweight flexible manipulator that are widely used in space applications due to their fast response and high speed of operation, is more complex due to their distributed structural flexibility. The structural flexibility associated with light-weight manipulators often results in mechanical vibrations and consequently leads to long settling times and position inaccuracy. This paper proposes the robust control of a single link flexible manipulator using the concept of Quantitative Feedback Theory (QFT). As a first step to achieve the control target, transfer function from input torque to output hub angle is established. The QFT is then adopted to formulate robust controller associated with the actuator. The QFT approach is based on classical ideas of frequency domain shaping of the open-loop transfer function. Simulation results envisage that the proposed QFT based robust controller tracks a desired tip trajectory while suppressing the tip deflection even in the presence of parametric uncertainties.

General Terms
Light-Weight Flexible Manipulator, Quantitative Feedback Theory

Keywords
Structural flexibility, QFT, flexible link manipulator, pre-filter.

1. INTRODUCTION
Robot manipulator technology is constantly improving by increasing efficiency and becoming more human interactive. The ever increasing utilization of robotic manipulators in various applications in recent years has been motivated by the requirements and demands of industrial automation. Among the rigid as well as flexible robotic manipulators, the latter is superior as it possesses many advantages such as lower energy consumption, faster response, smaller actuator requirement, safer operation, compliant structure and non-bulky design. Owing to these advantages, they are used in various applications such as sophisticated assistants for the disabled, to reduction of the launch cost in space exploration, handling waste material in hazardous plants where access to underground storage is limited [1]. Thus they are used from simple pick and place purposes to more complex tasks such as manufacturing high precision products.However, the control of flexible manipulators must take into account both the rigid degrees of freedom and elastic degrees of freedom due to its flexible nature. As the dynamics of flexible manipulators incorporate the effects of mechanical flexibilities in both the links and joints, the resulting dynamic equations are highly complex, and, in turn, the controller design based on mathematical model becomes more challenging than that for rigid robots [2]. Generally energy principles are used to derive the mathematical models of the manipulator and for a simple rigid manipulator the rigid arms store kinetic energy by virtue of their position in the gravitational field. However the flexible arms store potential energy by virtue of the deflection of its links, joints, or drives. Different schemes for modeling manipulators were studied by a number of researchers [3]. Each approach differs from another in view of complexity and accuracy such as Lagrange's equation and assumed mode method (AMM), Lagrange's equation and finite element method (FEM), Euler-Newton equation and AMM and Euler-Newton equation and FEM [4-6]. The dynamic model for a manipulator with flexible link and flexible joint (FLFJ) based on Euler-Lagrange formulation and AMM was explained in [2,7]. The non-linear dynamic model of FLFJ manipulator follows an Euler-Lagrange, assumed mode principle [9].

The control schemes applied to flexible robot include PD control, computed torque control, adaptive control, active damping control, neural network based control, sliding mode control, optimal and robust control. A hybrid PD-PID controller was developed for the control of flexible manipulators in [10]; PD controller for joint motion control and PID control for end-point vibration suppression. PID control was used to regulate the motion of manipulators [11]. Even though PID control being most widely used control in industrial applications, literature shows it is seldom used to control flexible manipulators compared to rigid counterparts and the reason being attributed to problems of common tuning methods showing sluggish response.Incapability of these controllers to account for model uncertainties lead to the necessity of devising robust control techniques. The widely used robust controllers are based on sliding mode and H_α norm. A sliding mode controller was designed to control actively the tip position of single link flexible manipulator subjected to parameter variations [12]. The drawback of this controller is the presence of chattering that cannot be completely reduced. A combination of LQG and H_α feedback control was used in [13] to control one link flexible manipulator. The drawback was that it does not guarantee general stability margin and also not all control problems can be devised in an H_α framework.. The controller based on QFT technique exhibits robustness against model uncertainty and disturbances [14]. QFT approach can be used to enhance the robust stability and tracking performance [15]. Robust controller for flexible manipulator using QFT had been designed in [16] where a QFT based multivariable control system that combines spatially discrete and distributed actuation is employed for an articulating flexible structure in the form of a single link flexible manipulator.In this paper, robust control of flexible link manipulator is proposed through the use of QFT for tracking and vibration suppression.

Section II gives the mathematical model of the system. Section III gives an account of the QFT control methodology. Section IV discusses the application of QFT control to flexible manipulator system.

2. DYNAMIC MODEL

The flexible link manipulator considered in this work is shown in Fig.1, where θ is the angle of flexible link hub and α

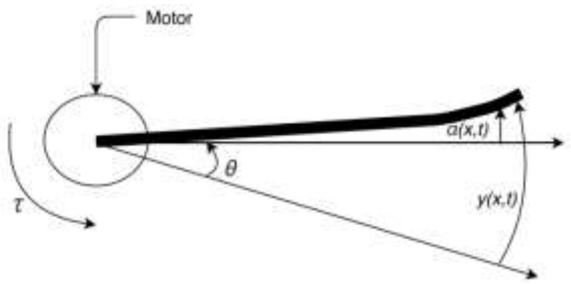

Figure 1: Flexible link manipulator

is the flexible link arm angle (i.e. arm's deflection). The base of flexible link manipulator which determines the tip angular position of the flexible link is driven by servomotor. The total work done on the system is the servomotor torque and is denoted by T_{output}. The Euler-Lagrange formulation is considered in characterizing the dynamic behavior of the system. Considering the motion of the flexible link system on a two-dimensional plane, the potential energy can be formulated as

$$V = \frac{1}{2} K_{stiff} \alpha^2 \tag{1}$$

Where K_{stiff} is the stiffness of the joint. The kinetic energy in the system arising from the the moving hub and the flexible link can be formulated as

$$T = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_{arm} (\dot{\theta} + \dot{\alpha})^2 \tag{2}$$

where J_{eq} and J_{arm} are the inertias of the motor hub and the link respectivel.

Using Lagrange equations, the equation of motion is obtained as

$$J_{eq} \dot{\theta} + J_{arm} (\dot{\theta} + \dot{\alpha}) = T_{output} - B_{eq} \dot{\theta} \tag{3}$$

$$J_{arm} (\ddot{\theta} + \ddot{\alpha}) + K_{stiff} \alpha = 0 \tag{4}$$

Output torque on the load from motor is

$$T_{output} = \frac{\eta_m \eta_g K_t K_g (V_m - K_t K_g \dot{\theta})}{R_m} \tag{5}$$

where η_m is the motor efficiency, η_g is gearbox efficiency, K_t is motor torque constant, K_g is high gear ratio, K_m is the motor back-emf constant and R_m is the armature resistance. Parameter values used are given in the Table I.

3. QUANTITATIVE FEEDBACK THEORY

QFT is a unified theory that emphasizes the use of feedback for achieving the desired system performance tolerances despite plant uncertainty and plant disturbances. QFT quantitatively formulates these two factors in the form of (a) the set $\mathcal{T}_R(s) = \{T_R(s)\}$ of acceptable command or tracking input-output relationships and the set $\mathcal{T}_D(s)=\{T_D(s)\}$ of acceptable disturbance input-output relationships and (b) a set $\mathcal{P}(s)=\{P(s)\}$ of possible plants which include the uncertainties.

Table 1: Parameters of Flexible Link System

Parameter	*Value*	*Uncertainty*
Main arm length (l)	30cm	
Armature Resistance (R_m)	2.6Ω	±12%
Motor back emf constant (K_m)	0.00767Nm	±12%
Motor torque constant (K_t)	0.00767Nm	±12%
Equivalent inertia (J_{eq})	0.0021kgm^2	±10%
High gear inertia (K_g)	70kgm^2	
Joint Stiffness (K_{stiff})	1.66	±10%
Equivalent viscous damping (B_{eq})	0.004Nm/rad/ sec	±20%
Gearbox efficiency (η_g)	0.9	
Motor Efficiency (η_m)	0.69	

The objective is to guarantee that the control ratio $T_R(s)=Y(s)/R(s)$ is a member of $\mathcal{T}_R(s)$ and $T_D(s)=Y(s)/D(s)$ is a member of $\mathcal{T}_D(s)$ for all plants P(s) which are contained in $\mathcal{P}(s)$.QFT employs a two degree-of-freedom control structure which uses output feedback, a cascade compensator G(s), and a pre-filter F(s) and is as shown in Fig.2. This structure is capable of reducing the variations of the plant output due to plant parameter variations.Using the open-loop transmission L(s)=G(s)P(s) and the following three conditions, a robust controller G(s) is synthesized such that the closed loop systems are stable for the whole set of plants.

3.1 Robust Stability Specification

Phase margin, gain margin, or the corresponding M_L contour on the Nichols Chart (NC) can be used for determing the stability margin. If one of the three stability requirements is specified, the remaining two can be calculated. The M_L contour places an upper limit on the magnitude of the closed loop frequency response. It is the stability specification used directly for QFT design technique.

$$\left|\frac{L(jw)}{1+L(jw)}\right| \le M_L \qquad (6)$$

The M_L contour forms a boundary and this boundary must not be violated by the plot of the open-loop transmission L(s)=G(s)P(s) [14].

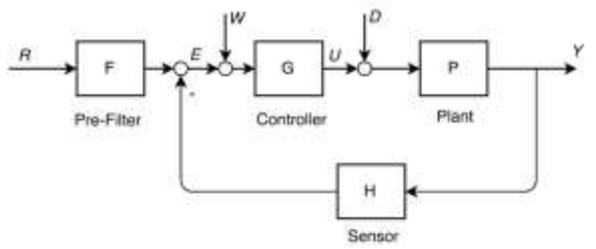

Figure 2: Feedback control system configuration for QFT

3.2 Tracking specification for the reference input

The QFT technique requires that the desired tracking control ratios be modeled in the frequency domain to satisfy the required gain and the desired time domain performance specifications for a step input.

$$|T_L(jw)| \le \left|\frac{L(jw)}{1+L(jw)}\right| \le |T_U(jw)| \qquad (7)$$

The parameters of $T_U(s)$ and $T_L(s)$ are selected so as to meet desired time-domain specifications for the step input such as settling time, overshoot etc. An additional pole that descends the lower tracking bound is added. Also an additional zero that raises the upper tracking bound in high frequency range is added so as to widen the region between the bounds [14].

3.3 Specification for plant input disturbance rejection

The upper limit of the frequency response of the closed loop system due to disturbance input is defined as:

$$\left|\frac{Y(jw)}{D(jw)}\right| \le \alpha_m \qquad (8)$$

where α_m is a constant that limits the output for the step plant input disturbance.These three specifications generate robust bounds on L(jw) (nominal loop transmission) at selected frequencies and are plotted on NC. Finally the designed L(jw) must lie on or just above the bound at each frequency to satisfy the required performance [14]. Then the pre-filter is designed so that the closed loop system satisfies the tracking specifications. This design is also carried out in frequency domain using Bode diagram.

4. QFT CONTROL OF FLEXIBLE MANIPULATOR

The single link flexible manipulator has parametric and non-parametric uncertainty which can be attributed to simplications in modeling of system, unknown external disturbances and variations in operating configuration. In this work only parametric uncertainty is considered. The effect of uncertainty is considered by varying the motor torque constant K_t within ±12%.

The performance requirement is to have an overshoot of 2% and settling time of 5 seconds. For the stability specification M_L is taken as 1.2. Using these specification, a controller is designed based on QFT using QFT Toolbox in Matlab. The loop transmission in frequency domain along with the bounds superimposed on NC is shown in Fig.3. The designed controller is obtained

$$G_c(s) = \frac{4.238*10^5(s+596.3)(s+8.891)}{(s+1680)(s+100.6)(s+880.5)} \qquad (9)$$

Figure 4 shows the Bode magnitude plot of the closed loop frequency response without pre-filter together with the tracking frequency response specifications plotted with dashed lines. The pre-filter is then designed so as to shape the response within the bounds and is obtained as

$$F(s) = \frac{3.22}{(s+2.87)(s+1.122)} \qquad (10)$$

Figure 5 shows the closed frequency response with pre-filter.

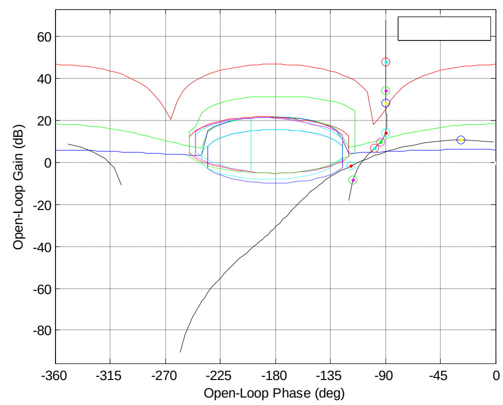

Figure 3: Nominal loop transmission for the link

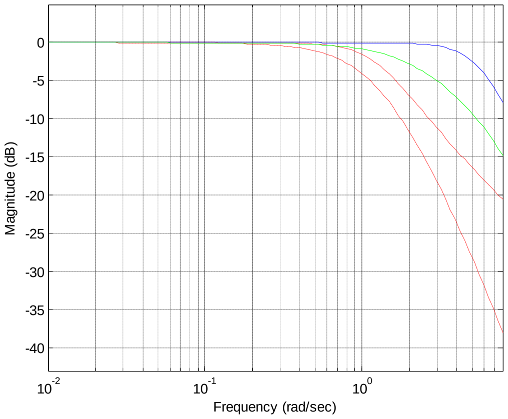

Figure 4: Closed loop frequency response without pre-filter

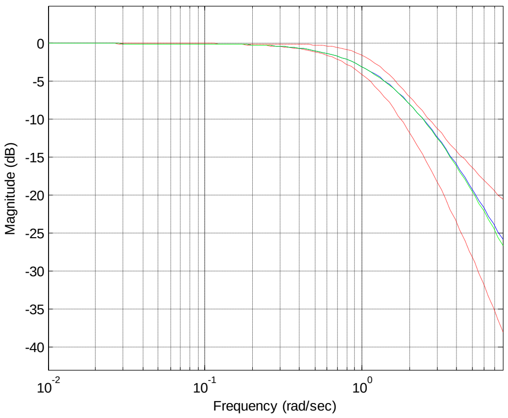

Figure 5: Closed loop frequency response with pre-filter

The tracking performance of the controlled system is analyzed for a square and sine wave input as the reference. The Fig.6 and Fig.7 show that tip perfectly tracks the pre-filter output for a square and sine input. This shows the acceptable performance of QFT control in dealing with parametric uncertainty.

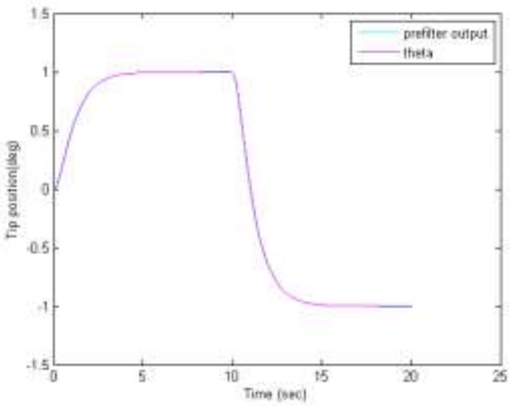

Figure 6: Square wave tracking

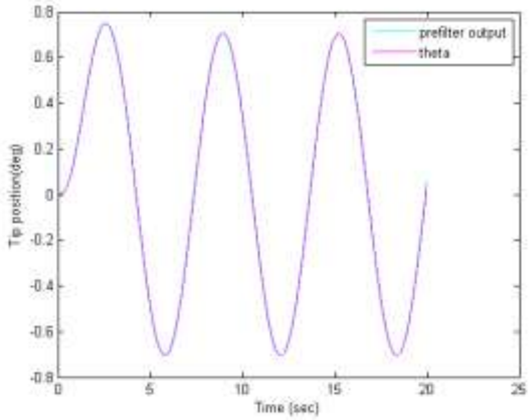

Figure 7: Sine wave tracking

5. CONCLUSION

The QFT methodology was employed to synthesize a control system in order to satisfy the quantitative time domain specifications on the tip position of a flexible link manipulator. Following the formulation of a control system model represented by transfer function, a QFT compensator was designed. Also a pre-filter was designed for the motor to improve the transient and steady state time responses. Simulation results demonstrated favorable tracking control performance of the controlled system over the range of plant uncertainty. The work is in progress to implement the controller on a hardware setup.

6. ACKNOWLEDGMENTS

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