Formulation of ABCD Matrix for Reflection and Refraction of Gaussian Light Beams on the Hemispherical Microlens Drawn on the Tip of Fiber

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ABSTRACT
We present the formulation of ABCD matrix for both reflection and refraction of Gaussian light beam by a spherical surface which separates media of different refractive indices. The analysis takes care of arbitrary angle of incidence. Based on optical phase matching on the interface, we derive expressions for spot sizes and wave-front radii and use them to obtain the ABCD matrix. The formulated ABCD matrix may subsequently be employed to study the launch optics involving laser diode to single-mode fiber coupling via hemispherical micro lens on the fiber tip. The execution of ABCD matrix formalism, as regards estimation of the excitation efficiency, involves little computation and consequently it will be user friendly with the system engineers who are working in the field of optical technology.

Keywords
Spot size; ABCD matrix; Hemispherical microlens; Single-mode circular core step index fiber.

1. INTRODUCTION
Microlenses are fabricated on the tip of the fiber in order to increase the laser diode to single mode fiber coupling efficiency [1-6].The microlenses are fabricated either in the conical or in the hemispherical shape. A microlens on fiber tip possesses the common advantage of being self-centered. Hyperbolic microlens on the fiber tip has large aperture so to collect the entire light emitted by laser diode and it is also free of spherical aberration and thus it emerges as the most efficient one in this context [3].Still, hemispherical microlens is used worldwide on account of the simplicity in its fabrication [3]. Numerous studies for optimum launch optics involving various types of microlens on the tip of mono mode fiber are available in literature [5-13]. It has been shown that the application of ABCD matrix formalism leads to prediction of the concerned coupling optics correctly but in a simple fashion [6-13]. The importance of graded index fiber is well known in view of its large bandwidth and insignificant sensitivity to macro as well as micro bending. This is why the study of graded index fiber in optimum launch optics is proliferating in literature. Very recently investigations on the coupling optics of hyperbolic microlens on graded index fiber tip have been reported [10]. The said ABCD matrix formalism regarding estimation of relevant coupling optics is based on paraxial approximation.

In this communication, we report the formulation of ABCD matrix for both reflection as well as refraction by a spherical interface which separates media of refractive indices n_1 and n_2 respectively. Our analysis is based on matching of transverse phase and amplitude of incident, reflected and refracted waves on the hemispherical interface. In this we use two assumptions namely beam diameter is negligibly small compared to the radii of curvature of optical wave front as well as the optical surface and one of the principal axes of the hemispherical surface should lie in the plane of incidence. This matrix takes care of incidence on the interface at any arbitrary angle of incidence [8, 14]. Accordingly, the use of this matrix in coupling optics will lead to realistic prediction in the sense that it will take care of all possible angles of incidence avoiding paraxial approximation. To the best of our knowledge, such study is not available in literature till date.

2. THEORY
2.1 Formulation of ABCD Matrix
In Figure 1, we have shown three separate three dimensional Cartesian coordinate system namely (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) in order to represent the incident, reflected and refracted waves respectively. The origins of the three system of axes have been selected at the point of incidence on the spherical interface. The choice of axes has been made in such a way that z axis in each case will be in the direction of propagation of the concerned beam, while all x and y axes will lie in the plane of incidence and perpendicular to the plane of incidence respectively in the manner such that each system defines right handed coordinate system.

The spherical interface of radius of curvature(a) separating media of refractive indices n_1 and n_2 is represented in (XYZ) axes system as follows:

$$X^2 + Y^2 + Z^2 = a^2 \quad (1)$$

Also, it is to be noted that the interface shown being a part of the sphere of revolution about Y axis, the incident plane happens to be XZ plane in view of the choice of principal axes. We also take angle of incidence and angle of refraction as θ_1 and θ_2 respectively. It is quite easy to have the following transformation equations connecting (XYZ) and (x_1, y_1, z_1):

$$X = x_1 + asin\theta_1, Y = y_1, Z = z_1 - acos\theta_1 \quad (2)$$

Employing the values of X, Y and Z as given in (2), the spherical surface represented by (1) can be expressed as:

$$(x_1 + asin\theta_1)^2 + {y_1}^2 + (z_1 - acos\theta_1)^2 = a^2 \quad (3)$$

Retaining terms up to 2nd power of transverse aperture in the form of ${x_1}^2$ and ${y_1}^2$ (This is practically applicable for a hemispherical lens interface) and taking care of the fact that Z coordinate on the interface is negative as per Figure 1, we solve for z_1 in (3) to get:

$$z_1 = x_1 sin\theta_1 + \frac{{x_1}^2+{y_1}^2}{2a} \tag{4}$$

The fields(U_i) associated with incident (i=1), reflected (i=2) and refracted (i=3) beams are given by:

$$U_i(x_i, y_i, z_i) = A_i exp[-j\varphi_i(x_i, y_i, z_i)], i = 1,2,3 \tag{5}$$

with the corresponding phase (φ_i) being given as:

$$\varphi_i(x_i, y_i, z_i) = k_i z_i + \frac{k_i}{2}\left(\frac{{x_i}^2}{q_{T1}} + \frac{{y_i}^2}{q_{S1}}\right) \tag{6}$$

where T and S represent tangential and sagittal planes respectively and the relation among the involved parameters are given below with λ_0 standing for the wavelength in free space, k_i the propagation constant, q_i the complex wave parameter, w_i the beam radius and R_i the radius of curvature of the corresponding wavefront:

$$k_1 = \frac{2\pi n_1}{\lambda_0} = k_2; k_3 = \frac{2\pi n_2}{\lambda_0}$$

$$\frac{1}{q_i} = \frac{1}{R_i} - \frac{j\lambda_0}{\pi n_i {w_i}^2} \tag{7}$$

Using (6) and putting the value of z_1 as given in (4), we obtain the complex phase of the incident wavefront on the interface in terms of x_1 and y_1 as follows:

$$\varphi_1(x_1, y_1) = k_1 x_1 sin\theta_1 + \frac{k_1 {x_1}^2}{2}\left(\frac{1}{q_{T1}} + \frac{1}{a}\right) + \frac{k_1 {y_1}^2}{2}\left(\frac{1}{q_{S1}} + \frac{1}{a}\right) \tag{8}$$

Further, employing the boundary condition that phases of incident, reflected and refracted waves must be identical on the interface, we get:

$$\varphi_1(x_1, y_1) = \varphi_2(x_1, y_1) = \varphi_3(x_1, y_1) \tag{9}$$

The aforesaid relations can be utilized to prescribe ABCD matrix for both reflection and refraction in both tangential and sagittal planes. We first consider refracted wave in this context.

2.1.1 Refracted wave

The equations of transformation concerned with incident and refracted coordinate systems are as follows:

$$x_3 = x_1 cos(\theta_1 - \theta_2) + z_1 sin(\theta_1 - \theta_2),$$

$$y_3 = y_1$$

$$z_3 = z_1 cos(\theta_1 - \theta_2) - x_1 sin(\theta_1 - \theta_2) \tag{10}$$

Further, we use (10) and (4) in (6) to get φ_3 (x_3, y_3, z_3) in terms of x_1 and y_1 on the interface in (x_1,y_1,z_1) system. This is given below:

$$\varphi_3(x_1, y_1) = k_3[x_1\{sin\theta_1 cos(\theta_1 - \theta_2) - sin(\theta_1 - \theta_2)\}] + \frac{k_3 {x_1}^2}{2}\left(\frac{1}{q_{T3}} + \frac{cos(\theta_1-\theta_2)}{a}\right) + \frac{k_3 {y_1}^2}{2}\left(\frac{1}{q_{S3}} + \frac{cos(\theta_1-\theta_2)}{a}\right) \tag{11}$$

Using the phase matching condition on the interface in the form $\varphi_1(x_1, y_1) = \varphi_3(x_1, y_1)$ as given in (9), we obtain from (8) and (11):

$$k_1 x_1 sin\theta_1 + \frac{k_1 {x_1}^2}{2}\left(\frac{1}{q_{T1}} + \frac{1}{a}\right) + \frac{k_1 {y_1}^2}{2}\left(\frac{1}{q_{S1}} + \frac{1}{a}\right)$$
$$k_3[x_1\{sin\theta_1 cos(\theta_1 - \theta_2) - sin(\theta_1 - \theta_2)\}] + \frac{k_3 {x_1}^2}{2}\left(\frac{1}{q_{T3}} + \frac{cos(\theta_1-\theta_2)}{a}\right) + \frac{k_3 {y_1}^2}{2}\left(\frac{1}{q_{S3}} + \frac{cos(\theta_1-\theta_2)}{a}\right) \tag{12}$$

Equating coefficient of x_1 on both sides of (12), we get:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{k_3}{k_1} = \frac{n_2}{n_1} \tag{13}$$

Equation (13) is actually Snell's law of refraction at an interface separating two media. Further, we equate coefficients of ${x_1}^2$ and ${y_1}^2$ on both sides of (12) to get:

$$\frac{1}{q_{T3,S3}} = \frac{1}{n q_{T1,S1}} + \frac{1 - n cos(\theta_1 - \theta_2)}{na} \tag{14}$$

Using (7) and (14), we can formulate the relationship of w, R parameters of refracted beam with those of incident beam in the following matrix form:

$$\begin{pmatrix} w_{T3,S3} \\ \frac{w_{T3,S3}}{R_{T3,S3}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1-n cos(\theta_1-\theta_2)}{na} & \frac{1}{n} \end{pmatrix} \begin{pmatrix} w_{T1,S1} \\ \frac{w_{T1,S1}}{R_{T1,S1}} \end{pmatrix} \tag{15}$$

It is seen from (15) that ABCD matrix for refraction at a spherical interface corresponding to any arbitrary angle of incidence is same in both tangential and sagittal planes.

2.1.2 Reflected wave

The following equations give transformations from incident wave coordinate systems to reflected wave coordinate systems:

$$x_2 = -x_1 cos2\theta_1 - z_1 sin2\theta_1,$$

$$y_2 = y_1,$$
$$z_2 = x_1 sin2\theta_1 - z_1 cos2\theta_1 \tag{16}$$

Using the equations of transformation given by (16), we get from (6)

$$\varphi_2(x_1, y_1) = k_2[x_1\{sin2\theta_1 - sin\theta_1 cos2\theta_1\}] + \frac{k_3 {x_1}^2}{2}\left(\frac{1}{q_{T3}} - \frac{cos2\theta_1}{a}\right) + \frac{k_3 {y_1}^2}{2}\left(\frac{1}{q_{S3}} - \frac{cos2\theta_1}{a}\right) \tag{17}$$

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Following (9) we can employ the phase matching condition on the interface in the form $\varphi_1(x_1, y_1) = \varphi_2(x_1, y_1)$

whereby w1e get from (8) and (17):

$$k_1 x_1 sin\theta_1 + \frac{k_1 {x_1}^2}{2}\left(\frac{1}{q_{T1}} + \frac{1}{a}\right) + \frac{k_1 {y_1}^2}{2}\left(\frac{1}{q_{S1}} + \frac{1}{a}\right)$$
$$k_2[x_1\{sin2\theta_1 - sin\theta_1 cos2\theta_1\}]$$

$$+\frac{k_2 {x_1}^2}{2}\left(\frac{1}{q_{T2}}+\frac{cos2\theta_1}{a}\right)+\frac{k_2 {y_1}^2}{2}\left(\frac{1}{q_{S2}}+\right.$$
$cos2\theta 1a$ (18)

Equating coefficient of x_{12} and y_{12} on both sides of (18) and taking care of the fact that k_2 and k_1 belong to the same medium, we have:

$$\frac{1}{q_{T2,S2}}=\frac{1}{q_{T1,S1}}+\frac{1-cos\,2\theta_1}{a}$$
(19)

Employing (7) and (19), we can express the relationship of w, R parameters of reflected beam with those of incident beam in the following matrix form:

$$\begin{pmatrix} w_{T2,S2} \\ \frac{w_{T2,S2}}{R_{T3,S3}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1+cos\,2\theta_1}{a} & 1 \end{pmatrix} \begin{pmatrix} w_{T1,S1} \\ \frac{w_{T1,S1}}{R_{T1,S1}} \end{pmatrix}$$
(20)

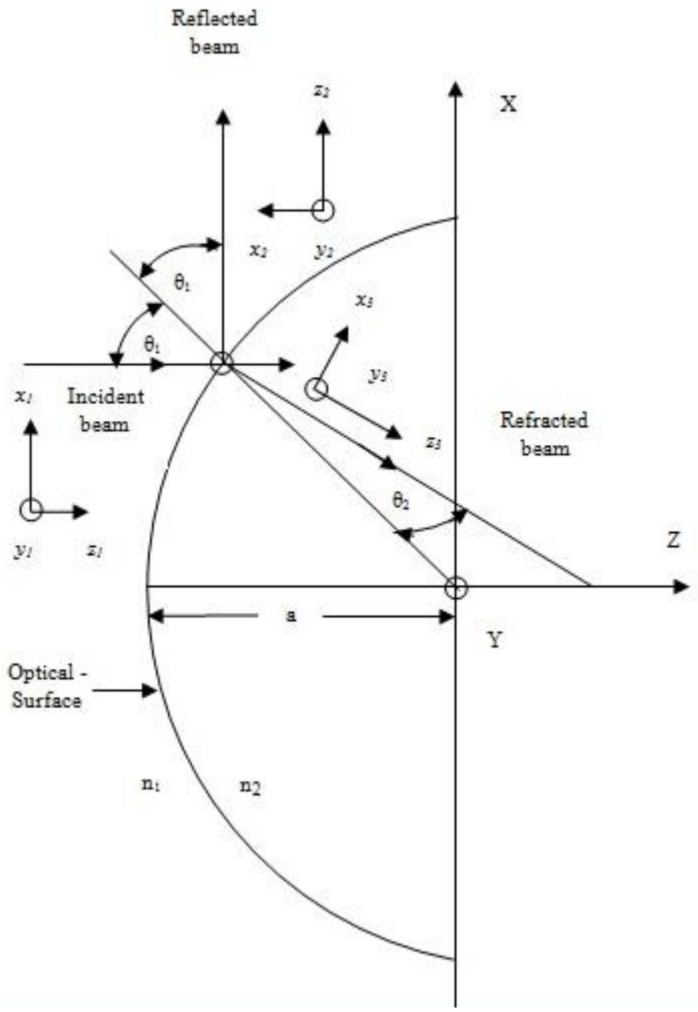

Fig 1: Different Co-ordinate systems used to describe the spherical interface and also the incident, reflected, and refracted beams

3. CONCLUSION

Conclusively, the ABCD matrix in case of both refraction and reflection of Gaussian laser beam at arbitrary angle of incidence from a spherical interface has been prescribed. The prescription of the matrix involves phase and amplitude matching. In the process, we have retained terms up to second order of smallness because of the assumptions made. It is relevant to mention here that the curvature matrix of the interface can also be found by creating a vector Taylor series expansion about the point where the optic axis and the interface intersect. Further, this method of evaluation of curvature matrix is simple enough compared to the available methods in literature. Employing this matrix, one can deal with many physical problems involving reflection or refraction from such surfaces for accurate prediction of the concerned parameters in simple fashion..

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5. REFERENCES

[1] J.H. Song, H.N. J. Fernando, B. Roycroft, B.Corbett , F. H. Peters, Practical design of lensed fibers for semiconductor laser packaging using laser welding technique, J. Lightwave Technol. 27 (2009),pp1533-1539.

[2] E. Li, Characterization of fiber lens, Opt. Lett. 31 (2006), pp169-171.

[3] H. M. Presby, C. A. Edwards, Near 100% efficient fibre microlenses, Electron Lett. 28 (1992), pp582-584.

[4] C. A. Edwards, H. M. Presby, C. Dragone, Ideal miccrolenses for laser to fiber coupling, J. Lightwave Technol. 11 (1993),pp 252-257.

[5] J. John, T. S. M. Maclean, H. Ghafouri-Shiraz , J. Niblett, Matching of single-mode fibre to laser diode by microlenses at 1.5 -1.3 μm wavelength, IEE Proc.Optoelectron. 141(1994),pp 178-184.

[6] H. M. Presby, C. A. Edwards, Efficient coupling of polarization maintaining fiber to laser diodes,IEEE Photon. Technol. Lett. 4 (1992),pp 897-899.

[7] S. Gangopadhyay , S. N. Sarkar, Laser diode to single-mode fibre excited via hyperbolic lens on the fibre tip: formulation of ABCD matrix and efficiency computation, Opt. Commun.132 (1996),pp 55-60.

[8] S. Gangopadhyay , S. N. Sarkar, ABCD matrix for reflection and refraction of Gaussian light beams at surfaces of hyperboloid of revolution and efficiency computation for laser diode to single-mode fiber coupling by way of a hyperbolic lens on the fiber tip, Appl. Opt.36 (1997), pp8582-8586.

[9] S. Mukhopadhyay, S. Gangopadhyay , S. N. Sarkar, Coupling of a laser diode to a monomode elliptic-core fiber via a hyperbolic microlens on the fiber tip: efficiency computation with the ABCD matrix, Opt. Engg. 46 (2007), pp1-5.

[10] S. Mukhopadhyay , S. N. Sarkar, Coupling of a laser diode to single mode circular core graded index fiber via hyperbolic microlens on the fiber tip and identification of the suitable refractive index profile with consideration for possible misalignments, Opt. Engg. 50 (2011), pp1-9.

[11] S.Mukhopadhyay , S. Gangopadhyay , S.N. Sarkar , Misalignment consideration in a laser diode to monomode elliptic core fiber coupling via a hyperbolic microlens on the fiber tip: efficiency computation by the ABCD matrix, Opt. Engg. 46 (2007), pp1-5.

[12] S.K. Mondal , S.N. Sarkar , Coupling of a Laser Diode to Single-Mode Fiber with an Upside -Down Tapered Lens End, Appl. Opt. 38 (1999),pp 6272-6277.

[13] S.Gangopadhyay, S.N.Sarkar, Misalignment Considerations in Laser Diode to Single-Mode Fibre Excitation Via Hyperbolic Lens on the Fibre Tip,Opt. Commun.146 (1998), pp 104-108.

[14] G. A. Massey, A. E. Siegman, Reflection and Refraction of Gaussian Light Beams at Tilted Ellipsoidal Surfaces, Appl. Opt.8 (1969), pp975-978.