Model Order Reduction-A Time Domain Approach

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ABSTRACT
Modeling of real time system posses a large number of problems. It is a challenging task to model accurately a large real time system. Modeling of large real time systems results in large number of differential or difference equations that lead to state variable or transfer function models that represents a higher order system. It is very difficult to handle such a higher order system model for the analysis and design purposes. This paper presents an overview of time domain techniques that can be used to reduce the higher order model to a lower order one. The requirement of model order reduction is that the reduced order model so obtained should retain the important and key qualitative and quantitative properties such as stability, transient and steady state response etc. of the original system.

1. INTRODUCTION
In many practical situations a fairly complex and higher order model is obtained while modeling a large real time system from theoretical considerations. For example consider a multi machine power system. The model for each synchronous generator including the prime mover, the governor and the exciter can be obtained in a manner but the order of the model turns out to be very high. This complexity often makes it difficult to obtain a good understanding of the behavior of a system. The exact analysis of such system is both tedious and costly, it posses a great challenge to both system analyst and control engineer. Therefore, the need arises to go for obtaining a simple and reduced order model of the original system for the following reasons [1].
1. To have better understanding of the system.
2. To reduce Computational Complexity.
3. To reduce Hardware Complexity.
4. To make feasible designs.
5. To generalize results established on a particular system to comparable system.
6. To improve the methodology of computer aided control system design.

2. STATEMENT OF MODEL ORDER REDUCTION PROBLEM

The problem of reduction of a high order system into its lower order approximants in time domain can be stated as:
Consider a linear time-invariant multivariable system described by the equations:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Where $x \in R^n$, $y \in R^m$ and $u \in R^p$.

The objective of model order reduction is to obtain the low-order model.

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t)$$
$$y_r(t) = C_r x_r(t)$$

Where $x_r \in R^r$ and r < n, such that $y_r(t) \in R^m$ is a close approximation to y_r(t) for all inputs u (t) [2].

3. METHODS OF MODEL ORDER REDUCTION
In this paper, we are discussing about time domain techniques, which are given below:
1) Modal Analysis Approach
2) Aggregation Method

1) Modal Analysis Approach
This method attempts to retain the dominant eigenvalues of the original system and then obtains the remaining parameter of the low order model in such a way that its response, to a certain specified input should approximate closely to that of high order system. In other words we neglect the effect of far off poles and zeros from the dominant poles and zeros. The methods proposed by Davison (1966), Marshall (1966), Mitra (1967) and Aoki (1968) all belong to this category [3].
The algorithm for this method is given below:
1. Compute the state-space realization (A, B, C, D) for a given higher order system as described below:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

2. Compute the eigenvalues for given higher order system.
3. Compute the modal matrix (M) for given higher order system as described below:

$$M = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}$$

Where M_1 and M_4 are matrices of dimension (r × r) and (n-r) × (n-r) respectively.
4. Compute the inverse of modal matrix (N) for given higher order system as described below:

$$N = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix}$$

Where N_1 and N_4 are matrices of dimension (r × r) and (n-r) × (n-r) respectively.
5. Compute the sub-matrices (A_1, A_2, A_3, and A_4) for given higher order system matrix (A) as described below:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

Where A_1 and A_4 are matrices of dimension (r × r) and (n-r) × (n-r) respectively.

6. Compute the reduced order system matrix (A_r) using equation as given below:

$$A_r = A_1 + A_2 M_3 M_1^{-1}$$

7. Compute the sub-matrices (B_1, B_2) for given higher order input matrix (B) as described below:

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Where B_1 is a matrix of dimension (r ×1) and B_2 is a matrix of dimension (n-r) × 1.

8. Compute the reduced order input matrix (B_r) using equation as given below:

$$B_r = B_1$$

9. Compute the sub-matrices (C_1, C_2) for given higher order output matrix (C) as described below:

$$C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$$

Where C_1 is a matrix of dimension (1 × r) and C_2 is a matrix of dimension 1 × (n-r).

10. Compute the reduced order output matrix (C_r) using equation as given below:

$$C_r = C_1$$

11. Compute the reduced order state-space realization (A_r, B_r, C_r, D_r) as given below:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t)$$

$$y_r(t) = C_r x_r(t)$$

The main advantage of this method is that it provides satisfactory approximation in both the transient and steady state region. Also its computational work require less time.

2) Aggregation Method

Aggregation is one of the most favorable techniques for reduced-order modeling. This method retains the important eigenvalues of the original system in the reduced order model. The main advantage is that some internal structural properties of the original system are preserved in the reduced model, which is useful not only in analysis but also in deriving state-feedback suboptimal control [4] [5].

The algorithm for this method is given below:

1. Compute the state-space realization (A, B, C, D) for a given higher order system as described below:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

2. Compute the modal matrix (M) for given higher order system as described below:

$$M = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}$$

Where M_1 and M_4 are matrices of dimension (r × r) and (n-r) × (n-r) respectively.

4. Compute the inverse of modal matrix (N) for given higher order system as described below:

$$N = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix}$$

Where N_1 and N_4 are matrices of dimension (r × r) and (n-r) × (n-r) respectively.

4. Compute a regular arbitrary matrix (M_0) using equation as given below:

$$M_0 = [I_r : 0_{r\times(n-r)}]$$

5. Compute the aggregation matrix (K) using equation as given below:

$$K = M_1 M_0 N$$

6. Compute the reduced order state-space realization (A_r, B_r, C_r, D_r) using equations as given below:

$$A_r = KAK^T \left[KK^T\right]^{-1}$$

$$B_r = KB$$

$$C_r = CK^T \left[KK^T\right]^{-1}$$

7. Compute the reduced order state-space realization (A_r, B_r, C_r, D_r) as described below:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t)$$

$$y_r(t) = C_r x_r(t)$$

The main advantage of this method is that its computational work requires less time. The main drawback of this method is that it is applicable to only those systems which are controllable and observable.

A Case Study

In this section an attempt has been made to apply the time domain model order reduction technique as explained earlier. The step response characteristics of the original as well as its reduced order model are compared.

1) Modal Analysis Approach

Consider the 4th order state-space realization (A, B, C, D) for a system as described below:

$$\dot{x}(t) = \begin{bmatrix} -1 & -1 & 6 & -2 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} x(t)$$

For given higher order system the eigenvalues are
(-1, -2, -3, -4).

From the explanation of above algorithm the reduced second order state-space realization (A_r, B_r, C_r, D_r) is obtained as given below:

$$\dot{x}_r(t) = \begin{bmatrix} -1 & -1 \\ 0 & -2 \end{bmatrix} x_r(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y_r(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_r(t)$$

The time-domain responses of both original and reduced second order model are as shown in figure 1 given below [6]:

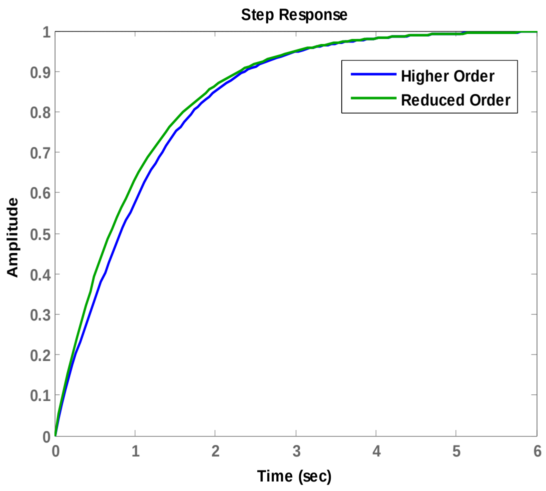

Figure 1:Time-domain responses

The time domain performance specifications of both original and reduced second order model are as shown in table-1 given below:

Table 1: Time domain performance specifications

S. No. / Name of System	Higher Order System	Reduced Order System
Peak Amplitude	0.999	1
Peak Time (sec)	6.75	9.06
Percentage Overshoot	0	0
Rise Time (sec)	2.26	2.2
Settling Time (sec)	3.93	3.91
Final Value	1	1

2) Aggregation Method

Consider the following 4th order state-space realization (A, B, C, D) for a system as described below:

$$\dot{x}(t) = \begin{bmatrix} -4 & -1 & 4 & 2 \\ 0 & -5 & -2 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} x(t)$$

From the explanation of above algorithm the reduced second order state-space realization (A_r, B_r, C_r, D_r) is obtained as given below:

$$\dot{x}_r(t) = \begin{bmatrix} -4 & -1 \\ 0 & -5 \end{bmatrix} x_r(t) + \begin{bmatrix} 2.6667 \\ -3 \end{bmatrix} u(t)$$

$$y_r(t) = \begin{bmatrix} 1.2857 & 0.1429 \end{bmatrix} x_r(t)$$

The time-domain responses of both original and reduced second order model are as shown in figure 2 given below [6]:

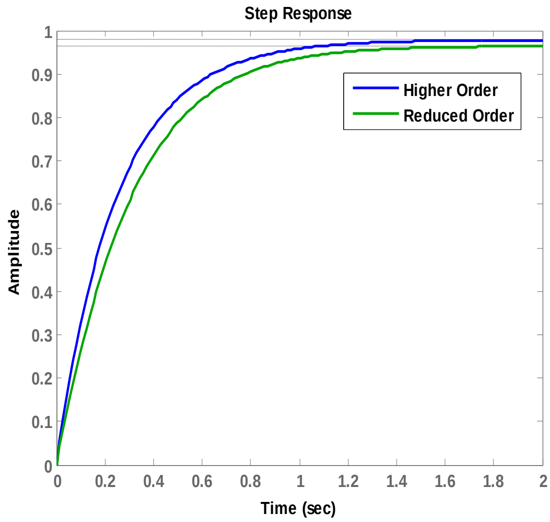

Figure 2: Time-domain responses

The time domain performance specifications of both original and reduced second order model are as shown in table-2 given below:

S. No. / Name of System	Higher Order System	Reduced Order System
Peak Amplitude	0.978	0.964
Peak Time (sec)	2.26	2.26
Percentage Overshoot	0	0
Rise Time (sec)	0.56	0.632
Settling Time (sec)	1.01	1.09
Final Value	0.978	0.964

Table 2: Time domain performance specifications

5. CONCLUSIONS

From the analysis of step responses for time domain (Modal Analysis approach and Aggregation Method) higher order system and its reduced second order model, it is found that peak amplitude, peak time, percentage overshoot, rise time, settling time and final value obtained in all methods is approximately equal to higher order system. Therefore the steady state error is less in all methods. Further model order reduction makes the design of controllers for the system easy.

6. REFERENCES

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