

# Constant False Alarm Rate Multipath Detector for Multiple Antenna based GNSS Receivers

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## ABSTRACT

The position solutions provided by the Global Navigation Satellite Systems (GNSS) for the possible Geodetic-grade applications can sometimes be severely corrupted by multipath. Thus it becomes necessary to estimate and/or mitigate the possible causes of the multipath. However less attention has been given to the detection of multipath. The multipath detection method proposed in this paper, targeted at multiple antenna GNSS receivers, is based on the relation between the arithmetic and the geometric means of the covariance matrix eigenvalues. This relation is used to build a metric, whose theoretical distribution is known in the absence of multipath. Comparison between the empirical and theoretical distributions is done by the Kolmogorov-Smirnov test, which is the basis of the proposed algorithm. It operates directly on the digitized signal, in parallel to tracking loops, and has no need of inferring the number of multipath components or computing their delays. The resulting detector is CFAR (i.e., Constant False Alarm Rate), meaning that it allows to set detection thresholds independently of the incoming noise power by adjusting the false alarm probability.

## General Terms

Wireless Communication, Global Navigation Satellite System, Multipath Mitigation.

## Keywords

Array signal processing, satellite navigation system, multipath channels, statistics.

## 1. INTRODUCTION

The performance of the Global Navigation Satellite Systems (GNSS) is known to be severely corrupted in multipath scenarios, providing a biased position solution which could jeopardize the possible geodetic-grade applications. Indeed, multipath estimation and/or mitigation have attracted the attention of many researchers in the recent years. But, less attention has been given to the detection of multipath. The problem of multipath mitigation has been extensively researched in both single and multiple antenna configurations. In the case of single-antenna receivers several DLL-based methods have been proposed, such as the Narrow Correlator[1]; the Pulse Aperture Correlator (PAC) [2]; Strobe Correlator[3]; the Double Delta Correlator[4] and its variants[5] [6], and the Multipath Elimination Technology[7]. More sophisticated techniques include the Multipath Estimating Delay Lock Loop (MEDLL)[8], the Multipath Mitigation Technique (MMT)[9] also the use of particle filtering for multipath mitigation[10]. However, multipath Mitigation in single-antenna receivers has inherent drawbacks

such as the difficulty to resolve close reflections. Meaning, acknowledging that a given scenario is corrupted by multiple propagation paths[11], and thus using adequate techniques to combat its effect[12],[13]. Specifically, the detector resorts to the estimated noise covariance matrix to calculate a statistic that measures the dispersion of its eigen values. The theoretical distribution of such statistic is known. This theoretical distribution is used by the proposed method to perform a Kolmogorov-Smirnov one sample test to assess departures from the theoretical distribution, with the resulting detector being the Constant False Alarm Rate. This allows the threshold value of the detector to be changed continuously until desired value is obtained.

This provided an estimation regarding its behavior under synchronization errors. A statistical-based test for close multipath detection using antenna-array based GNSS receivers is proposed. The atmospheric-dependant sources (i.e., delays that depend on the ionosphere and troposphere conditions) can to a great extent be mitigated by differential systems external to the receiver's operation. However the multipath effect that is location-dependant remains as the most important cause of accuracy degradation in time-delay estimation, and consequently in position estimation, thus becoming a signal processing challenge.

For multiple-antenna receivers, spatial diversity has been evaluated as an effective tool in the design of adaptive multipath mitigation techniques for GNSS receivers [14],[15] [16]. The application of such methods in scenarios where multipath is actually not present implies a waste of computational load, and thus power consumption, at no benefit. Thus, there is a need of multipath detectors operating as close as possible to the antennas, that is, just at the output of the ADC converters.

In this paper, we present methods of detecting cycle slips and multipath in GNSS (global navigation satellite system) for RTK-PPP (real time kinematic - precise point positioning)[17]. After detecting the received signals with cycle slips and multipath from the GNSS satellites, we remove these measurement data of pseudoranges and L1 and L2 carrier phases, for maintaining the accuracy of GNSS RTK-PPP. We have already developed PPP algorithms based on GR (GNSS Regression) models. Our derived RTK precise point positioning algorithm achieved the positioning accuracy in decimeter level. The algorithm does not require the real-time transmitted information such as from WAAS etc [18]-[22]. Therefore our derived PPP algorithm may be easily implemented without any external online received data.

## 2. PROPOSED METHODOLOGY

The proposed method operates at the output of the correlators, It operates in parallel to the tracking loops[23],[24]. According to the detector results, the GNSS receiver can

adapt its correlation strategies. This is possible since the Software Defined Radio[25]-[27] approach allows the coexistence of different algorithms for synchronization that can be stored in memory and applied as required. In this work, we considered coherent multipath as the worst case for multipath detection/estimation.

The idea of using the estimated spatial covariance matrix to construct a metric was introduced. The same idea was proposed in some previous works [28], and applied to the covariance matrix of a temporally averaged block of data, which implies the use of much larger matrices [29]. In this paper, we provide a more comprehensive statistical analysis. The remainder of the paper presents the signal model at the bourns of the antenna elements and discusses the relation between the Maximum Likelihood Estimator (MLE) of synchronization parameters and the estimated covariance matrix.

INPUT :- We use two types of GNSS signals, one GPS signal, one carrier frequency and BPSK modulation technique.

OUTPUT:- In the output we get a graph of probability of multipath detection versus relative delay separation.

## 2.1 Project Methodology

The proposed statistical metric was tested in realistic scenarios by computer simulation, considering two types of GNSS signals. On one hand, we considered the civilian GPS navigation signal, referred to as L1 C/A code and transmitted at a carrier frequency of  $1540 \cdot rc$  MHz, where  $rc = 1.023$  MHz,  $Tc = 1/rc$ , and is BPSK-modulated. On the other hand, we considered a BOC (1,1) modulated signal, which is compatible to the E1 band Galileo signal. The modulation technique used is BPSK modulation. After BPSK modulation we transmit this signal with carrier frequency. The GNSS signal model is quite liberal. Thus it can encompass for instance, both GPS [30], [31] and Galileo [32] signal structures.

The detector's significance level was set to 0.05 and 0.001, allowing thus 5% and 0.1% Type I errors, respectively. It operated with  $L = 1$  sample to construct the EDF, thus making it suitable in applications where multipath appears and disappears quickly.

The receiver consisted of a half-wavelength 8-element circular antenna array, with a pre-correlation bandwidth of 4 MHz and a sampling frequency of  $f_s = 10$  MHz.

## 2.2 Synchronisation of GNSS signals using antenna arrays.

A GNSS antenna receives measurements which are considered to be a superposition of plane waves corrupted by noise and, possibly, interferences and multipath. An antenna receives  $M$  scaled, time-delayed, and Doppler-shifted signals with known waveform structure. These signals correspond to the LOSSs of  $M$  visible satellites. The received complex baseband signal is shown below:-

$$Y(t) = \sum_{i=1}^M a_i(t)s(t - T_i); fd_i(t) + V(t)$$

where  $s(t)$  is the transmitted complex baseband low-rate navigation signal spread by the pseudorandom code of the  $i$ -th satellite, considered known. Signal parameters are  $a_i(t)$ , its

complex amplitude;  $T_i$ , the time-delay; and  $fd_i(t)$ , the Doppler deviation. Finally,  $V(t)$  is a zero-mean, temporally white, additive Gaussian process that gathers thermal noise and all other non-modeled terms. In the sequel, we focus on a single satellite's signal, thus neglecting the contribution of the rest of satellites. This assumption we made is realistic, considering that GNSS systems use pseudorandom noise (PRN) codes with a high processing gain and relatively small cross-correlation among satellite codes. Therefore, the influence of other satellites can be considered as Gaussian noise and included in the thermal noise term.[33]-[37] The receiver architecture proposed in this paper is as shown below. (Fig 1). In this paper, we operate at the output of a bank of correlators. After integration-and-dump, the receiver operates with a set of accumulated signals. The accumulation interval typically set to the duration of the code period.

These correlators are matched to the satellite of interest through the known spreading sequence. We consider  $K$  correlator outputs per arm. This is referred to as multi-correlator architecture.

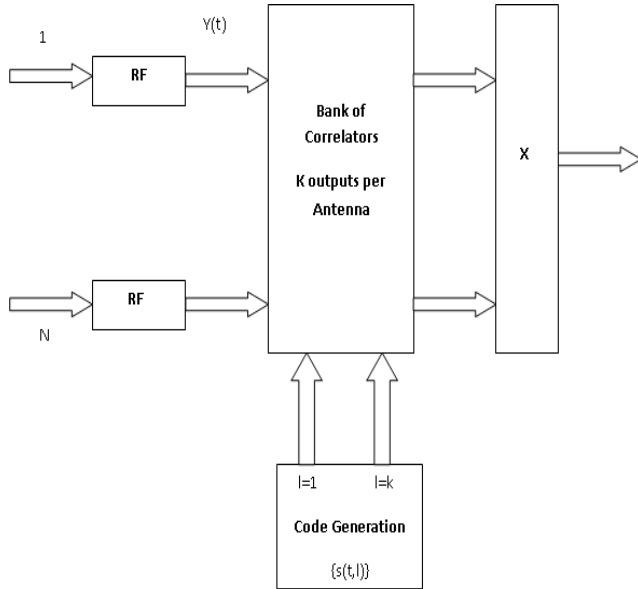
In compact form, the  $K$  snapshot signal model can be represented as :-

$$X = a g(\theta, \varphi) d(\tau, fd) + N$$

Where  $d(\tau, fd)$  is known as the basis function matrix and  $N$  is the matrix containing all undesired contributions of  $X$ . The above signal model is referred to as the structured signal model as it imposes a structure on the signal due to steering vector form. The unstructured signal model approach considers that spatial signatures and complex amplitudes can be rearranged into an equivalent channel vector  $h$  which is assumed arbitrary and unknown.

$$X = h d(\tau, fd) + N$$

Where  $h$  takes into account the possible modeling errors, the structure of which being arbitrary. The unstructured signal model is more preferable than the structured signal model because in many GNSS applications it becomes reasonable to consider the direction of the impinging signals as a priori known. However this knowledge implies somewhat complex DOA estimation algorithm must be used and that a nearly perfect calibration is required [38]. The latter represents a technological challenge and many of errors can jeopardize the operation of the algorithm possible sources. The above calibrations are gathered in  $h$  which is treated as a channel vector and it is estimated accordingly. This is possible because DOAs and amplitudes do not force  $h$  to have a certain structure. However the drawback is its inability to estimate carrier phases since the spatial reference is lost in the case of unstructured model



**Fig 1 :- Reciever implementation considered in this paper.**

Here we consider Multiple antenna receiver, consisting of  $N$  Radiating elements ( $1 \dots N$ ) as shown in the above figure. Each of the antenna element receives a replica of the complex baseband signal. However, the phase of each signal will be different depending on the array geometry and the Direction Of Arrival (DOA) of the signal [39] – [42].

Thus focusing on a single satellite, we get the following signal

$$y(t) = g(\theta(t), \hat{\theta}(t)) a(t)s(t-T); fd(t) + V(t)$$

Where  $y(t)$  is observed signal vector  $g(\theta(t), \hat{\theta}(t))$  is known as the steering vector [43], and  $\theta(t), \hat{\theta}(t)$  are the time varying azimuth and elevation of the desired signal respectively. Lastly,  $v(t)$  are samples of process  $V(t)$ .

### 2.3 Maximum likelihood synchronization and the covariance matrix

Parameter estimation has been thoroughly covered in literature [48], [49]. The Maximum Likelihood (ML) principle presents an optimal paradigm to obtain a parameter estimator that asymptotically

The algorithm operates as follows

First the statistic is initialized to zero, since  $L > 1$ , the algorithm needs previous values of the statistic which are not yet available. Then each time, a record of  $K$  samples is provided by  $X_k$ , the covariance matrix is computed and so are its eigen values, thus obtaining the instantaneous value  $S_k$ . The KS test is used to assess whether the sample  $S_{k-L+1:K}$  follows the  $X^2(N^2 - 1)$  distribution or not.

### ALGORITHM 1: STATISTICAL MULTIPATH DETECTOR

The Maximum Likelihood Principle (MLE) attains its lower variance bound, i.e., the Cramér-Rao Bound CRB), as the number of samples goes to infinity. The ML solution is based on the maximization of the distribution of measurements

conditional on the value of the parameter. as an implementation of the estimator based on the Accelerated Random Search (ARS) algorithm proposed in [44], [45] [46]. The ARS algorithm is a stochastic optimization method that finds global optimum of non-convex functions ensuring convergence. In this work, we use the classical Kolmogorov-smirnov non parametric test [48]-[50] which is based on the maximum likelihood difference between the EDF and the CDF of the reference distribution. The one sample KS statistic is

$$DL(FL(S), F(S)) = \sup FL(S) - F(S)$$

Where  $FL(S)$  is the empirical estimate of  $F(S)$  computed from the sa  $S_{1:L} = \{S_1 \dots S_L\}$  under the null hypothesis.

To sum up, the proposed algorithm is sketched in algorithm 1. The algorithm requires the setting of the threshold value, which is tabulated as a function of the desired significance value or miss probability. Typical  $\alpha$  values range from 0.1 to 0.001.  $L$  must be chosen in accordance to the statistics of the multipath detection. Our metric can operate from  $L = 1$ .

Require  $d_\alpha, L, N$ .

1.  $K = 1$  to  $\infty$  do
2. Initialize  $S_{1-L:0} = 0$
3. **For** Store  $k$  samples in  $X_k$
4. Estimate  $W_k$
5. Estimate  $S_k$  from  $W_k$
6. Compose  $F_L(S)$  from  $S_{k-L+1:K}$
7. Compute statistic  $D_L$
8. if  $\sqrt{LD_L} > d_\alpha$  then,
9. Reject the null hypothesis  $F_L(S) \neq F(S)$
10. Implies multipath corrupted scenario.
11. **Else if**  $\sqrt{LD_L} < d_\alpha$  then,
12. Accept the null hypothesis  $FL(S) = F(S)$
13. Implies multipath free scenario
14. end if
15. end for

### 2.4 Statistical multipath detector algorithm based on $R_{xx}$

Require  $d_\alpha, L, N$ .

1. Initialize  $S_{1-L:0} = 0$
2. **For**  $K = 1$  to  $\infty$  do
3. Compute  $R_{xx}$  using  $R_{xx} = 1/K(XX^H)$
4. Evaluate  $S_K = 2L(Nd = 1)$
5. Compose  $F_L(S)$  from  $S_{k-L+1:K}$
6. Compute  $DL(FL(S), F(S))$  where  $F(S)$  is the reference CDF corresponding to a  $X^2((N-1)^2-1)$  distribution.
7. if  $\sqrt{LD_L} > d_\alpha$  then **f**,
8. Reject the null hypothesis  $F_L(S) \neq F(S)$   
Implies multipath corrupted scenario.  
end for

### 3. RESULTS

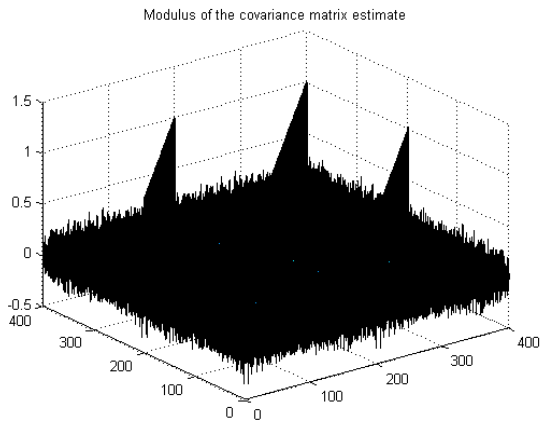
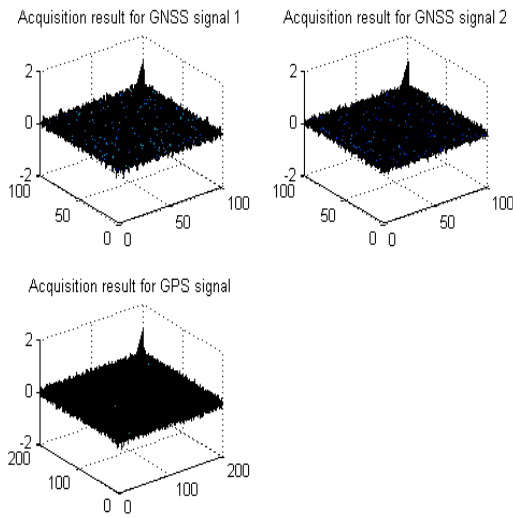


Fig 2. Result of the acquisition of the covariance matrix estimate is shown in first three graphs and the modulus of the covariance matrix estimate is shown in the last graph.

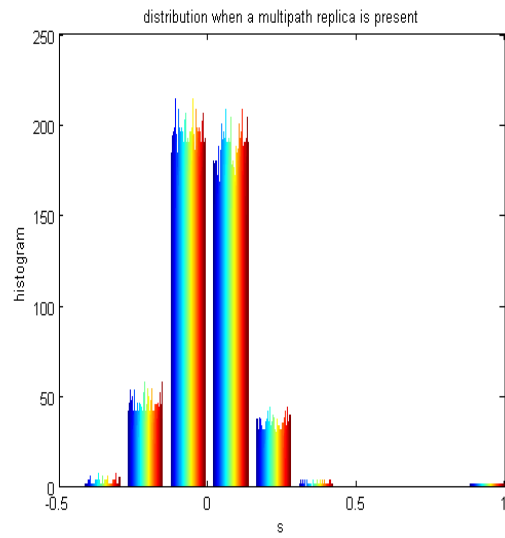


Fig 3. Distribution of the multipath replica

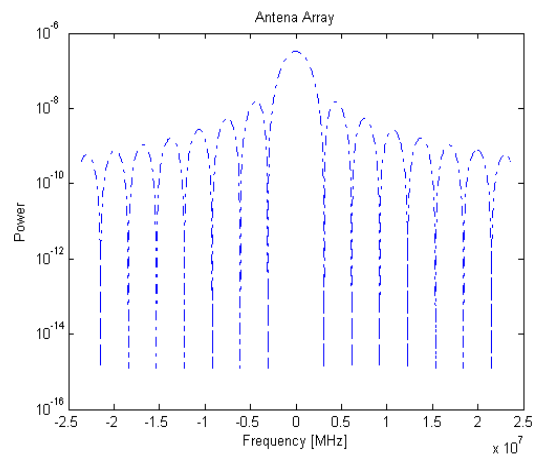


Fig 4. Antenna array output.

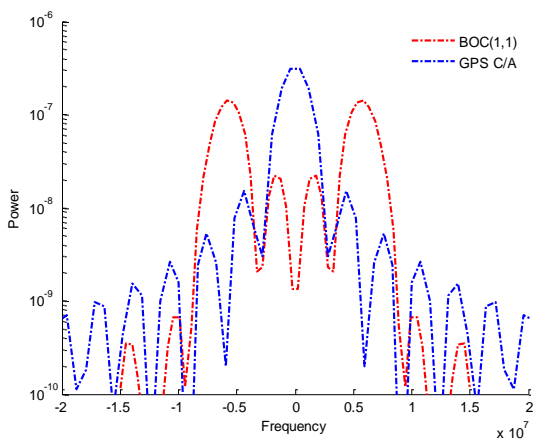


Fig 5. Acquisition of GPS and BOC Signal

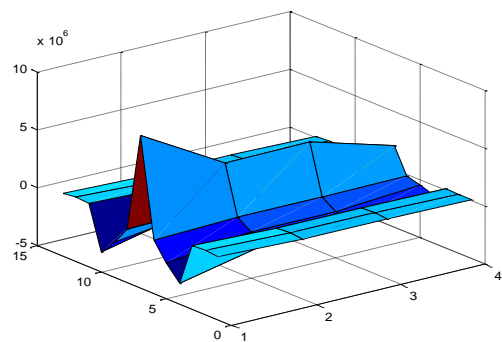


Fig 6. Multipath corrupted Scenario

The proposed algorithm can be used to design a device that could adapt its correlation strategies according to the results of the scenario sensing. The software defined radio approach allows the co existence of different algorithms for the synchronization that can be stored in the memory and applied as required. The formulation of this new detector is generic and thus it can be used as a metric to assess the existence of the LOSS echoes in the scenario.

The use of correlator comes at no additional cost to the receiver. The computational cost is more in the first algorithm. Thus this algorithm is only recommended in high precision GNSS receivers, being highly dependent on these errors.

For the other applications, we have found out that the other algorithm proposed does not require synchronization, can be used.

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