Effect of Cosmological Constant on Higher Dimensional Husain Space-Time

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ABSTRACT

In the present work, we study the effect of cosmological constant on higher dimensional Husain space-time. We show that singularities arising in higher dimensional asymptotically flat space-time and in cosmological solution are naked but depend on the choices of the parameters. The naked singularities are found to be gravitationally strong, violating cosmic censorship hypothesis.

Keywords- Cosmic censorship, gravitational collapse, higher dimension, strong curvature naked singularity.

1. INTRODUCTION

It is widely believed that under physically reasonable conditions, a sufficiently massive collapsing body will undergo continual gravitational collapse, resulting in the formation of a gravitational singularity. This singularity may or may not be visible to a faraway observer. However, there is some debate as to whether such a singularity will be naked or covered by an event horizon. The cosmic censorship hypothesis (CCH) by Penrose [1], states that the space-time singularities arising in the gravitational collapse must be hidden behind the event horizon. In other words, the singularities forming in the gravitational collapse of a massive star are never naked. There are two versions of this hypothesis. The strong version suggests that no singularities are visible to any observers (i.e. local or faraway) while the weak CCH states that the singularities formed in the gravitational collapse are hidden inside the black holes and cannot be seen by an observer at infinity.

Over the last 25 years or so, classical relativists are interested in formulating a proper provable version of CCH. Despite several attempts made by many researchers, this hypothesis remains unproven till date. Various models studied in the past years show that either naked singularities or black holes may form during the gravitational collapse. These models include dust [2]-[6], radiation [7]-[12], perfect fluid [13]-[14], null strange quark fluids [15]-[16] etc. Among all these solutions Vaidya solution [17] is one of the most important solution. This solution can yield a naked singularity. This was shown first by Papapetrou [18], since then this solution is being used to analyze the scenario of gravitational collapse in general relativity.

Our investigation is mainly focusing on the singularity that may possibly form during the collapse, at the center. Hawking and Penrose has stated the singularity theorem according by, the collapsing massive body which develops a singularity, need not necessarily lead to a black hole. The naked singularity may be the other possibility. There could be chances of formation of naked singularities when the center of the collapsing star gets trapped before its boundary has entered the Schwarzschild radius [19]. Anzhong Wang [20]- [24] has generalized the Vaidya solution which includes most of the known solutions to the Einstein equation such as anti-de Sitter-Charged Vaidya solution, Husain solution. Husain solution of null fluid with $P = k\,\rho$ have been lately used as the formation of black hole with short hair [21]. To discuss the nature of the singularities forming in higher dimensional Husain solution after using the cosmological constant is our ultimate aim in this paper.

The paper is organized as follows: In section II, we briefly describe the five dimensional Husain space-time. In section III, we investigate the formation and the nature of the singularities in higher dimensional asymptotically flat space-time. In section IV, we discuss the gravitational collapse of cosmological solution. Finally, the paper ends with concluding remarks.

2. HUSAIN SOLUTION IN FIVE DIMENSIONAL SPACE-TIME

General spherically symmetric line element in five-dimensional (5D) space-time [22]- [23] is

$$ds^2 = -\left[1-\frac{m(v,r)}{r^2}\right]dv^2 + 2dvdr + r^2\left(d{\theta_1}^2 + sin^2\theta_1 d{\theta_2}^2 + sin^2\theta_1 sin^2\theta_2 d{\theta_3}^2\right), \qquad (1)$$

where $m(v,r)$ is usually called the mass function, v is an advanced Eddington time coordinate, in which r decreases towards the future along a ray v = constant.

$d{\theta_1}^2 + sin^2\theta_1 d{\theta_2}^2 + sin^2\theta_1 sin^2\theta_2 d{\theta_3}^2$ is a line element on unit 3-sphere.

Non-vanishing components of the Einstein tensor are given by

$$G_0^0 = G_1^1 = -\frac{3m'(v,r)}{2r^3}, \quad G_0^1 = \frac{3\dot{m}(v,r)}{2r^3}, \quad G_2^2 = G_3^3 = G_4^4 = -\frac{m''(v,r)}{2r^2}, \qquad (2)$$

where

$\{x^i\} = \{v, r, \theta_1, \theta_2, \theta_3\}$, $(i = 0,1,2,3,4)$ and

$\dot{m}(v,r) = \frac{\partial m(v,r)}{\partial v}$, $m'(v,r) = \frac{\partial m(v,r)}{\partial r}$

Einstein field equations are

$$G_{\mu v} = KT_{\mu v}, \qquad (3)$$

where $G_{\mu v}$ is Einstein tensor, K is a gravitational constant and $T_{\mu v}$ is energy momentum tensor [24]-[25] given by

$$T_{\mu v} = T_{\mu v}^{(n)} + T_{\mu v}^{(m)}, \qquad (4)$$

where

$$T_{\mu v}^{(n)} = \sigma l_\mu l_v, \quad T_{\mu v}^{(m)} = (\rho + P)\left(l_\mu n_v + l_v n_\mu\right) + P g_{\mu v} \qquad (5)$$

Using (3), (4), (5) we can write the expressions for $\sigma,\ \rho\ and\ P$ as

$$\sigma = \frac{3\dot{m}(v,r)}{2Kr^3}\ ,\ \rho = \frac{3m'(v,r)}{2Kr^3},\ p = -\frac{m''(v,r)}{2Kr^2} \qquad (6)$$

Here ρ , p are energy density and pressure, while σ is the energy density of the Vaidya null radiation.

We have considered null vectors l_μ , n_μ such that

$$l_\mu = \delta_\mu^0\ ,$$

$$n_\mu = \frac{1}{2}\left[1 - \frac{m(v,r)}{r^2}\right]\delta_\mu^0 - \delta_\mu^1$$

$$l_\lambda l^\lambda = n_\lambda n^\lambda = 0\ ,\ l_\lambda n^\lambda = -1\ , \qquad (7)$$

This fluid in general belongs to the type II fluids for which the energy conditions are given by [24]-[26]

(i) The weak and strong energy conditions:

$$\sigma > 0\ ,\ \rho \geq 0\ ,\ P \geq 0\ ,\ (\sigma \neq 0)\ , \qquad (8)$$

(ii) The dominant energy conditions:

$$\sigma > 0\ ,\ \rho \geq P \geq 0\ ,\ (\sigma \neq 0)\ , \qquad (9)$$

Following [22, 26] we define the mass function in five dimensional Husain solution:

$$m(v,r) = f(v) - \frac{g(v)}{(3k-1)r^{3k-1}}\ ,\ k \neq \frac{1}{3}\ ,$$

$$= f(v) + g(v)\ln r\ ,\ k = \frac{1}{3}\ , \qquad (10)$$

where $f(v)$and $g(v)$ are arbitrary functions (which are restricted by the energy conditions).

Let us introduce a term with cosmological constant in the above mass function. So the mass function $m(v,r)$ becomes,

$$m(v,r) = \frac{\Lambda r^4}{12} + f(v) - \frac{g(v)}{(3k-1)r^{3k-1}}\ ,\ k \neq \frac{1}{3}\ ,$$

$$= \frac{\Lambda r^4}{12} + f(v) + g(v)\ln r\ ,\ k = \frac{1}{3}\ , \qquad (11)$$

where $f(v)$and $g(v)$ are arbitrary functions (which are restricted by the energy conditions).

It can be observed that for $k = \frac{1}{3}$, energy conditions are not always satisfied for all r [26], hence in the present work, we consider the first case only (i.e. we will not consider the case $k = \frac{1}{3}$).

Using mass function (11) into (6), we can find that

$$P = k\rho = k\left(\frac{\Lambda}{2K} + \frac{3g(v)}{2Kr^{3(k+1)}}\right), \qquad (12)$$

$$\sigma = \frac{3}{2Kr^3}\left[\dot{f}(v) - \frac{\dot{g}(v)}{(3k-1)r^{3k-1}}\right]\ , \qquad (13)$$

Husain solution is characterized by the equation of state (12), $P = k\rho$, where $0 < k < 1$, due to which, we always have $\rho \geq P \geq 0$ and thus dominant energy conditions hold. It can be observed from (11) , (12) and (13) that , to satisfy weak and strong energy conditions we must have $g(v) \geq 0$, and to ensure dominant energy conditions we would expect $\dot{m}(v,r) > 0$. (This has been discussed in details in [24, 26].

Inserting the expression for $m(v,r)$ from (11) into (1), we write the Husain metric in five dimensional space-time:

$$ds^2 = -\left[1 - \frac{\Lambda r^2}{12} - \frac{f(v)}{r^2} + \frac{g(v)}{(3k-1)r^{3k+1}}\right]dv^2 + 2dvdr$$

$$+ r^2\left(d\theta_1{}^2 + sin^2\theta_1 d\theta_2{}^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3{}^2\right)\ , \qquad (14)$$

It can be observed that above metric is asymptotically flat for $k > \frac{1}{3}$ and cosmological for $k < \frac{1}{3}$.

Defining now,

$K^a = \frac{dx^a}{dk}$ as tangent vector to null geodesics, where k is an affine parameter, it follows that $\xi^a K_a$ is constant along radial null geodesics and hence a constant of motion:

$$\xi^a K_a = vK_v + rK_r = A\ , \qquad (15)$$

where A is a constant.

Geodesics equations of motion for metric (1) on using the null conditions $K^a K_a = 0$, takes the form

$$\frac{dK^v}{dk} + \left[\frac{m}{r^3} - \frac{m'}{2r^2}\right](K^v)^2 = 0\ , \qquad (16)$$

$$\frac{dK^r}{dk} + \left[\frac{m}{r^3} - \frac{m'}{2r^2} - \frac{m^2}{r^5} + \frac{mm'}{2r^4} + \frac{\dot{m}}{2r^2}\right](K^v)^2 + \left[\frac{m'}{r^2} - \frac{2m}{r^3}\right]K^v K^r = 0\ , \qquad (17)$$

Let us denote $K^v = \frac{R}{r}$. (18)

And, from the null condition, we obtain

$$K^r = \frac{R}{2r}\left[1 - \frac{m(v,r)}{r^2}\right]\ , \qquad (19)$$

where R satisfies the differential equation

$$\frac{dR}{dk} - \frac{R^2}{2r^2}\left[1 - \frac{3m}{r^2} + \frac{m'}{r}\right] = 0\ , \qquad (20)$$

Analysis of the structure of the singularity is initiated by a study of transverse radial null geodesic defined by $ds^2 = 0$. Thus equation for outgoing null geodesics for the metric (14) is given by

$$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{\Lambda r^2}{12} - \frac{f(v)}{r^2} + \frac{g(v)}{(3k-1)r^{3k+1}}\right]\ , \qquad (21)$$

In general, above equation does not yield analytic solution for general values of $f(v)$ and $g(v)$. In the next sections we analyze the gravitational collapse of the higher dimensional space times in both the types of solutions (i.e. asymptotically flat as well as cosmological).

3. GRAVITATIONAL COLLAPSE OF THE HIGHER DIMENSIONAL ASYMPTOTICALLY FLAT SPACE-TIME

For asymptotically flat space-time we consider $\frac{1}{3} < k < 1$ (i.e. $k > \frac{1}{3}$)

Let us take $k = \frac{1}{2}$. Then the mass function (11) becomes

$$m(v,r) = \frac{\Lambda r^4}{12} + f(v) - \frac{2g(v)}{r^{1/2}} , \qquad (22)$$

In order to simplify the calculations we choose $f(v) = \lambda v^2$ and $g(v) = \mu v^{5/2}$ where λ and μ are positive constants. Then the mass function (22) becomes

$$m(v,r) = \frac{\Lambda r^4}{12} + \lambda v^2 - \frac{2\mu v^{5/2}}{r^{1/2}} , \qquad (23)$$

With this choice of the functions, metric (14) becomes

$$ds^2 = -\left[1 - \frac{\Lambda r^2}{12} - \frac{\lambda v^2}{r^2} + \frac{2\mu v^{5/2}}{r^{5/2}}\right] dv^2 + 2dvdr$$

$$+ r^2\left(d\theta_1{}^2 + sin^2\theta_1 d\theta_2{}^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3{}^2\right) , \qquad (24)$$

It can be observed that the metric (24) is self-similar admitting a homothetic killing vector ξ^a given by

$$\xi^a = v\frac{\partial}{\partial v} + r\frac{\partial}{\partial r} , \qquad (25)$$

which satisfies

$$L_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab} , \qquad (26)$$

where L denotes the Lie-derivative.

Using (18), (19) and (23) into (15), we find the solution of differential equation (20) as,

$$R = \frac{2A}{2 - X + \frac{\Lambda r^2}{12}X + \lambda X^3 - 2\mu X^{7/2}} , \qquad (27)$$

Where we have defined $X = \frac{v}{r}$, and is known as self-similarity variable.

To investigate the structure of the singularity, we need to consider the radial null

geodesics defined by $ds^2 = 0$. Thus equation for radial null geodesics for the metric (24) is given by

$$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{\Lambda r^2}{12} - \lambda X^2 + 2\mu X^{5/2}\right] , \qquad (28)$$

It can be observed that the above differential equation (i.e.(28)) has singularity at $r = 0$, $v = 0$.

For the geodesic tangent to be uniquely defined and exist at this point we must have [27]

$$X_0 = \lim_{\substack{v\to 0\\ r\to 0}} X = \lim_{\substack{v\to 0\\ r\to 0}} \frac{v}{r} = \lim_{\substack{v\to 0\\ r\to 0}} \frac{dv}{dr}$$

$$= \frac{2}{1 - \lambda X_0{}^2 + 2\mu X_0{}^{5/2}} , \qquad (29)$$

i.e.

$$2\mu X_0{}^{7/2} - \lambda X_0{}^3 + X_0 - 2 = 0 , \qquad (30)$$

The above algebraic equation decides the nature of the singularity. If the above equation has a real and positive root, then there exist future directed radial null geodesics originating from $r = 0$, $v = 0$. In this case the singularity will be naked. If (30) has no real and positive root, then the singularity will be covered and the collapse proceeds to form a black hole.

If we substitute $X_0 = y^2$, then (30) becomes

$$2\mu y^7 - \lambda y^6 + y^2 - 2 = 0 , \qquad (31)$$

To analyze the nature of the root of (31) the following rule in the *'theory of equations'* may be useful:

Every equation of odd degree has at least one real root whose sign is opposite to that of its last term, the coefficient of the first term being positive.

As in (31) the coefficient of the first term (i.e. 2μ) is positive and the last term is negative, the equation must have at least one positive root.

If we take $\mu = 0.001$ then the roots of (31) obtained for different values of λ for asymptotically flat space-time are shown in the following table.

TABLE I

λ	y	X_0
0.01	1.43615	2.06252
0.02	1.47498	2.17557
0.03	1.53270	2.34916
0.04	1.68290	2.83215

In particular, for $\lambda = 0.01$ and $\mu = 0.001$ one of the positive roots to (31) is y = 1.43615. Using this value in $X_0 = y^2$, we get $X_0 = 2.06252$. Thus we have obtained a real and positive root to (31), which ensures that the singularity is naked.

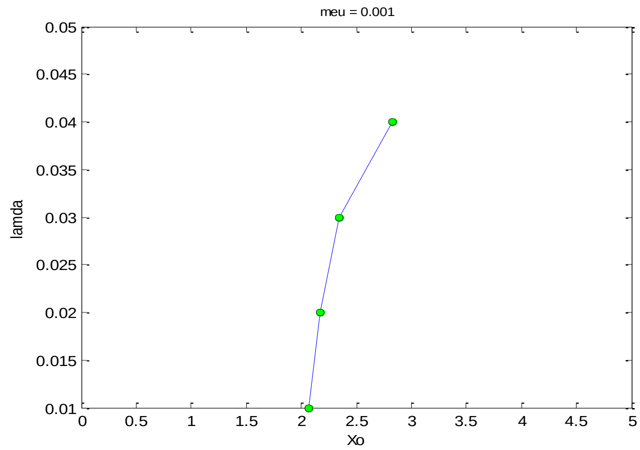

Figure 1: Graph of the values of X_0 against the values of λ for fixed value of μ.

If we observe the above graph we see that the values of X_0 increases as we increase the values of λ .

If we take $\lambda =\ 0.01$ then the roots of (31) obtained for different values of μ are shown in the following table.

TABLE II

μ	y	X_0
0.001	1.43615	2.06252
0.002	1.42696	2.03621
0.003	1.41849	2.01211
0.004	1.4106	1.98979
0.005	1.40328	1.96919
0.006	1.39639	1.94990
0.007	1.38989	1.93179

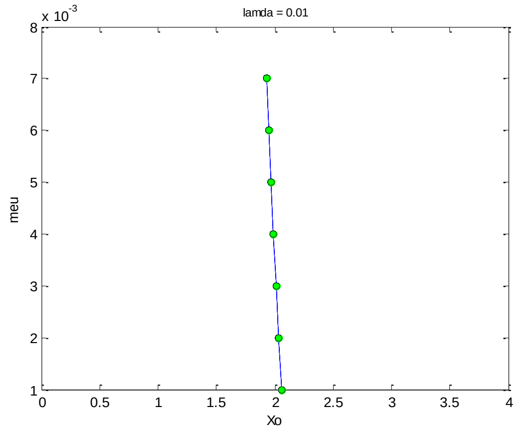

Figure 2: Graph of the values of X_0 against the values of μ for fixed value of λ.

If we observe the above graph we see that the values of X_0 decreases as we increase the values of μ .

A. STRENGTH OF THE NAKED SINGULARITY

A singularity is said to be a strong if the collapsing objects do get crushed to a zero volume at the singularity, and a weak singularity if they do not. According to Clarke and Krolak [28], a sufficient condition for a singularity to be strong in the sense of Tipler [29], is that, at least along one radial null geodesic (with affine parameter k), we should have, in the limit of the singularity

$$\lim_{k\to0} k^2\psi = \lim_{k\to0} k^2 R_{ab}K^aK^b > 0 \quad , \tag{32}$$

where K^a is the tangent to the null geodesics and R_{ab} is the Ricci tensor.

Using the expression for K^r and K^v we write

$$k^2 R_{ab}K^aK^b = X\left[3\lambda - \tfrac{15}{2}\mu X^{1/2}\right]\left(\tfrac{kR}{r^2}\right)^2 , \tag{33}$$

Using the fact that as singularity is approached, $k \to 0$, $r \to 0$, $X \to X_0$ and using L'Hospital's rule, we find that

$$\lim_{k\to0} \frac{kR}{r^2} = \frac{1}{1-\lambda X_0^{\,2} + 2\mu X_0^{\,5/2}} \quad , \tag{34}$$

Using (34), (33) becomes

$$\lim_{k\to0} k^2 R_{ab}K^aK^b = \frac{X_0\left(3\lambda - \frac{15}{2}\mu X_0^{\,1/2}\right)}{\left(1-\lambda X_0^{\,2} + 2\mu X_0^{\,5/2}\right)^2} \quad , \tag{35}$$

Thus the singularity will be strong if

$$3\lambda - \tfrac{15}{2}\mu X_0^{\,1/2} > 0$$

For our particular case (i.e. $\lambda = 0.01$, $\mu = 0.001$, $X_0 = 2.06252$) we find that

$$3\lambda - \frac{15}{2}\mu X_0^{\,1/2} > 0$$

Thus the Clarke and Krolak condition for the strong curvature singularity is satisfied, hence the naked singularity arising in this case is a strong curvature one.

4. GRAVITATIONAL COLLAPSE OF HIGHER DIMENSIONAL COSMOLOGICAL SOLUTION

The space-time (14) will become cosmological for $k < \frac{1}{3}$. Hence let us take $k = \frac{1}{5}$.

To get analytical solution choose $f(v) = \lambda v^2$ and $g(v) = \mu v^{8/5}$.

With these choices, the mass function (11) becomes

$$m(v,r) = \frac{\Lambda r^4}{12} + \lambda v^2 + \frac{5}{2}\mu v^{8/5} r^{2/5} , \tag{36}$$

Using the above mass function, the metric (1) becomes

$$ds^2 = -\left[1 - \frac{\Lambda r^2}{12} - \frac{\lambda v^2}{r^2} - \frac{5}{2}\frac{\mu v^{8/5}}{r^{8/5}}\right]dv^2 + 2dvdr$$

$$+ r^2\left(d\theta_1^{\,2} + sin^2\theta_1 d\theta_2^{\,2} + sin^2\theta_1 sin^2\theta_2 d\theta_3^{\,2}\right) \quad , \tag{37}$$

Equation of outgoing radial null geodesics, then can be written as

$$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{\Lambda r^2}{12} - \frac{\lambda v^2}{r^2} - \frac{5}{2}\frac{\mu v^{8/5}}{r^{8/5}}\right] \quad , \tag{38}$$

In order to determine the nature singularity at $r = 0$, $v = 0$, we let

$$X_0 = \lim_{\substack{v\to0\\ r\to0}} \frac{v}{r} = \lim_{\substack{v\to0\\ r\to0}} \frac{dv}{dr} = \frac{2}{1-\lambda X_0^{\,2} - \frac{5}{2}\mu X_0^{\,8/5}} \quad , \tag{39}$$

i.e.

$$\lambda X_0^{\,3} + \frac{5}{2}\mu X_0^{\,13/5} - X_0 + 2 = 0 \quad , \tag{40}$$

With the substitution $X_0 = y^5$, above equation becomes

$$\lambda y^{15} + \frac{5}{2}\mu y^{13} - y^5 + 2 = 0 \quad , \tag{41}$$

One of the positive roots of the above equation for $\lambda = 0.0001$ and $\mu = 0.001$ is $y = 1.150564$, from which we obtain $X_0 = 2.01629$, which shows that the central singularity arising in the higher dimensional cosmological Husain space-time is also naked.

A. STRENGTH OF THE NAKED SINGULARITY

Further, following the method discussed in the previous section it can be checked that

$$\lim_{k\to 0} k^2\psi = \lim_{k\to 0} k^2 R_{ab}K^a K^b$$

$$= \frac{X_0\left(3\lambda + 6\mu {X_0}^{-2/5}\right)}{\left(1 - \lambda {X_0}^2 - \frac{5}{2}\mu {X_0}^{8/5}\right)^2} > 0 \quad , \qquad (42)$$

which shows that the singularity arising in this case is also a gravitationally strong curvature singularity.

5. CONCLUDING REMARKS

Cosmic censorship conjecture has become a challenging and most significant open problem in a general relativity. This problem remains unproven till date though many researchers tried to prove it. Five dimensional Husain Solution has been studied analytically in detailed keeping this fact in a mind and also we have studied the effect of cosmological constant on it. We have considered both asymptotically flat as well as cosmological solutions and shown that naked singularities do occur as the end stage of gravitational collapse in both the solutions. Again it is found that in both cases, the naked singularities arising in Tipler sense [29] are strong curvature naked singularities under certain restriction on the mass function.

Formation of the strong curvature naked singularities in the asymptotically flat as well as cosmological solutions even after using cosmological constant indicate that, the cosmological constant does not play any fundamental role in the formation of the naked singularity in five dimensional Husain space-time as well, which violates the cosmic censorship hypothesis.

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