Second Law Analysis of Thermodynamics in Free Convective MHD Flow Past an Infinite Vertical Porous Plate with Thermal Radiation & Heat Source

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ABSTRACT
The aim of the present paper is to analyze the second law of thermodynamics in term of entropy generation in electrically conducting viscous incompressible natural convective fluid flow past an infinite vertical porous plate in the presence of thermal radiation, transverse magnetic field and heat source with constant suction. The effects of various fluid parameters on velocity and temperature field are discussed. In addition, the entropy generation due to viscous dissipation, fluid friction and magnetic field is analyzed. It is concluded that the entropy of the flow system near the plate can be regulated by high intensity of applied magnetic field and heat source, and reverse behavior is observed far away from the plate.

Keywords
Entropy generation, Heat source, Thermal radiation

1. INTRODUCTION
The study of MHD natural convective flow through porous material in the presence of thermal radiation has attracted the attention of many researchers due to the numerous potential applications in nuclear reactor, filtrations of solid from liquids, geophysical sciences, petroleum technology, and power technology etc. A. K. Singh [1], Ali M.M. et al. [2], Raptis et al. [3] etc. are the few of the mathematician who made the contribution in the area of study. Radiation effect on unsteady MHD free convective flow with hall current and mass transfer through viscous incompressible fluid flow past a vertical porous plate immersed in porous medium with heat source/sink was discussed by Sharma et al. [4]. Sharma et al. [5] studied that the influence of chemical reaction and thermal radiation on unsteady MHD free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source. Radiation and chemical reaction effects on free convective MHD flow through a porous medium bounded by vertical surface was inspected Reddy et al. [6]. In the same ideology, the researchers like [7, 8] also elaborated the MHD natural convective fluid flow system in different geometry along with different flow conditions.

In last half century, the study of entropy generation has drawn the attention of researchers due to its application in area of engineering and industrial fields such as; heat exchanger, turbo-machinery, cooling of devices, cooling of nuclear reactor etc. it is basically based on the second law of thermodynamics. Cengel et al. [9] stated that as entropy generation takes place, the quality of energy decreases. Entropy generation in convective heat transfer was discussed by Bejan [10]. An analytical investigation into the Nusselt number and entropy generation rate of film condensation on horizontal plate was surveyed by Chang et al. [11]. Further Butt et al. [12] discussed the entropy generation in hydrodynamics slip flow over a vertical plate with convective boundary. Later Butt et al. [13, 14] extended their work for study the entropy generation in fluid flow systems.

In view of the above cited work, the main objective of the present work is to analyze the entropy generation in MHD viscous incompressible, natural convective fluid flow along infinite vertical porous plate in the presence of radiation and heat source. The present problem is considered as a specific case when the externally applied magnetic field is present with thermal radiation and heat source. The main purpose of the study is to controls and regulate the total entropy generation rate in the system and improve the performance of the devices that are working in the above stated flow model.

2. MATHEMATICAL FORMULATION
Let us consider steady, two dimensional natural convective viscous incompressible electrically conducting fluid flow through a porous medium bounded by the infinite vertical porous plate in the presence of thermal radiation. For the geometry of the problem, we consider that the vertical plate is taken along the x^* axis in the opposite direction of the gravity and y^* is taken normal to it. A uniform magnetic B_0 is assumed to be applied in the transverse direction of the fluid flow i.e., normal to it. Due to the infinite length of vertical plate it is supposed that all the variables do not to be subject to the vertical coordinate's axis x^*. Under the above assumption, the governing equations of flow for the stated problem are given as:

$$\frac{\partial v^*}{\partial y^*} = 0, \qquad ...(1)$$

$$\upsilon^* \frac{\partial u^*}{\partial y^*} = \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} - + g\beta_T\left(T^* - T_\infty^*\right) - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta}{k_p} u^*, \qquad ...(2)$$

$$\upsilon^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\vartheta}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + \frac{\sigma B_0^2}{\rho C_p} u^{*2} + + \frac{Q^*}{\rho C_p}\left(T^* - T_\infty^*\right), \qquad ...(3)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} y = 0;\ u^* = 0,\ T^* = T_w \\ y \to \infty;\ u^* \to 0,\ T^* \to T_\infty \end{array}\right\}, \qquad ...(4)$$

Equation (1) gives $v^* = -V_0$

Using Cogley et al. [15], the expression for the radiative heat flux is:

$$\frac{\partial q_r}{\partial y^*} = 4\left(T^* - T_\infty\right)\int_0^\infty K_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda = 4I\left(T^* - T_\infty\right), \qquad ...(5)$$

Introducing the following non-dimensional quantities:

$$u = \frac{u^*}{V_0},\ y = \frac{y^* V_0}{\vartheta},\ T = \frac{T^* - T_\infty^*}{T_w - T_\infty^*},\ \Pr = \frac{\mu C_p}{\kappa},\ M = \frac{\sigma B_0^2 \vartheta}{\rho V_0^2},$$
$$K = \frac{V_0^2 k_p}{\vartheta^2},\ Gr = \frac{\vartheta g \beta_T\left(T^* - T_\infty^*\right)}{V_0^3},\ Ec = \frac{V_0^2}{C_p\left(T_w - T_\infty^*\right)}, \qquad ...(6)$$
$$Q = \frac{Q^* \vartheta}{\rho C_p V_0^2},\ R = \frac{4 I_1 \vartheta^2}{\kappa V_0^2},$$

The dimensionless form of the governing equations (2) and (3) reduce as:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - M_1 u = -GrT, \qquad ...(7)$$

$$\frac{\partial^2 T}{\partial y^2} + \Pr\frac{\partial T}{\partial y} - (R - Q\Pr)T = -\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2 - M\Pr Ecu^2 \qquad ...(8)$$

Where $M_1 = \left(M + \frac{1}{K}\right)$,

The corresponding boundary conditions are reduced to:

$$\left.\begin{aligned} &y = 0;\ u = 0,\ T = 1 \\ &y \to \infty;\ u \to 0,\ T \to 0 \end{aligned}\right\}, \qquad ...(9)$$

Where u^* and v^* are the velocity components in the x^* and y^* directions, respectively; g the acceleration due to the gravity, T^* fluid temperature, T_∞ temperature of the fluid far away from the plate, β the thermal expansion coefficient, σ electrical conductivity of the fluid, ρ the fluid density, ϑ kinematic viscosity of fluid, k_p the permeability of the medium, κ the thermal conductivity, C_p specific heat at constant pressure, Q^* the heat source parameter, T_w temperature at plate, V_0 the suction velocity.

3. METHOD OF SOLUTION

The equations (7) and (8) are coupled non-linear partial differential equations that are not possible to solve into the closed form, therefore to reduce these equations in the set of ordinary differential equations, the velocity and temperature of the fluid can be represented in the nbd of the plate as:

$$\left.\begin{aligned} u &= u_0 + Ecu_1 + O(Ec^2) \\ T &= T_0 + EcT_1 + O(Ec^2) \end{aligned}\right\}, \qquad ...(10)$$

Substituting (10) in equations (7) and (8) and equating the harmonic and non-harmonic term and after neglecting the higher order term of $O(Ec^2)$, equations (7) and (8) become:

3.1 Zeroth order equations:

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - M_1 u_0 = -GrT_0, \qquad ...(11)$$

$$\frac{\partial^2 T_0}{\partial y^2} + \Pr\frac{\partial T_0}{\partial y} - RT_0 + Q\Pr T_0 = 0, \qquad ...(12)$$

3.2 First order equations:

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - M_1 u_1 = -GrT_1, \qquad ...(13)$$

$$\frac{\partial^2 T_1}{\partial y^2} + \Pr\frac{\partial T_1}{\partial y} - RT_1 + Q\Pr T_1 = -\Pr\left(\frac{\partial u_0}{\partial y}\right)^2 - M\Pr u_0^2, \qquad ...(14)$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &y = 0;\ u_0 = 0,\ T_0 = 1,\ u_1 = 0,\ T_1 = 0 \\ &y \to \infty;\ u_0 \to 0,\ T_0 \to 0,\ u_1 \to 0,\ T_1 \to 0 \end{aligned}\right\}, \qquad ...(15)$$

With the help of boundary conditions (15), the differential equations (11) to (14) are solved and the expression for velocity u and temperature T are known and not given here due to sake of brevity.

4. SKIN FRICTION COEFFICIENT

The coefficient of skin friction at plate is given by

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{at\ y=0}, \qquad ...(16)$$

the effect of various parameters on skin friction are shown through the table 1.

5. NUSSELT NUMBER

Rate of heat transfer in term of Nusselt number at the plate is given by

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{at\ y=0}, \qquad ...(17)$$

the effect of various parameters on Nusselt number are shown through table 2.

6. ENTROPY GENERATION

The entropy generation rate per volume in the existence of magnetic field is given by [Woods (16)].

$$S_{gen}^{'''} = \frac{\kappa}{T_0^{*2}}\left[\frac{\partial T^*}{\partial y^*}\right]^2 + \frac{\mu}{T_0^*}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{u^{*2}}{k_p}\right] + \frac{\sigma B_0^2}{T_0^*}u^{*2}, \qquad ...(18)$$

Where T_0^* is the reference temperature. In the right-hand side of the equation (18); the first term shows the entropy generation due to heat transfer through temperature difference, second term is due to viscous dissipation and fluid friction and third term due to presence of applied magnetic field. The dimensionless entropy generation is expressed as:

$$Ns = \left[\left(\frac{\partial T}{\partial y}\right)^2 + \Pr EcT_0\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \frac{u^2}{K}\right\} + \Pr EcT_0 Mu^2\right], \qquad ...(19)$$

7. RESULT AND DISCUSSION

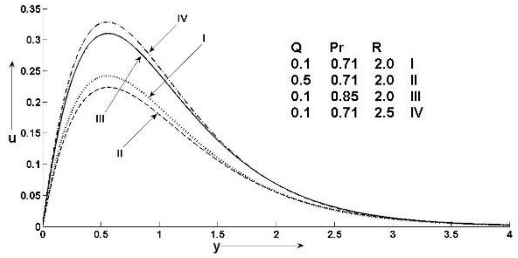
In the present study, an analytical solution is obtained for the velocity, temperature, entropy and Bejan number for the viscous incompressible free convective fluid flow through plate with thermal radiation in the presence of heat source. Figure 1 illustrates that the increase in radiation parameter and Prandtl number enhances the fluid velocity while the increase of heat source parameter is evident for the reverse effect. Figure 2 depicts that the velocity of fluid decreases with increase the magnetic field number M, while the mass buoyancy parameter Grashof number Gr, Eckert number Ec, permeability parameter K effect reversely on the fluid flow. It is clear from the figure that velocity of the fluid increases rapidly near the plate and then gradually comes to minimum for the free stream. Further, figures 3 and 4 show that an increase in mass buoyancy parameter Gr and permeability parameter K boosts the fluid temperature, while the increase in magnetic field parameter M and Prandtl number $\Pr$ decreases the fluid temperature. It is also evident that an increment in the Eckert number Ec and heat source parameter Q enhances the temperature of the fluid, while the temperature of the fluid flow decreases with increase of radiation parameter R. The effects of various governing parameters on entropy generation are presented through graphs 5 to 6. It is apparent from the figure 5 that the influence of magnetic number and heat source Q, decline the entropy generation number Ns while the Prandtl number $\Pr$ shows the reverse effect. Figure 6 revels that an increase in radiation parameter R supports the entropy generation, whereas dominance of permeability parameter K, Eckert number Ec and buoyancy parameter Gr declines the entropy generation. Further figure 7 to 8 illustrate the influence of several physical parameters on Bejan number Be. It is observed from the figure 7 that an increase in the heat source parameter Q and Prandtl number $\Pr$ decreases the Bejan number, while the magnetic field parameter M enhances it. Figure 8 shows that the radiation parameter R supports the Bejan number whereas permeability parameter K, Eckert number Ec and buoyancy parameter Gr decrease the Bejan number. Table 1 illustrates the effect of Nusselt number with respect to all the related physical parameters respectively.

8. CONCLUSION

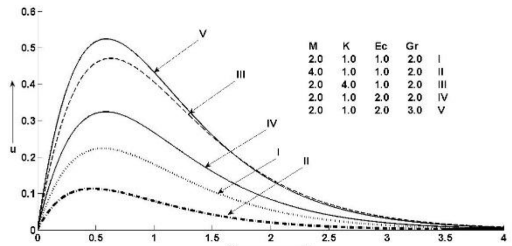
An investigation is motivated for the theoretical study of entropy generation in viscous incompressible free connective fluid flow in the presence of transverse magnetic field with thermal radiation and heat source. The study concludes that:

- Growth in the strength of applied transverse magnetic field parameter declines the fluid velocity; it is due to the reason of Lorentz force that retards the fluid velocity throughout the fluid flow model.
- Magnetic field parameter and thermal radiation parameter play an important role to regulate the fluid velocity, temperature and entropy generation in the fluid model.
- Increment in the magnetic field parameter, heat source parameter, permeability parameter declines the entropy generation number Ns near the plate and upsurges far away from the plate.
- Dominance of Prandtl number and thermal radiation parameter supports the entropy generation.
- The value of Bejan number Be increases with the increase of magnetic number and thermal radiation, while increase of heat source parameter and Prandtl number effect adversely. It is evident that the value of Bejan number is very close to 1.
- Increment in the magnetic field parameter drops the skin friction coefficient while rate of heat transfer gets increased.
- Rise in the thermal radiation parameter increases the skin friction coefficient and Nusselt number.

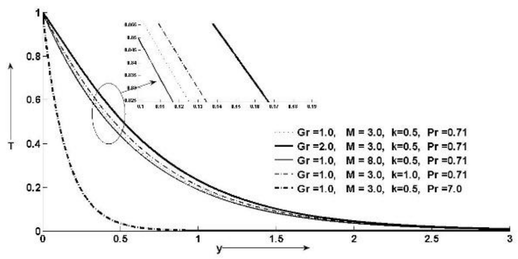
The present investigation will motivate for the future study to regulate and control the fluid velocity, high temperature and entropy generation due to irreversibility process in the system and their minimization will be the challenges in the engineering and industries discipline. Cooling of nuclear reactor, MHD power generator, geothermal devices, cooling of electronic devices, petroleum engineering, agriculture engineering etc. are the few well known area of study, where the magnetohydrodynamics (MHD) plays an outstanding role of investigation.



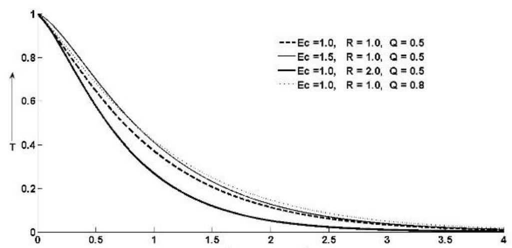
$Fig\ 01: u\ versus\ y, when\ Gr = 2.0, M = 2.0, K = 1.0, Ec = 1.0$



$Fig\ 02: u\ versus\ y, when\ Q = 0.1, Pr = 0.71, R = 2.0$



$Fig\ 03: T\ versus\ y, when\ Ec = 1.0, R = 2.0, Q = 0.5$



$Fig\ 04: T\ versus\ y, when\ Gr = 1.0, M = 3.0, K = 0.5, Pr = 0.71$

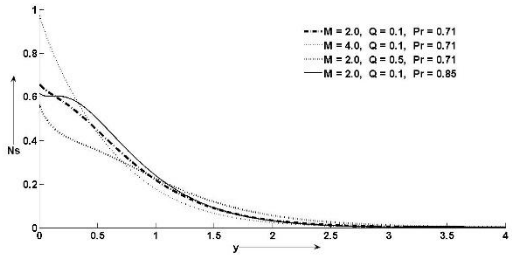


Fig 05 : $\mathbf{Ns}$ versus $\mathbf{y}$, when $\mathbf{K = 1.0, Ec = 2.0, Gr = 1.0,}$

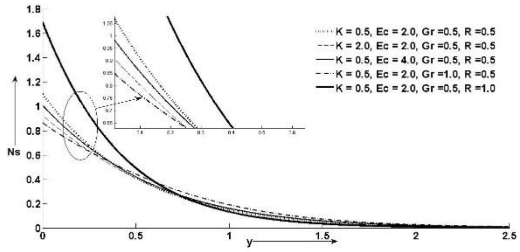


Fig 06 : $\mathbf{Ns}$ versus $\mathbf{y}$, when $\mathbf{M = 4.0, Q = 0.5, Pr = 0.71}$

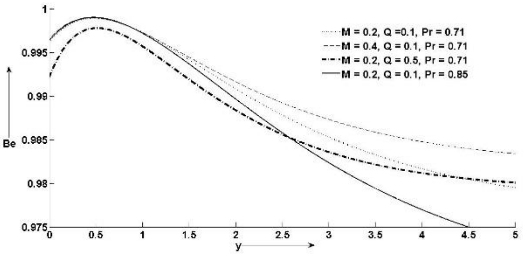


Fig 07 : $\mathbf{Be}$ versus $\mathbf{y}$, when $\mathbf{K = 1.0, Ec = 2.0, Gr = 0.1}$

$\mathbf{R = 0.5}$

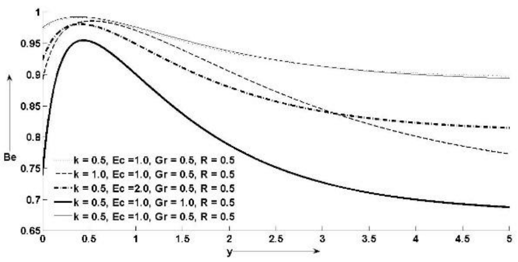


Fig 08 : $\mathbf{Be}$ versus $\mathbf{y}$, when $\mathbf{M = 0.4, Q = 0.1, Pr = 0.71}$

Table 1. Numerical value of skin friction coefficient at the plate for the value of various physical parameters

S.No.	Pr	R	Ec	Q	Gr	M	K	Cf
1.	0.71	2.0	1.0	0.1	2.0	2.0	1.0	1.0941
2.	**0.85**	2.0	1.0	0.1	2.0	2.0	1.0	1.3484
3.	0.71	**2.5**	1.0	0.1	2.0	2.0	1.0	1.4287
4.	0.71	2.0	**2.0**	0.1	2.0	2.0	1.0	1.5413
5.	0.71	2.0	1.0	**0.5**	2.0	2.0	1.0	1.0168
6.	0.71	2.0	1.0	0.1	**2.5**	2.0	1.0	1.6821
7.	0.71	2.0	1.0	0.1	2.0	**3.0**	1.0	0.7506
8.	0.71	2.0	1.0	0.1	2.0	2.0	**2.0**	1.6879

Table 2. Numerical value of Nusselt Number at the plate (rate of heat transfer coefficient) for the value of various physical parameters

S.No.	Pr	R	Ec	Q	Gr	M	K	Nu
1.	0.71	0.5	0.5	0.1	1.0	3.0	0.5	1.0486
2.	**0.85**	0.5	0.5	0.1	1.0	3.0	0.5	1.1295
3.	0.71	**1.0**	0.5	0.1	1.0	3.0	0.5	1.3199
4.	0.71	0.5	**1.0**	0.1	1.0	3.0	0.5	0.9971
5.	0.71	0.5	0.5	**0.8**	1.0	3.0	0.5	0.5407
6.	0.71	0.5	0.5	0.1	**2.0**	3.0	0.5	0.8942
7.	0.71	0.5	0.5	0.1	1.0	**5.0**	0.5	1.0701
8.	0.71	0.5	0.5	0.1	1.0	3.0	**1.0**	1.0233

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