Comparative Analysis of Image Compression Techniques using SVD, DCT and DWT

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ABSTRACT
Digital image processing has become a topic of great interest in recent years. In order to use digital image effectively some algorithms are used to compress an image with no or less degradation in the image quality. Some of these algorithms are said to be lossy algorithms because some information is permanently lost in these techniques. In this paper, the performance of some popular lossy image compression algorithms has been compared in terms of peak signal to noise ratio (PSNR), mean square error (MSE) and compression ratio (CR).

Keywords
Image Compression, Singular Value Decomposition, Discrete Cosine Transform, Haar Wavelet, Peak Signal to Noise Ratio.

1. INTRODUCTION
These days, large amount of data transmit in the form of images, videos or graphics. These types of data require a large amount of storage capacity. Therefore, compression concept is used to reduce the size of data. For compression two types of techniques are used Lossy techniques and Lossless techniques. Lossy techniques are used in this paper for the image compression. In such techniques, some components are permanently removed from original image. So, compressed image is the modified version of the original image with reduced size [1].

The rest of the paper is organised as follow: Section 2, 3 and 4 explain the theoretical aspects and methodology of image compression by using singular value decomposition (SVD), discrete cosine transform (DCT) and discrete wavelet transform (DWT) respectively. Simulation results are given in section 5 and then conclusion is given in section 6.

2. SINGULAR VALUE DECOMPOSITION
Singular value decomposition is a technique which is used to compress an image. It performs better in case where the images are of higher pixel quality. In this technique a given linear matrix transformation K is decomposed into three component matrices i.e. L, D and R.

$$K = LDR^T \qquad (1)$$

where, $K = m \times n$ matrix

$D = m \times n$ matrix is a diagonal matrix which contains the singular values of K as its diagonal values

$L = m \times m$ is a matrix with left singular values of K

$R = n \times n$ is a matrix with right singular values of K

both L and R are orthonormal matrices [2].

$$LL^T = 1 \; \& \; RR^T = 1 \qquad (2)$$

from equation (1), $K = LDR^T$

so, $K^T = \left(LDR^T\right)^T = LDD^T L^T$

$$KK^T = LDD^T L^T = LD^2 L^T \qquad (3)$$

Therefore matrix L is obtained by Eigen vectors of . Similarly, Eigen vectors of gives the matrix R. When image is compressed using SVD, only a few singular values are retained while others are not used [3]. So these singular values must be in descending order. Various steps for above technique are given below.

- Read the image.
- Split the image into RGB components.
- Decompose each image using SVD.
- Select β value and discards diagonal values of D matrix that are not required and construct the image.

From the above algorithm, it can be say that as there is change in the value of β, there is corresponding change in clarity of the image. Optimum value of β should be selected such that there is no damage to image quality and at the same time storage space occupied by image is reduced. So, the value of β is chosen in such a way that good amount of compression is achieved.

3. DISCRETE COSINE TRANSFORM
Discrete cosine transform is well known transform for image compression in lossy manner. Discrete cosine transform is a tool which transforms the input signals into its frequency components. Discrete cosine transform is nothing but the real part of Fast Fourier transform.

The JPEG is mostly used algorithm for image compression which works on the DCT. Joint Photographic Experts Group abbreviated as JPEG standard has been put in place by ISO (International Standards Organization) and IEC (International Electro-Technical Commission). Since it is lossy technique, so some information is perpetually lost [4]. In this method, image is break into parts of different frequencies. After this quantization is done on each part in which actually the compression occurs. Less important frequencies are rejected in the quantization hence the term 'Lossy' is used. The frequencies which meant for less important information are discarded by quantization and the important frequencies used to retrieve the image in decompression step [5].

Another method of DCT image compression is to set value of coefficients. This method gets high PSNR value. The algorithm performs the DCT on the coefficients of image. The process involves the following steps.

- Read the image which has to be compressed.
- Convert the RGB image into the gray scale image.
- Forward DCT is applied to the image and get DCT processed matrix. DCT changes the spatial domain to frequency domain.
- Square the resulting matrices after applying DCT and sort the linear order.
- These values of index are stored in the variable.
- Using coefficients all these variables are processed in iteration to discard the elements from the DCT processed matrix.
- Apply IDCT to get the compressed image with high PSNR [6].

4. DISCRETE WAVELET TRANSFORM

The Fast Fourier transform and discrete wavelet transform both are linear operations that are used to generate a data vector of length from a data structure containing segments of various lengths by filling and transforming. These operations of filling and transforming are done on the mathematical properties of matrices. Similarity in both transforms is that the localized frequency is calculated by mathematically calculations of the power contained by the frequency interval (power spectra). One way to see the time frequency resolution differences between the two transforms is to look at the function coverage of the time frequency plane [7].

Applications of wavelets are: Compression of fingerprints, De-noising the data, Detecting self-similar behaviour in a time series, Computer and human vision.

DWT analyse discrete signals or image containing discrete signals in time for its frequency contents. For this purpose wavelet functions are used. The different types of wavelet transform.

- Haar Wavelet.
- Symlet transform.
- Daubechies.
- Bi-orthogonal.

4.1 Haar wavelet transform

Haar wavelet consists of haar functions which were introduced by Hungarian mathematician Alfred Haar and it was first used in 1910 [8].Wavelets has many applications but also used in image compression. In image compression Haar wavelet is used because it is very simple technique for compression.

In Haar wavelet very simple principle is followed to compress an image by calculating averages and differences of adjacent pixels of an image. Haar wavelet is computationally more efficient than the discrete cosine transform. By implementation of Haar wavelet the image is converted into two parts: as observed in Figure1, during the summing and differencing row wise the image is converted into left and right part containing sums on the left side of the image and differences on the right side of the image as shown in Figure 1(a). During the summing and differencing column wise the image is converted into top and bottom part containing sums on the top side and differences on the bottom side of the image as shown in Figure 1(b) [9].

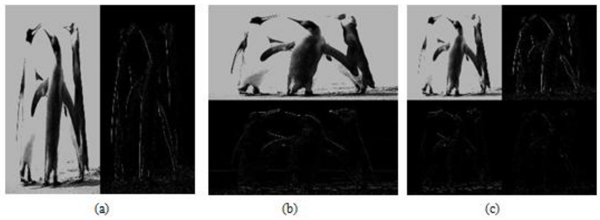

Figure 1. (a) Row wise calculations, (b) Column wise calculations, (c) 1-D Haar wavelet image

For an example the resultant images of 1-D Haar wavelet as shown in Figure 1(c). In case of 2-D Haar wavelet the iterations are done until small enough results obeserved. In 2-D wavelet the iterations are done times according to size of input image [10].

For example concentrate on the first row i.e.

$$r_1 = (84\ 88\ 78\ 75\ 64\ 67\ 75\ 87)$$

This transformation process occurs in different steps. First group all the row values in pairs.

$$[84, 88], [78, 75], [64, 67], [75, 87]$$

Now replace first four values of with the average of pairs and last four values of with half of the difference of pairs. Results will write in new row as:

$$r_1 s = (86\ 76.5\ \ 65.5\ 81\ \text{-}2\ 1.5\ \text{-}1.5\ \text{-}6)$$

By this process calculate the results of remaining row of image. After row wise calculations repeat the process for column wise. In order to achieve the image compression by using Haar wavelet the steps are followed:

- Calculate sum and difference of each row of image.
- Calculate sum and difference of each column of image.
- Apply hard thresholding or quantization on the final matrix (image).
- Write the matrix out to binary file.

By the process one should have to decide whether using 1-D or 2-D Haar wavelet. For 2-D wavelet repeat the step for times until get small enough results of image.

5. PERFORMANCE PARAMETERS

Performance of image compression is based on the two factors: compression ratio and quality of compressed image which is given by PSNR.

Compression ratio

Compression ratio can be defined as ratio of the size of original image to the compressed image [11].

$$Cr = \frac{s_o}{s_c} \tag{4}$$

where, $Cr =$ compression ratio

$s_o =$ size of original image

$s_c =$ size of compressed image

For instance if we are taking the image of 250 kb and after compression the size of compressed image is 100 kb then we can say the compression ratio is 2.5 or 40%.

Mean square error

Mean square error is a measure of the deformity in the reformed image.

$$MSE = \frac{1}{UV}\sum_{i=1}^{U}\sum_{j=1}^{V}[X(i,j) - Y(i,j)]^2 \tag{5}$$

Peak signal to noise ratio

It tells about the quality of the image. It is estimate of peak error if we have high value of PSNR then the quality of image is high. PSNR can be calculated as:

$$PSNR = 10\log_{10}\frac{(255)^2}{MSE} \tag{6}$$

6. SIMULATION RESULTS

The comparative analysis of image compression techniques SVD, DCT and DWT is shown in Table 1.

Table 1. Performance evaluation of image compression techniques

Technique for compression	Parameters			
	Levels	**PSNR**	**MSE**	**Compression Ratio**
SVD	N=50 N=150 N=250	28 33.36 37.86	92.8 29.9 10.63	8.7 2.9 1.75
DCT	Coefficient=20k Coefficient=30k Coefficient=40k	76.40 77.21 77.87	.0015 .0012 .0011	9.13 8.27 7.81
DWT	Level=3 Level=4 Level=5	32.80 36.29 36.37	34.10 15.28 14.98	6.4 3.35 3.30

In this comparison, there are different levels for different techniques and corresponding there will be different values of PSNR, MSE and CR.

7. CONCLUSION

When implementation of the image compression is done using SVD at different singular values N there are different values of PSNR, MSE and CR. As the value of N increases, corresponding compression ratio reduces and there is increase in PSNR. MSE and PSNR are inversely related to each other, therefore MSE gets reduced. When implementation of the image compression is done using DCT there will be low value of MSE and high PSNR at moderate compression levels. Low MSE means that the distortion is less and high PSNR ensures that reconstructed image has good quality. Discrete wavelet transform provides confined compression ratio without much degrade the image quality. When decomposition level is changed in Haar wavelet transform there is not much change in the compression ratio. The above work provides the complete overview of image compression using SVD, DCT and DWT. It also gives the necessary information regarding compression ratio, MSE, PSNR. In future other techniques of image compression available in literature can be study and more comparative analysis can be done by comparing these techniques.

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