Drifting Effect of Electron in Multi-ion Plasmas with Non Extensive Distribution of Electrons

Sheenu Juneja
Department of Mathematics, Statistics & Physics,
Punjab Agricultural University, Ludhiana.141004

Parveen Bala
Department of Mathematics, Statistics & Physics,
Punjab Agricultural University, Ludhiana.141004

ABSTRACT
In the present research work, effect of drifting velocity of electrons have been studied in plasma systems consisting of cold positive and negative ions and electron beam. The electrons are assumed to obey non-extensive velocity distribution. Standard reductive perturbation method is used to derive dispersion relation, which comes out to be a polynomial of four degree in phase velocity that corresponds to four ion-acoustic modes. The expression for critical velocity is found to be the function of various parameters including nonextensive parameter q. The nonextensivity and electron beam parameters play crucial role in the characterization of solitons. We have taken (Ar^+F^-), (H^+ H^-), ($H^+O_2^-$) plasma systems for our study.

General Terms
Nonlinear wave structures

Keywords
Multi-ion plasma, Nonextensive distribution, Reductive Perturbation method.

1. INTRODUCTION
The nonlinear wave structures are described by a quantum of energy that can be propagated as travelling wave in nonlinear media. These waves arise due to the delicate balance of nonlinearity and dispersion [1]. Ion acoustic solitary waves or solitons are the most studied ones among all nonlinear wave structures that exist in ionosphere, inter-galactic environment and plasmas etc. Electron beam is found to observe frequently in the region of space where ion-acoustic waves exist. Observational studies in auroral zone have confirmed electron and ion beam associated with solitary waves [2]. The high speed electron have an appreciable influence on the excitation of nonlinear waves in earth's magnetosphere and interplanetary space [3]. An important work with electron beam was done by Yadav et. al. [4], where they predicted the formation of four soliton branches above a certain critical velocity. However Bala et. al. [5], showed the formation of six soliton branches above a critical velocity while taking the case of multicomponent plasma model with positive, negative ions, electron beam and nonthermal electrons. In the past, the studies of solitary wave with electron beam has become the source of interest for many researchers in variety of plasma systems [4, 5, 6-10]. Multi-ion plasmas such as (H^+, O_2^-), (H^+, H^-) and (Ar^+, F^-) plasma composition occur in D-region of ionosphere, where negative ions are found [11].

One of the universally accepted particle distribution is the Maxwellian or extensive distribution, which is valid for systems in equilibrium. But for systems with long range interactions such as in plasma where nonequilibrium states exist, the non-extensive distribution becomes important. Over the last two decades there is an increasing interest to study a new statistical approach based on the derivations of Boltzmann–Gibbs-Shanon (BGS) entropic measures, also known as Tsallis distribution [12]. It has been further observed that ions play a dominant role in the nonextensive effects in comparison with the electron [13]. Here, the parameter q is a measure of nonextensitivity and q=1 leads to extensive distribution. For q<1(>1), high energy states are more (less) probable than in extensive case. The Tasllis q-distribution has been used with some success in a number of research work in plasma physics [14-22]. The aim of present investigation is to study drifting effect of electrons in multi-ion plasma model consisting of positive, negative ion and electron beam with nonextensitively distributed electrons. Regarding the organization of paper, basic equations related to our plasma model have been given in section 2, using nonextensive distribution of electrons and standard reductive perturbation method, a nonlinear wave equation known as Korteweg-de Vries equation (KdV) has been derived. Section 3 is devoted to the discussion of numerical results and finally the conclusions are made

2. FORMULATION OF PROBLEM
We have considered the collisionless, unmagnetized, multi-ion plasma model containing cold positive and negative ions and electron beam. Further, the electrons are assumed to obey nonextensive distribution. The number density of the electron fluid associated with non-extensivity of electrons is given by

$$n_e = [1+(q-1)\phi]^{(q+1)/2(q-1)} \tag{1}$$

Here q is the nonextensive parameter. The nonlinear behaviour of ion-acoustic waves is present in multispecies plasma model is governed by the following set of normalized continuity, momentum and Poisson equations:

$$\frac{\partial n_j}{\partial \mathrm{t}} + \frac{\partial (n_j v_j)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} = \frac{1}{Q_j}\frac{\partial \phi}{\partial x} \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \alpha_b n_b - \frac{n_1}{B} + \frac{\alpha \varepsilon_z}{B} n_2 \tag{4}$$

Where, j=1,2,b, 1 stands for positive ions, 2 stands for negative ions, b stands for electron beam. Here, $Q_1=-\delta$, $Q_2=\eta\delta/\varepsilon_z$, $Q_b=\eta_e\delta Z_1$ and

$$B = (1-\alpha\varepsilon_z)/(1+\alpha_b), \alpha_b = \frac{n_b^{(0)}}{n_e^{(0)}}, \alpha = \frac{n_2^{(0)}}{n_1^{(0)}}, \eta = \frac{m_2}{m_1},$$

$$\eta_e = \frac{m_e}{m_1}, \varepsilon_z = \frac{Z_2}{Z_1} \delta = \frac{1+\alpha\varepsilon_z^2/\eta + \alpha_b/\eta_e Z_1}{B}$$

Here $(n_1, v_1), (n_2, v_2)$and(n_b, v_b) are the densities and fluid velocities of positive and negative ion species and electron beam respectively. $n_1{}^{(0)}, n_2{}^{(0)}, n_b{}^{(0)}$ are the equilibrium densities of two ion components and beam respectively. In equations (2) to (4), velocities (v_1, v_2, v_b), ϕ (potential), time(t) and space coordinates (x) have been normalized with respect to the ion- acoustic speed in the mixture, $C_s = (\frac{T_e \delta Z_1}{m_1})^{\frac{1}{2}}$, thermal potential T_e/e, inverse of ion plasma frequency is the mixture $\omega_{pi}{}^{-1} = (\frac{m_1}{4\pi n_{e0}\delta Zl})^{\frac{1}{2}}$, Debye length $\lambda_D = (\frac{T_e}{4\pi n_{eo} e^2})^{\frac{1}{2}}$ respectively. Ion densities (n_1, n_2, n_b) are normalized with their corresponding equilibrium values, whereas electron densities are normalized by$n_e{}^{(0)}$.

To study small but finite amplitude ion acoustic solitary waves in our multispecies plasma model, we construct here a weakly nonlinear theory of ion-acoustic waves which leads to scaling of the independent variables through the stretched co-ordinates ξ and τ as:

$$\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t) \quad (5)$$
$$\tau = \varepsilon^{3/2} t \quad (6)$$

Where ε is small parameter measuring the weakness of the dispersion and λ is the phase velocity of wave. Now to strike balance between nonlinear and dispersive terms, we use reductive perturbation technique where we expand all dependent quantities in equations (2) to (4) around the equilibrium values in power of ε in the following form:

$$\begin{pmatrix} n_j \\ v_j \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ k \\ 0 \end{pmatrix} + \sum_{r=1}^{\infty} \varepsilon^r \begin{pmatrix} n_j^{(r)} \\ v_j^{(r)} \\ \phi^{(r)} \end{pmatrix} \quad (7)$$

Here $k=0$ for positive and negative ions and $k=v_0$ for electron beam, is the initial electron beam velocity. Using equations (5), (6) and (7) into Poisson's equation (4), to the lowest order of ε, we get the following dispersion relation

$$\frac{2(\eta + \alpha\varepsilon_z^2)}{\eta\delta\lambda^2 Z_1 B} + \frac{2\alpha_b}{(v_0 - \lambda)^2 \eta_e \delta Z_1} - q = 1 = F(\lambda) \quad (8)$$

The dispersion relation (8) is a four degree polynomial in λ thereby giving four modes propagating with different phase velocity. It may be further mentioned that in the limit q→1, our expression for phase velocity reduces to that of Yadev *et. al.* [4] and Bala *et. al.* [5] for $\beta = 0$.

To the next power in ε, after a long algebraic but straightforward manipulations, the following nonlinear equation known as Korteweg-de Vries (KdV) equation is obtained.

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)} \frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \quad (9)$$

Where A=2Q/P is nonlinearity coefficient and B=2/P is dispersion coefficient. Also coefficient of $\partial\phi^{(1)}/\partial\tau$

$$P = \frac{4\delta\lambda Z_1^2}{B}\left(\frac{\eta + \alpha\varepsilon_z^2}{(\eta\delta Z_1\lambda^2)^2}\right) - \frac{4\alpha_b(v_0 - \lambda)}{(\eta_e\delta Z_1(v_0 - \lambda)^2)^2} = -\frac{dF}{d\lambda} \quad (10)$$

$$Q = \left(\frac{3\eta\delta\lambda^2(1 - \alpha\varepsilon_z^3)}{B(\eta\delta\lambda^2)^3}\right) - \frac{\alpha_b 3\eta_e\delta(v_0 - \lambda)^2}{(\eta_e\delta(v_0 - \lambda)^2)^3} + \frac{(3-q)(q+1)}{4} \quad (11)$$

3. DISCUSSION

The critical beam velocity is found numerically from equations (8) and (10) with the condition that $\frac{\partial F}{\partial\lambda} = 0$ for F(λ)=1 [4]. The analytical expression for v_{cr} is given by:

$$v_{cr} = \sqrt{\frac{2}{(1+q)\delta B}}\left(\left(\frac{\alpha_b B}{\eta_e Z_1}\right)^{1/3} + \left(\frac{\eta + \alpha\varepsilon_z^2}{\eta}\right)^{1/3}\right)^{3/2} \quad (12)$$

It is clear that critical beam velocity depends on electron beam concentration α_b, negative ion concentration α, mass ratios η and $\eta_e$respectively. It may be noted that in the Maxwellian limit (q→ 1), our impression for v_{cr} becomes same as that of Bala et. al. [5] for β=0, where nonthermal electrons were reported.

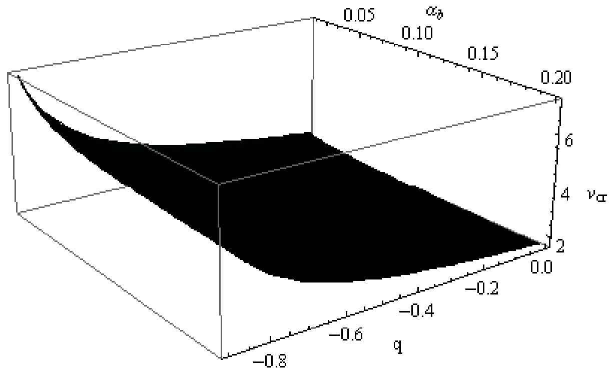

Fig 1: For the range q<0, 3D plot of critical velocity as a function of beam density α_b and nonextensive parameter q with α=0.1, η=1, η_e=1/1836 and ε_z=Z_1=Z_2=1

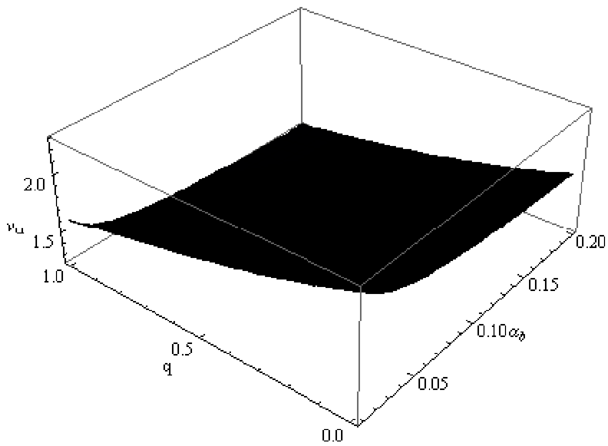

Fig 2: For the range 0<q<1, 3D plot of critical velocity as a function of beam density α_b and nonextensive parameter q with α=0.1, η=1, η_e=1/1836 and ε_z=Z_1=Z_2=1

For $H^+H^-(\eta = 1)$ plasma system, the 3D variation of critical velocity v_{cr} as a function of nonextensive parameters q and electron beam density α_b in figure 1 (for q<0), figure 2 (for 0<q<1) and figure 3 (for q>1) respectively.

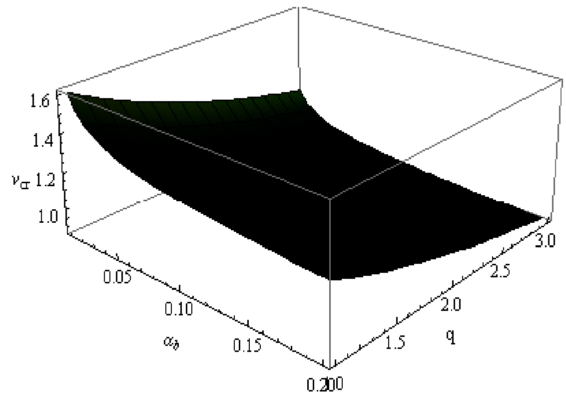

Fig 3: For the range q>1, 3D plot of critical velocity as a function of beam density α_b and nonextensive parameter q with α=0.1, η=1, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

The value of v_{cr} decreases with nonextensivity q for all three ranges. Further v_{cr} is also found to decrease with the electron beam density α_b. This behavior is opposite to that observed by Yadev et. al. [4] and Bala et. al. [5]. Hence all ranges of q predict the similar behaviour as is clear from figures 1, 2, 3 respectively. A similar kind of behavior has been observed for the plasma systems $H^+O_2^-$ (η=32) and Ar^+F^- (η=0.476) (not shown here).

In order to investigate the effect of mass of negative ions on the critical velocity we have plotted v_{cr} vs α_b for three different mass ratios i.e η=0.476, η=1, η=32 (figure 4). The other parameters are taken as $\eta_e = \frac{1}{1836}$, q=1.3(>1). The critical beam velocity is found to decrease with increase in mass of negative ions.

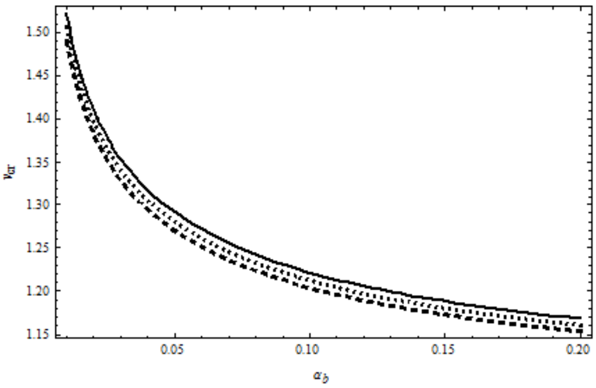

Fig 4: Variation of critical velocity v_{cr} with α_b for three different mass ratios i.e η=0.476 (solid curve), η=1 (dotted curve), η=32 (dashed curve), $\eta_e = \frac{1}{1836}$, $q = 1.3$, α=0.1, and $\varepsilon_z=Z_1=Z_2=1$

From boundary conditions [4], the soliton existence condition is $u\frac{\partial F}{\partial \lambda} < 0$. Here, u is small constant velocity. Taylor expansion gives the velocity of soliton M=λ+u as:

$$F(M) = F(\lambda) + u\frac{dF}{d\lambda} \tag{13}$$

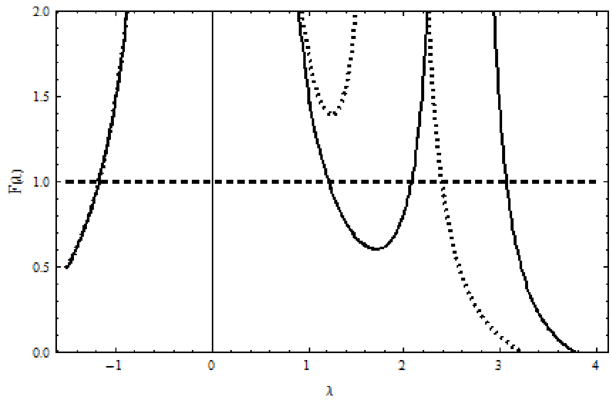

Fig 5: Plot of function F(λ) as a function of λ for α=0.8, $\alpha_b = 0.001$, η=0.476, q=0.3, $v_0 = 2.6$ (dotted), v_0 = 1.9 (dashed), solid line corresponds to F(λ)=1.

The absence of convective instability of a linear wave is described by the condition F(M)<1 [4]. Zank and Mckenzie [23] were the pioneer in the putting forward similarity of two criteria. Further F(M)=F(λ)=1 corresponds to the dispersion curve of linear wave. In figure 5, A plot of function F(λ) as a function of λ have been given where F(λ)=1 corresponds to dispersion relation. Here F(λ) intersects the line F(λ)=1 at four different places for $v_0 > v_{cr}$, thereby indicating the existence of four real roots that correspond to four linear modes. Among them one mode corresponds to wave mode moving in negative direction. Two of them corresponds to slow modes with phase velocities smaller than drift velocity and fourth one corresponds to faster mode with $\lambda > v_0$.

From figure 5, we see that for the fast mode, dF/dλ (slope) is negative and soliton corresponding to this mode moves with supersonic speed. Out of two slow modes, one moves with subsonic speed and the other move with supersonic speed based on the sign of dF/dλ. However, for $v_0 < v_{cr}$, (dashed lines) two real roots are possible, other two may be complex ones. These four modes corresponds to four solitons branches that may be of rarefactive or of compressive type. The next step in the present investigation is to study the effect of q-parameter on the soliton profile of four branches.

4. CONCLUSION

In the present investigation, effect of drift velocity of electrons in multi-ion plasma system has been presented. The electrons are taken as non extensive ones. The dispersion relation and KdV equation has been derived using standard reductive perturbation method. The critical velocity of electron beam was found to be a function of non extensivity q, beam concentration α_b, negative ion concentration α, mass ratios η and η_e respectively. It is found that above a critical velocity of electron beam, four soliton branches appear corresponding to four different modes. Out of which, three move with supersonic velocity while one moves with subsonic speed based on the sign of dF/dλ. Further, the critical velocity decreases with non extensive parameter q and electron beam concentration α_b.The present work is further extended to study the soliton profiles of four branches $v_0 > v_{cr}$.

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6. REFERENCES

[1] Dodd, R. K.,Eilbeck, J. C., Gibbon, J. D. and Morris, H. C.1984Solitons and Nolinear wave equation Academic Press, New York.

[2]Das, G. C. 1979 Ion-a cousticsolitons and shock waves in multicomponent plasmas. Phys Plasmas21, 257-265.

[3] Nejoh, Y. 1996 Large amplitude ion-acoustic waves in a plasma with a relativistic electron beam. Plasma Phys.56**,** 67-76.

[4] Yadav, L.L., Tiwari R.S. and Sharma S. R.1994 Ion-acoustic compressive and rarefactivesolitons in a electron beam plasma system.Phys Plasmas1,559-566.

[5] Bala P., Gill, T. S. and Kaur H. 2010 Localized nonlinear electrostatic structures in a multispecies plasmas. J. Phys. ConferenceSeries208**,** 012076.

[6]Moslem, W.M. 1998 Propagation of ion acoustic waves in a warm multicomponent plasma with an electron beam. J. Plasma Phys., 61, 177-189.

[7]Bethomier, M., Pottelette, R., Malingre, M. and Khotyaintsev, Y. 2000 Electron-acoustic soliton in electron beam plasma system. Phys. Plasmas 7, 2987-2994.

[8]El-Taibany, W.F. and Moslem, W.M. 2005Higher order nonlinearity of electron-acoustic solitary waves with vortex like electron distribution and electron beam.Phys.Plasmas 12, 032307.

[9] Esfandyari, A.R., Kourakis, I. and Shukla, P.K. 2008 Ion acoustic waves in a plasma consisting of adiabatic warm ions, nonisothermal electrons and a weakly relativistic electron beam:Linear and higher order nonlinear effects. Phys.Plasmas 15, 022303.

[10] Singh,S.V., Lakhiana, G.S., Bharuthramand, R. and Pillay,S.R. 2011 Electrostatic solitary structures in presence of non-thermal electrons and a warm electron beam on the auroral field lines. Phys. Plasmas18,122306.

[11] Swider, W. 1988 Ionospheric Modeling edited by J.N. Korenkov, Birkhauser, Basel 403.

[12] Tsallis, C. 1988 Possible generalization of Boltzmann-Gibbs entropy. J.Stat.Phys.52, 479.

[13] Zhipeng, L., Liyan, L., and Jiulin, D. 2009 A nonextensive approach for the instability of current-drive ion-acoustic waves in space plasma.Phys. Plasmas 16, 072111.

[14] Rossignoli, R. and Canosa, N. 1999 Non additive entropies and quantum statistics. Phys.Lett.A281,148-153.

[15] Abe, S., Martinrez, S.and Pennini, F. and Plastino, A. 2001 Nonextensive thermodynamic relations. Phys. Lett. A281, 126-130.

[16] Wada,T.,2002On the thermodynamic stability of Tsallis entropy. Phys.Lett.A.297, 334-337.

[17] Reynolds,A.M. and Veneziani, M.2004 Rotational dynamics of turbulence and Tsallis statistics. Phys.Lett.A.327, 9-14.

[18] Sattin, F. 2005 Non-Extensive Entropy from Incomplete Knowledge of Shannon Entropy. Phys.Scripta71, 443-446.

[19] Wu, J. and Che, H. 2007 Fluctuation in nonextensive reaction–diffusion systems. Phys.Scripta75,722-725.

[20] Tribeche, M. and Djebarni, L. and Amour, R. 2010 Ion acoustic solitary waves in a plasma with a q-nonextensive electron velocity distribution. Phys.Plasmas 17, 04211

[21] Bains, A.S., Tribeche, M. and Gill, T.S. 2011 Modulational instability of ion acostic waves in a plasma with q-nonextensive electron velocity distribution. Phys. Plasmas. 18, 022108.

[22]Akhtar, N. and Al-Taibany, W.F. and Mahmmod, S. 2013 Electrostatic double layers in a warm negative ion plasma with nonextensive electrons. Phys.Lett.A, 377, 1282-1289.

[23] Zank, G.P. and Mckenzie, J.F. 1988 Solitons in an ion-beam plasma. J. Plasma Phys 39,183-191.