Social Network as a Complex Network Modeling

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Abstract
Social Network Analysis is current emerging area of importance in finance, politics, defense and security. Complexity of social network is tried to explain with random graph models to understand dynamics of social behavior. Synchronization and randomness are two opposite attributes of social dynamics are discussed with their importance and significance in social network analysis.

Keywords
Social Network Analysis; Random Graph Model; Social Dynamics.

[1] Introduction
A graph is a collection of objects and these objects are connected to one another with the help of links. In terms of mathematics objects are called vertices and the links that connects these vertices are called edges. A graph is an ordered pair G= (D, H) comprising a set 'D' of vertices and a set 'H' of edges [1].

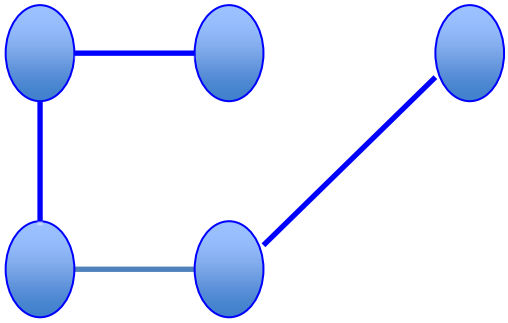

Fig. 1: Simple graph

Social Network is also a form of graph in which nodes represent the system components (person, groups) and edges are the links between these components, in general these links formed a social network.

Graphs can be defined in many ways on the basis of their properties and these properties are weights associated with edges, direction of edges, in degree, out degree, etc.

A. Types of Graph

1) Undirected Graph (Independent Interaction)
An undirected graph is a graph in which edges have no direction. An undirected graph is that in which the interaction of node i with node j also implies that j interact with i also.

Fig. 2: Undirected graph

2) Directed Graph (Bounded Interaction)
Directed graph is a graph in which interaction takes place in a specific direction. It is an ordered pair G=(D,H) a node i interacts with the node j does not means that node j also points to that node.

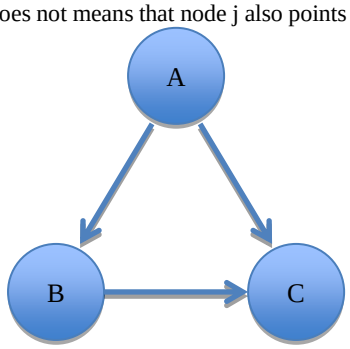

Fig. 3: Directed graph

3) Weighted Graph (Strength of Relation)
A graph is a weighted graph if its edges contain some values called weights. These values can be seen as cost, length, capacity, etc. depending on the problem.

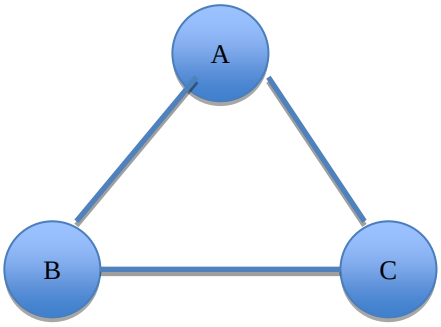

Fig. 4: Weighted graph

4) *Mixed Graph (Multiway Relation)*
It is a graph in which some edges are directed and some are undirected.

B. Topology

1) Degree

The degree or connectivity d_i of a node is the number of edges connected to it and it is defined with the help of adjacency matrix A as

$$d_i = \sum_{j \in D} a_{ij} \quad (1)$$

The degree of a node is calculated in two parts when the graph is directed, the number of outgoing links $d_i^{out} = \sum_j a_{ij}$ which is defined as out degree of that node and the number of ingoing links $d_i^{in} = \sum_j a_{ji}$ referred to as in degree of that node.
The total degree [2] is the summation of in degree and out degree$d_i = d_i^{out} + d_i^{in}$. (2)

2) *Degree Distribution*

The degree distribution P(d) [3] of a graph or network is defined as the fraction of nodes in that network with degree d. Suppose if there are 'r' nodes in a network and r_d nodes having the degree d, then degree distribution is $P(d) = \frac{r_d}{r}$. (3)

3) *Expected mean degree*

For analyze the network we have to calculate expected mean degree.

$$EMD = \sum_{d=0}^{m} d \binom{m}{d} p^d (1-p)^{m-d} = mp \quad (4)$$

4) *Variance of degree distribution*

$$Var\big(d(v)\big) m(m-1)p^2 = \sum_{d=0}^{m} d^2 \binom{m}{d} p^d (1-p)^{m-d} \quad (5)$$

An important use of degree correlation in a given network is to measure the average nearest neighbor degree of a node i

$$d_{nn,i} = \frac{1}{d_i} \sum_{j \in d_i} d_j = \frac{1}{d_i} \sum_{j=1}^{d} a_{ij} d_j \quad (6)$$

5) *Clustering Coefficient*

The measure of the degree to which nodes in graph tent to cluster together is known as clustering coefficient. A cluster of nodes having more than two nodes can be used to find cluster coefficient. If in a cluster three nodes are such connected that they are forming a triangle then it is called a triplet. A triplet is of two types open triplet and closed triplet. In open triplet, the three nodes are connected by two edges whilein close triplet there are three edges to connect three vertices. The Global clustering coefficient is calculated as the number of closed triplets from all (closed or open) triplets in a graph.

6) Average Path Length

Average path length is calculated by finding shortest path between all pair of nodes, then add the values of all these shortest path and divide by number of pairs of node.

7) Connectedness

a) Giant Component

A graph is composed of nodes and edges. Giant Component is defined as a single connected component which contains most of the nodes present in the network.

b) Connected Component

- Weakly connected component- It simply shows a path from any node to any other node in the graph.
- Strongly connected component- It shows the directed path from a node to any other node.

[2] Social Network as a Complex Network Modeling

Social Network is random complex network which has dynamic structure and random behavior (functions).

Only structure of Network is not sufficient for analyzing the Social Network, while functions are also important. So the collections of functions and structural attributes used to analyze the social network correctly. Social network is similar to neuronal system. So Social Network can be analyzed as neuronal system analyzed by using network topologies and dynamic behavior. But the challenge is to synchronize the topologies and behavior of social network, so there isneed to study of synchronization process.

A. Synchronization Processes

Synchronization process [2] provides the technique of coupling the network structure and functions. Like in Neuronal system synchronization system also applied for coupling billions of neurons connected with each other and makes a complete network. Social Networking is also a complex network like neuronal system i.e. social network has complex structure, sharing small world and scale free features. Neuronal System is composed by two types of neurons excitatory principal cells and inhibitory interneurons. Synchronization system used to understand the relation between complex network structure i.e. link structure and dynamic behavioral properties of complex networks. After studying many researchers work it can be calculated that synchronization is highly dependent on degree of the nodes in the network and independent network size[15]

1) Synchronization of Kuramoto Oscillators

Kuramoto Oscillator is the simplest and most efficient model for ynchronization of complex network like social network. For pplying this model, it assumes that each network component is an oscillator and the information exchange between them follows the ule of Kuramoto Model.[14]

$$\frac{d\theta_i}{dt} = \omega_i + \sum_j \sigma_{ij} a_{ij} \sin\big(\theta_j - \theta_i\big) \; (i = 1, \ldots\ldots, N). \quad (7)$$

$$r_{link} = \frac{1}{2N_l} \sum_i \sum_{j \in \Gamma_i} \left| \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{t_r}^{t_r + \Delta t} e^{i[\theta_i(t) - \theta_j(t)]} dt \right| \quad (8)$$

B. Network Models

1) *Erdos-Renyi Random Graph Model*

This Model is used for generating random graphs in which edges are set between nodes with equal probability. Two parameters are used to generate this model [4].
N-number of nodes in the graph generated.
p-probability that a link is formed between any two nodes.
Now we can derive a constant k with these parameters $k = {}^{2E}/_{N} = p(N-1)$
Since there is no bias for a particular node in this model, the degree distribution is binomial

$$P(\deg(v) = d) = \binom{n-1}{d} p^k (1-p)^{n-1-k}$$

2) Watts-Strogatz Small World Model

This model generates the graph with small world property. Each node is initially linked to its <k> closest neighbors. Each edge has a probability p that it will be rewired to the graph as random edges.Expected number of rewired links is $pE = pN < k > /$ 2.This model have very high clustering coefficient.

3) Barabasi-Albert (BA) Preferential Attachment Model

This model shows "rich get richer" effect. An edge is most likely to attach to nodes with higher degrees. In BA model, new nodes are added to network one at a time. Each new node is connected to m

existing nodes with probability $P_i = \frac{k_i}{\sum_j k_j}$ where k_i is the degree of node i.

[3] Modeling in Different Applications

1) Friendship network analysis

Now a day the number of students studying in abroad increases day by day. With the help of SNA we can analyze the friendship pattern of these students who are studying in other country. They show a unique friendship combination by making friends in their home country, host country and in other countries also. SNA helps to analyses the homesickness, social connectedness, satisfaction and contentment. Similar work is done by B. Hendrickson et al [5].

2) Analysis of a Concept Map

A group of people or stakeholders working on a particular issue have many thoughts, ideas and actions which can be visually represented with the help of Concept mapping. Similarly Social Network analysis is also a method of visual representation of data with the help of network in which people are considered as nodes and connections can be made between those people working on the same issue. While the motive of both the methods is different but the data structure used is same. This concept is also discussed by Daniel HcLindenc [6].

3) SNA used in Downloading

These days many website contain malicious content which can infect a user's personal computer who visit those websites. This becomes a major threat to sensitive information and data. Since many websites contain dynamic content, disabling all of them is not the solution. Social Network Analysis can play an important role to overcome this problem. The URL given by the user is examined and its reputation is checked by calculating number and quality of links on the web point back to that url. If the reputation level reaches to that threshold value then it mark that url as trusted and do not block its dynamic content [7].

4) Health

The study of relationship links (patterns) among actors in the network is known as Social Network Analysis. SNA helps in health department also by making network of doctors, nurses, technicians, hospital staffs and management staff, medical stores etc. In these kinds of network relations build according to the authority or exchange of resources like information, medicines, money etc. Network analysed by separating the network in different levels and representing network in matrix form. This is mainly done by analyzing the information content (like email, any details of patients, medicine etc) flow over network by maintaining log of numbers, lengths and timing of message. Many researcher work in area like J.G. Aaderson[8], A.G. Dunn et al [9], G Fattore et al [10], F. Griffiths et al [11].

5) Social Networking sites

It analyze the social Networking Sites by taking some set of data by conducting surveys, interviews and other collection procedures and analyze that data by applying Social Network Analyses methods. Many Social Networking Sites like Facebook is used for research like A. Mayer et al [12], Charles Steinfield et al[13].

[4] Conclusion

Currently SNA (Social Network Analysis) is proved to be important and interdisciplinary research area in every event of life i.e. politics, entertainment, marketing, defence, anti terrorism, etc. Challenge of SNA is the dynamics of society which changes with space and time. Complexity and dimensionality of social network also play vital role in analysis. Social dynamics is governed by random behavior of elements of complex network. Social network is a diverse field which has many application areas. So, there is a lot of space to work in SNA. There are the challenges of dynamicity of these models. All the above models are not perfect , all of them have some limitations like some model are used only for small size network and mostly models can not make exactly same random graph as real world graphs. So these are some issues in which future work would be done.

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