International Journal of Computer Applications (0975 – 8887)
Advanced Computing and Communication Techniques for High Performance Applications (ICACCTHPA-2014)

A Particle Filter based Neural Network Training Algorithm for the Modeling of North Atlantic Oscillation

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ABSTRACT
Chaotic dynamical systems are present in the nature in various forms such as the weather, activities in human brain, variation in stock market, flows and turbulence. In order to get a detailed understanding of a system, the modeling and analysis of the system is to be done in an effective way. A recurrent neural network (RNN) structure has been designed for modeling the dynamical system. The neural network weights are estimated using the Particle Filter algorithm. There are various natural systems, which can be represented by chaotic dynamical systems. But closed form mathematical equations for such systems are not readily available for generating such time series. The North Atlantic oscillations are one such system which is modeled with the selected RNN model structure and Particle Filter algorithm. While the model faithfully reproduces the given time series, the phase plane generated unravels the dynamics of the system. The characterization of the natural chaotic systems is done in the time domain by Embedding Dimension, Phase plots and Lyapunov Exponents.

General Terms
Dynamical systems, modeling, natural systems, neural network, algorithms, Artificial Neural Network, Recurrent Neural Networks, Particle Filter

Keywords
Chaotic systems, North Atlantic Oscillation, Embedding dimension, Lyapunov exponent, Phase plots,

1. INTRODUCTION
The chaotic systems are of interest to many researchers over the years. Chaotic systems are complex and unpredictable phenomena which occur in nonlinear systems which are sensitive to initial conditions [1]. The modeling of chaotic systems, based on output time series is quite challenging. Fortunately artificial neural networks have the required self-learning capability to tune the network parameters (ie. weights) to identify highly nonlinear systems [2]. There are a number of weather systems which are chaotic in nature. The Sunspot time series, the Sea clutter, the Venice. lagoon time series, the North Atlantic Oscillations etc are a few examples. In the present paper the North Atlantic Oscillations is modeled and analyzed with a Recurrent Neural Network (RNN) model trained with Particle Filter algorithm According to the universal approximation theorem [3] any non-linear dynamical system can be approximated to any accuracy by a recurrent neural network, with no restrictions on the compactness of the state space, provided that the network has enough sigmoidal hidden units. From the analysis important features are extracted and certain observations are made on the three weather systems which will contribute to the further analysis of these systems.

2. RECURRENT NEURAL NETWORKS
Recurrent neural networks could be built with multi-layer networks, adding feedback in their hidden layer. The Information flow is multidirectional. Such networks inherently possess sense of time and memory. Hence the networks could be used in creating models of highly nonlinear and chaotic systems [4]. The simplest form of fully recurrent neural network simply has the previous set of hidden unit activations feeding back into the network along with the inputs. Any non-linear dynamical system can be approximated to any accuracy by a recurrent neural network, with no restrictions on the compactness of the state space, provided that the network has enough sigmoid hidden units. This underlies the computational power of recurrent neural networks. The recurrent networks have the potential to be used in unison in systems with dynamic elements and feedback [5] feedback.

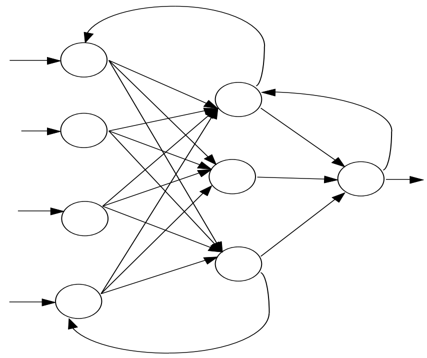

Figure 1: Recurrent neural network

3. TRAINING THE RNN
There are many efficient training algorithms for training RNN. Back propagation, Least mean square algorithm ,conjugate gradient, Kalman Filter, Particle Filter etc are a few to mention [6] [7]. In the work reported here the Particle Filter algorithm along with Sampling Importance Resampling(SIR) technique is tested for chaotic system modeling. The parameters of the neural network are estimated using the SIR Particle Filter algorithm, by choosing the weights of the neural network as the particles.

3.1 System representation
Consider a discrete time non linear dynamic system, described by a vector difference equation with additive white Gaussian noise that models “unpredictable” disturbances. The dynamic system equation is given by the following nonlinear equations

$$x_k = f(x_{k-1}, u_k, w_{k-1}) \tag{1}$$

where x_k is an n dimensional state vector u_k, is an m dimensional known input vector, and w_k is a sequence of independent and identically distributed zero mean white Gaussian process noise with covariance

$$E(ww^T) = Q \quad (2)$$

The measurement equation is

$$z_k = h(x_k, v_k) \quad (3)$$

where v_k is the measurement noise with covariance

$$E(vv^T) = R \quad (4)$$

The functions f and h and the matrices Q and R are assumed to be known. The neural network models the chaotic time series and the state variables continue to generate the state space evolution of the system, responsible for generating the time series. Once the training error comes down to an acceptable limit, the system driven by exogenous noise free wheels to generate the state space. The Lyapunov exponents, which characterize the behavior of the system are also calculated from the state space evolution and verified. The presence of feedback loops has a profound impact on the learning capability of the network and its performance.

3.2 Particle filters

Particle filters are suboptimal filters. They perform Sequential Monte Carlo (SMC) estimation based on point mass (or "particle") representation of probability densities. The SMC ideas in the form of sequential importance sampling had been introduced in statistic back in the 1950s. Although these ideas continued to be explored during the 1960s and 1970s, they were largely overlooked and ignored. Most likely the reason for this was the modest computational power available at the time. In addition, all these early implementations were based on plain sequential importance sampling, which degenerates over time. The major contribution to the development of the SMC method was the inclusion of the re-sampling step, which, coupled with ever faster computers, made the particle filters useful in practice. [8] Since then research activity in the field has dramatically increased, resulting in many improvements of particle filters and their numerous applications.

3.2.1 Monte Carlo Integration

Monte Carlo integration is the basis of SMC methods. Suppose we want to numerically evaluate a multidimensional integral

$$I = \int g(x)dx \quad (5)$$

Where . $x \in R^{nx}$. Monte Carlo (MC) methods for numerical integration factorize g(x) =f(x).π(x) In such a way that π(x) is interpreted as a probability density satisfying π(x) ≥0 and $\int$ π(x) dx=1. The assumption is that it is possible to draw N>>1 samples {x^i; i = 1… N}distributed according to π(x). The MC estimate of integral

$$I = \int f(x)\,\pi(x)\,dx \quad (6)$$

Is the sample mean

$$I_N = \tfrac{1}{N}\sum_{i=1}^{N} f(x^i) \quad (7)$$

If the samples x^i are independent then I_N is an unbiased estimate and according to the law of large numbers I_N will almost surely converge to I. If the variance of f(x),

$$\sigma^2 = \int (f(x) - I)^2 \pi(x)dx \quad (7)$$

Is finite, then the central limit theorem holds and the estimation error converges in distribution:

$$\lim_{n\to\infty}\sqrt{N}(I_N - 1) \sim N(0,\sigma^2) \quad (8)$$

The error of the MC estimate, e = I_N-I, is of order 0($N^{-1/2}$), meaning that the rate of convergence of the estimate is independent of the dimension of the integrand. In contrast, any deterministic numerical integration has a rate of convergence that decreases as the dimensions n_x increases. This useful and important property of MC integration is due to the choice of samples {x^i, i =1… N}, as they automatically come from regions of the state space that are important for the integration result. In the Bayesian estimation context, density π(x) is posterior density. Unfortunately, usually it is not possible to sample effectively from the posterior distribution, being multivariate, nonstandard, and only known up to a proportionality constant [8]. A possible solution is to apply the importance sampling method.

3.2.2 Importance Sampling

Ideally we want to generate samples directly from π(x) and estimate I using (7). Suppose we can only generate samples from a density q(x), which is similar to π(x). Then a correct weighting of the sample set still makes the MC estimation possible. The PDF q(x) is referred to as the importance or proposal density. Its "similarity" to π(x) can be expressed by the following condition:

$$\pi(x)>0 \Rightarrow q(x)>0 \text{ for all } x \in R^{nx} \quad (9)$$

which means that q(x) and π(x) have the same support. I can be written as

$$I = \int f(x)\pi(x)dx = \int f(x)\left(\tfrac{\pi(x)}{q(x)}\right)dx \quad (10)$$

provided that π(x)/q (x) is upper bounded. A Monte Carlo estimate of I is computed by generating N>>1 independent sample { x^i ,i = 1… N}distributed according to q(x) and forming the weighted sum:

$$I_N = \tfrac{1}{N}\sum_{i=1}^{N} f(x^i)\,\overset{"}{w}(x^i) \quad (11)$$

where $\overset{"}{w}(x^i) = \frac{\pi(x)}{q(x)}$ are the importance weights. If the normalizing factor of the desired density π(x) is unknown, we need to perform normalization of the importance weights. Then we estimate I_N as follows;

$$I_N = \frac{\frac{1}{N}\sum_{i=1}^{N} f(x^i)\overset{"}{w}(x^i)}{\frac{1}{\frac{N}{N}\sum_{i=1}^{N} i\,\overset{"}{w}(x^i)}} \quad (12)$$

where the normalized importance weights are given by:

$$w(x^i) = \frac{\overset{"}{w}(x^i)}{\sum_{j=1}^{N}\overset{"}{w}(x^j)} \quad (13)$$

This technique is applied to the Bayesian framework, where $\pi(x)$ is the posterior density.

3.2.3 Sequential Importance Sampling (SIS)

Importance sampling is a general MC integration method that we now apply to perform nonlinear filtering specified by the conceptual solution. The resulting sequential importance sampling (SIS) algorithm is a Monte Carlo method that forms the basis for most sequential MC filters developed over the past decades; this sequential Monte Carlo approach is known variously as bootstrap filtering, the condensation algorithm, particle filtering, interacting particle approximation, and survival of the fittest. It is a technique for implementing a recursive Bayesian filter by Monte Carlo simulations. The key idea is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these examples and weights. As the number of samples becomes very large, this Monte Carlo characterization becomes an equivalent representation to the usual functional description of the posterior PDF, and the SIS filter approaches the optimal Bayesian estimator:

SIS Particle Filter Algorithm [9]

$$\begin{aligned}
&\textit{At time } n = 1 \\
&\textit{Sample } X_1^i \sim q_1(x_1) \\
&\textit{Compute the weights } w_1(X_1^i) \textit{ and } w_1(X_1^i) \\
&\textit{At time } n \geq 2 \\
&\textit{Sample } X_n^i \sim q_n(x_n \mid X_{1:n-1}^i) \\
&\textit{Compute the weights} \\
&w_n(X_{1:n}^i) = w_{n-1}(X_{1:n-1}^i)\alpha_n(X_{1:n}^i) \\
&w_n^i \propto w_n(X_{1:n}^i)
\end{aligned}$$

Algorithm .1

3.2.4 Resampling

SIS provides estimates whose variance increases with 'n'. Employing the technique of resampling this problem can be solved. Consider an IS approximation $\hat{\pi}_n(x_{1:n})$ of the target distribution $\pi_n(x_{1:n})$. This approximation is based on the weighted samples from $q_n(x_{1:n})$. This approximation does not provide samples distributed according to $\pi_n(x_{1:n})$. To obtain approximate samples from $\pi_n(x_{1:n})$, sample from its IS approximation $\hat{\pi}_n(x_{1:n})$. This operation is called resampling as it corresponds to sampling from an approximation $\hat{\pi}_n(x_{1:n})$ which was itself obtained from sampling. In order to obtain N samples from $\hat{\pi}_n(x_{1:n})$ resample N times from $\hat{\pi}_n(x_{1:n})$ and associate a weight of $\frac{1}{N}$ with each sample. The approximate measure of $\hat{\pi}_n(x_{1:n})$ is given by

$$\bar{\pi}(x_{1:n}) = \sum_{i=1}^{N} \frac{N_n^i}{N} \, \delta x_{1:n}^i \; (x_{1:n}) \tag{14}$$

Sequential Importance Sampling – Resampling (SIR) Particle Filter Algorithm [10]

$$\begin{aligned}
&\textit{At time } n = 1 \\
&\textit{Sample } X_1^i \sim q(x_1 \mid y_1) \\
&\textit{Compute the weights } w_1(X_1^i) = \frac{\mu(X_1^i) g(y_1 \mid X_1^i)}{q(X_1^i \mid y_1)} \textit{ and } W_1^i \propto w_1(X_1^i) \\
&\textit{Resample} \left\{W_1^i, X_1^i\right\} \textit{ to obtain N equally weighted particles} \left\{\frac{1}{N}, \bar{X}_1^i\right\} \\
&\textit{At time } n \geq 2 \\
&\textit{Sample } X_n^i \sim q(x_n \mid y_n, \overline{X_{n-1}^i}) \textit{ and set } X_{n-1}^i \leftarrow (\overline{X_{1:n-1}^i}, X_n^i) \\
&\textit{Compute the weights} \propto_n (X_{n-1:n}^i) = \frac{g(y_n \mid X_n^i) f(X_n^i \mid X_{n-1}^i)}{q(X_n^i \mid y_n, X_{n-1}^i)} \textit{ and } W_1^i \propto w_1(X_{n-1:n}^i) \\
&\textit{Resample} \left\{W_1^i, X_{1:n}^i\right\} \textit{ to obtain N equally weighted particles} \left\{\frac{1}{N}, \overline{X_{1:n}^i}\right\}
\end{aligned}$$

Algorithm .2

4. MINIMUM EMBEDDING DIMENSION

There are three important dimensions for a dynamic system- geometric dimension or box counting dimension, attractor dimension or state space dimension and embedding dimension (d_e). The first two are invariant sets, calculated from the dynamic equations of the system. The embedding dimension of a dynamic system is the smallest integer for which the system states can be embedded into, without intersecting itself [13]. An efficient model of a dynamic system can be derived by selecting a proper embedding dimension capable of embedding all the properties of actual dynamic system [14]. Reconstructed system may preserve only some of the properties, if the selected dimension is not optimum and does not preserve the geometric shape of structures in phase space.

Taken's theorem [13] [14] states that the original dynamic properties of the attractor can be retained as long as the embedding dimension $d_e \geq 2d+1$ where d is the correlation dimension of the attractor, equivalent to Kaplan Yorke dimension. It is sufficient to find the minimum embedding dimension so as to reconstruct the dynamic system with all the properties. The minimum embedding dimension can be obtained from the following algorithm based on The Method of False Nearest Neighbours [15]:

1. Dimension of the attractor is assumed as'd=1' and the k^{th} state is assumed as 'x(k)'.
2. Each state x(k) is sampled, in dimension 'd', into time lagged set - {(s(k), s(k+T), s(k+2T), . . . , s(k+(d-1)T)} - where T is a small time lag.
3. Each state x(k) has a Nearest Neighbour (NN), $x^{NN}(k)$ with nearness in the sense of distance function norm, i.e, $R_d^2(k) = [x(k)-x^{NN}(k)]^2$
4. $R_d^2(k)$ is calculated in terms of time lagged sets as.

$$\begin{aligned}
R_d^2(k) = {}& [s(k) - s^{NN}(k)]^2 + [s(k+T) - s^{NN}(k+T)]^2 \\
& + \ldots + \\
& [s(k+(d-1)T - s^{NN}(k+(d-1)T]^2
\end{aligned} \tag{15}$$

5. Dimension is incremented asd=d+1.Corerespondingly the new state is x(k+dT) and its nearest neighbour is x^{NN}(k+dT). Then the distance is changed due to the $(d+1)^{st}$ samples as s(k+dT) and s^{NN}(k+dT) The new distance is calculated as

$$R_{d+1}(k)^2 = R_d(k)^2 + [s(k+dT) - s^{NN}(k+dT)]^2 \quad (16)$$

   Relative change in distance can be used to check whether the points are really close together or a projection from a higher state space.

6. The criteria for false nearest neighbours is chosen as the threshold given by

$$R_T < \frac{\left|[x(k+dT) - x^{NN}(k+dT)]\right|}{R_d(k)} \quad (17)$$

Using this criterion all the sequence of samples is tested. The sample where percentage of false nearest neighbours goes to zero is calculated. A graph is plotted between the percentage of false nearest neighbours and embedding dimension. The lowest point in the graph gives the minimum embedding dimension [2].

5. PHASE PLOTS AND STRANGE ATTRACTORS

The phase plot of a dynamical system is the plot of the state variables of the system. In such a diagram time is implicit and each axis represents one dimension of the state. The trajectory traversed by the states is called a phase trajectory [1]. A phase plot for a given system may depend on the system parameters and initial conditions. The geometrical shape of phase plots give valuable information about the nature of the system. It can be points, circle-like curves called limit cycles or strange attractors. Once a trajectory enters the attractor, it will stay in it forever if there is no external perturbation. The attractor of a system is an invariant set [18]. The state space analysis is done and change in dynamics of the systems is described in the different time intervals. The states of the systems are exactly reproduced by the RNN Particle Filter model.

6. LYAPUNOV EXPONENTS (LE)

The Lyapunov Exponents of a system are a set of invariant geometric measures that describe the dynamical content of the system. It is the most important quantity of chaotic systems.. Lyapunov Exponents quantify the average rate of convergence or divergence of nearby trajectories in a global sense. A positive exponent implies divergence of trajectories and a negative one implies convergence [20]. The more positive the exponent, the faster the trajectories move apart. Similarly, for negative exponents, the trajectories move together. If there are both positive and negative exponents, it indicates that neighboring orbits separate exponentially in average, so it is a signature of chaos. [21]. The number of exponents is equal to the number of states of the system. A system with m states has m Lyapunov exponents with λ_1, $\lambda 2$,..., ,l λ_m in descending order. As such, it can be seen that the Lyapunov Exponents describe the average rate of exponential change in the distance between trajectories in a set of orthonormal directions within the embedding space sense.

Mathematically Lyapunov Exponent can be defined by

$$\lambda_i = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{i-1} \ln\left[\frac{\partial f(x_k)}{\partial x_k}\right] \quad (18)$$

7. NORTH ATLANTIC OSCILLATIONS

The NAO is characterized by an oscillation of atmospheric mass between the Arctic and the subtropical Atlantic [21]. It is usually defined through changes in surface pressure. A permanent low-pressure system over Iceland (the Icelandic Low) and a permanent high-pressure system over the Azores (the Azores High) control the direction and streng and positions of these systems vary from year to year and this variation is known as the NAO NAO measures the strength ofthe westerly winds blowing across the North Atlantic Ocean between 40°N and 60°N. Studies reveal that the NAOaccounts for 31% of the variance in hemispheric winter surface air temperature north of 20°N. There is an index for the NAO as the difference between normalised mean winter (December to February) sea level pressure (SLP)anomalies at Ponta Delgadas, Azores and Akureyri, Iceland. The normalisation is achieved by dividing the SLP anomalies at each station by the long term (1864-2014) standard deviation[28][29]. A large difference in the pressure at the two stations (NAO+) leads to increased westerlies and, consequently, cool summers and mild and wet winters in Central Europe and Atlantic areas. In contrast, if the index is low (NAO-), westerlies are suppressed, these areas suffer cold winters and storms[23]. The NAO strongly affects the Atlantic ocean by inducing substantial changes in surface wind patterns .Changes in NAO have a wide range of effects on marine and terrestrial ecosystems, including distribution and population of fish, flowering dates of plants, growth, reproduction and demography of many land animals.[25] The modelling and analysis of NAO index considered as a time series is an important problem. In the forthcoming section the modelling and analysis of chaotic origins of the NAO is presented.

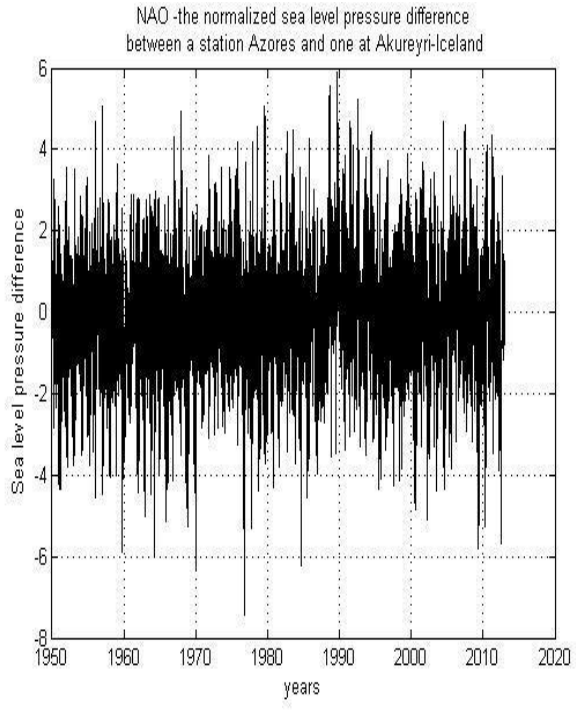

Figure 2: NAO time series

7.1 Estimation of minimum embedding dimension of NAO time series

In order to model the NAO time series with RNN Partcle Filter model structure, it is required to estimate the minimum embedding dimension. As explained in section 4 the minimum embedding dimension of the NAO timeseries is estimated using the method of false nearest neighbours.

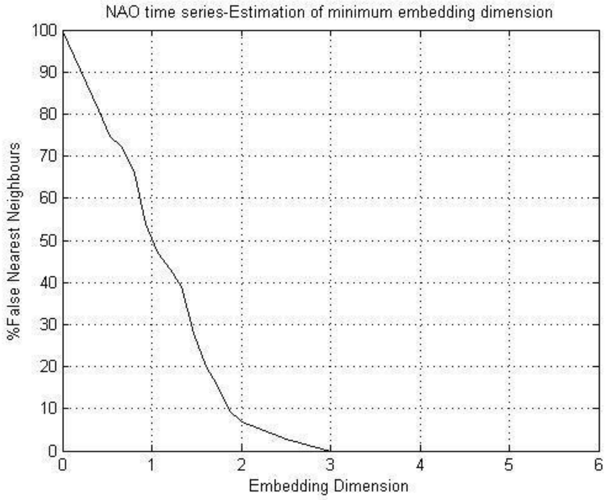

Figure 3. Minimum embedding dimension-NAO

7.2 Phase plots of NAO time series

After estimating the minimum embedding dimension the given timeseries is modeled using the RNN Particle Filter model structure. The system is modeled with three states. The recurrent neural networks are trained with a single channel time series data of the NAO time series. All the three sets of weights are updated using the SIR Particle Filter equations.. The training is continued until the modelling error comes to an appreciable level of $2.54x10^{-6}$ as shown in Figure.4 Further, the phase plots of the three states of the given time series are plotted.

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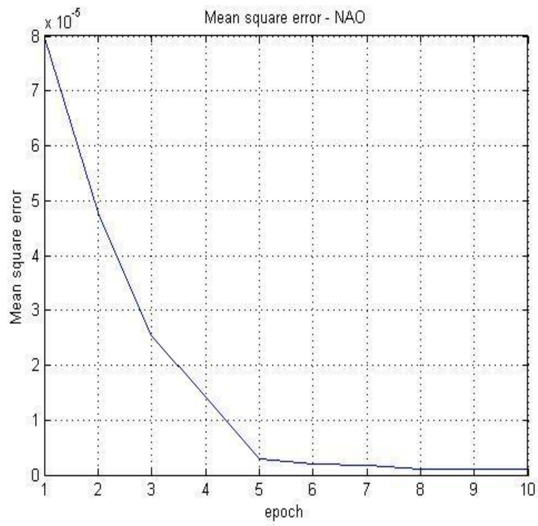

Figure 4. Mean square error-NAO

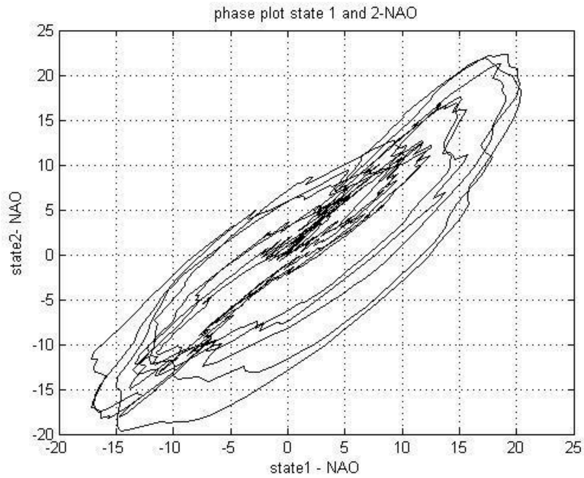

Figure 5. Phase plots: states 1&2- NAO

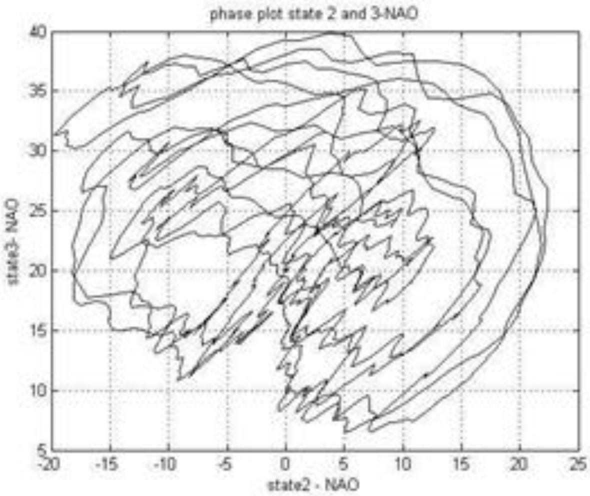

Figure 6. Phase plots: states 2&3- NAO

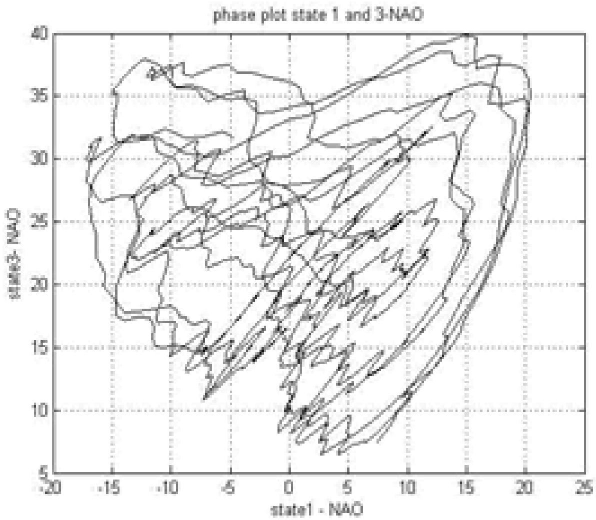

Fig.7.Phase plots states 1 and 3 NAO

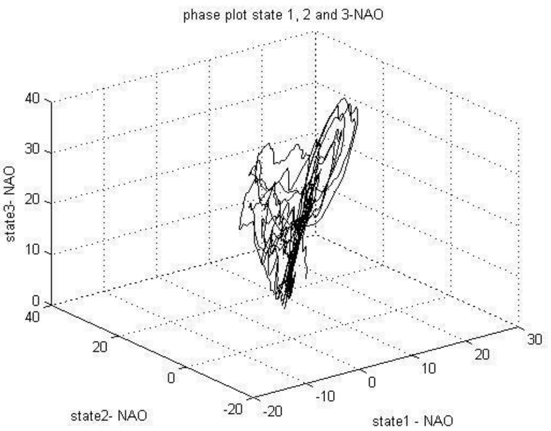

Figure 8. Phase plots: states 1, 2& 3: NAO

The phase plots of NAO time series reveals strange attractors revealing chaotic nature.

7.3 Lyapunov Exponents of NAO

The The Lyapunov Exponents of the NAO time series is calculated using the method described in section 6 and is given in table 1.

Table 1. Lyapunov Exponents: NAO

-15.2446	0	0.00127

It can be seen that one of Lyapunov exponent is negative, one is zero and the other one is positive and very small . Thus it can be proved that the NAO time series is chaotic in nature.

8. CONCLUSIONS AND FUTURE SCOPES

The analysis and characterization of an important weather system is presented in this paper. The North Atlantic Oscillations are an important atmospheric index which control the climate of Europe. The NOA is modelled using the RNN Particle Filter model structure with a very low modelling error. The important characteristics of these systems like embedding dimension, phase plots, strange attractors and Lyapunov exponents are calculated.. It is seen that the NAO index time series is found to have an embedding dimension of three. Its phase plots show strange attractors signifying a chaotic nature. While calculating the three Lyapunov exponents it was noticed that one gives a large negative value, one is zero and third one is a small positive value verifying the chaotic nature as observed from the phase plots.With the observations at hand, the analysis of NAO can be extended to prediction. Also use of other efficient learning algorithms like Rao Blackwellised particle filter, auxiliary particle filter, Kalman filter etc. can be used for training the RNN. Another important feature worth exploring is the frequency domain behaviour of chaotic systems.

9. REFERENCES

[1] Devany, " A first course in chaotic dynamical systems: theory and experiments", Persues publications, 1992 .

[2] S Chen, S A Billings, "Neural networks for nonlinear system modeling and identification", International journal of control, vol.56, issue2, pp.319-346, 1992.

[3] Simon Haykin, " Neural networks-A comprehensive foundation", 2nd edition, Pearson education, 1999.

[4] Puskorius G V, L A Feldkamp, "Neurocontrol of nonlinear dynamical systems with Kalman filter trained recurrent networks

[5] A Bullineria, " Neural computations: Lecture 2", 2014.

[6] Puskorius G V, L A Feldkamp, "Neurocontrol of nonlinear dynamical systems with Kalman filter trained recurrent networks",

[7] Si-Zhao, Quin Hong, Thomas J Mc Avoy "Comparison of four Neural network learning methods for dynamic system modeling", IEEE Transactions on neural networks, vol.3, pp 192-198, 1992

[8] A. Doucet, A. M. Johansen, "A Tutorial on Particle Filtering and Smoothing : Fifteen years later," Version 1.1, 2008.

[9] Afonso, Manya. "Particle filter and extended kalman filter for nonlinear estimation: a comparative study." IEEE Transactions on Signal Processing,pp1- 10, 2008.

[10] M. S. Arulampalam, S. Maskell, N. Gordon, T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," IEEE Transaction on Signal Processing., vol. 50, no. 2, pp. 174–188, 2002.

[11] Torma, Péter, and Csaba Szepesvári. "Combining local search, neural networks and particle filters to achieve fast and reliable contour tracking." Proceedings of the IEEE International Symposium on Intelligent Signal Processing. 2003

[12] Q. Wen and P. Qicoiig, "An improved particle filter algorithm based on neural network,"IFIP International Federation for Information Processing, vol. 228, pp. 297–305, 2006.

[13] Takens F "On the numerical determination of dimension of an attractor", Springer Lecture notes on Mathematics, vol. 898 pp.230-241, 1981

[14] Takens F "Detecting strange attractors in turbulence" Springer Lecture Notes in Mathematics", vol. 898, pp. 366–381, 1981

[15] P Hong, C Peterson. "Finding the embedding dimension and variable dependencies in time series", Neural Computation, vol.6, no.3, pp.509-520, 1994

[16] Liangyue Cao, "Practical method for determining the minimum embedding dimension from a scalar time series", Physica D: Nonlinear Phenomena, vol. 1, no.10, pp. 43-50, 1997

[17] J. C. Robinson, "Takens' embedding theorem for infinite-dimensional dynamical systems", Nonlinearity, vol2, pp. 1–10,1999

[18] Arun V. Holden, "Chaotic behavior in systems" - Manchester University Press, 1986

[19] Lennart Ljung, "System identification – Theory for the user" , Prentice Hall, 1987

[20] M. T. Rosenstein, J. J. Collins, C. J. De Luca, "A practical method for calculating largest Lyapunov exponents from small data sets", Physica D: Nonlinear Phenomina, vol.65, no.1, pp. 117-134, 1993

[21] M Bask, R Gencay, "Testing chaotic dynamics via Lyapunov exponents", Physica D, vol. 114, pp. 1–2,

1998

[22] R J Greatbatch, "The North Atlantic Oscillation," Stochastic Environmental Research and Risk assessment, no.14, pp. 213-242, 2000

[23] Collette, Christophe, M. Ausloos. "Scaling analysis and evolution equation of the North Atlantic oscillation index fluctuations." International journal of modern physics C 15.10 , pp. 1353-1366, 2004

[24] E. P. Gerber, "A Dynamical and Statistical Understanding of the North Atlantic Oscillation and Annular Modes," Ph.D Thesis, Princeton University, 2006

[25] S. M. Osprey, U. Kingdom, M. H. P. Ambaum, and U. King-, "Evidence for the Chaotic Origin of Northern Annular Mode Variability", Geophysical Research letters, 2011.

[26] Archana R, A Unnikrishnan, R Gopikakumari, " Modelling of Venice lagoon time series with improved Kalman filter based neural networks", International journal for computer applications, Special issue ACCTHPCA, 2012

[27] Archana R, A Unnikrishnan, R Gopikakumari, "Computation of state space evolution of chaotic systems from time series of output, based on neural networks", International Journal for engineering research and development, vol.2, Issue.2, pp 49-56, 2012

[28] NAO data: Climatic Research Unit, University of East Anglia, http://www.cru.uea.ac.uk/cru/data/nao/nao.dat

[29] National Oceanic and Atmospheric Administration(NOAA), United State Department of Commerce, http://oceanservice.noaa.gov