Integrated 3-echolen Supply Chain Inventory Models for Perishable Items for Quadratic Demand and Production Rate Dependant on Demand and Two Parametric Deterioration

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ABSTRACT
Supply chain inventory models (i.e. producer, distributor and retailer inventory models) have been developed for perishable items. In these models, demand rate is taken as quadratic function of time and the production rate is taken as demand dependent and variable holding cost which is realistic for newly launched products in the supermarket. Shortages are allowed and partial backlogged.

General Terms
Supply Chain Management, Inventory.

Keywords
SCM, Multi-Echolen.

1. INTRODUCTION
On the most important topics in the study of the management of contemporary manufacturing and distribution is supply chain management (SCM). The effective management of supply channel inventories is perhaps the most fundamental objective of SCM. Manufactures procure raw material and process them in to finished goods, and sell the finished goods to distributors, then to retailer and/or customer. When an item moves through more than one stage before reaching the final customer, it forms a “multi-echelon” inventory system. A large amount of researches on multi-echelon inventory control has appeared in the literature during the last decades. **Clark and Scraf (1960)** presented the concept of serial multi-echelon structures to determine the optimal policy. **Baneriee (1986)** derived a joint economic lot size model for a single vendor, single buyer system where the vendor has a finite production rate [4]. **Chakravarty and Martin (1988)** developed a scheme of joint cost sharing between the seller and the buyers. An algorithm was developed to determine both the discount price and the replenishment interval for any determining materials requirement for all materials at every stage in the supply chain.

Although a number studies have been done on the supply chain system but still most of the researchers consider a continuous decrease in the inventory levels of the producer and the distributor in spite of the fact that the lot is sent off in discrete units and not continuously. This is a major drawback in the studies done till date since, the model which is formulated in such studies is ultimately faculty due to the misleading assumptions and as a result the observation and the reading done on the basis of such model are wrong and very ambiguous.

Pake and Cohen (1993) extended the above study to consider for stochastic sub systems to explore the supply chain system. **Rau et al. (2003)** investigated a multi-echelon supply chain for a deteriorating item to derive an optimal joint total cost from an integrated perspective among the supplier, the producer and the buyer.

Furthermore, when the shortages occur, some customers are willing to wait for backorder and others would turn to buy from other sellers. **Wee (1995)** considered the constant partial backlogging rates during the shortage period in their inventory models. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not [7]. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment. **Chang and Dye (1999)** investigated an EOQ model allowing shortage and partial backlogging. It is assumed that the backlogging rate is variable and dependent on the length of the waiting time for the next replenishment. Recently, many researchers have modified inventory policies by considering the “time-proportional partial backlogging rate” such as **Abad (2000), Papachristos and Skouri (2000), Chang and Dye (2001), Wang (2002), Papachristos and Skouri (2003)** etc.

Maintenance of inventories of deteriorating items is a problem of major concern in the supply chain of almost any business organizations. In reality, the constant demand is not always applicable to real situation. For instance, it is usually observed in the supermarket that display of the customer goods in large quantities attracts more customers and generates higher demand. The holding cost is explicitly assumed to be varying over time in only few inventory functions [8]. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the holding cost will be higher.

Balki and Benkherouf (2004) consider an inventory model for deteriorating items with stock dependent and time-varying demand rates. Zhou and Yang (2005) developed two-warehouse inventory model for items with stock-level-dependent demand rate. **Ahmed et al. (2007)** have recently coordinated a two-level supply chain in which they considered production interruptions for restoring of the quality of the production process. **Chen and Kang (2007)** considered vendor buyer cooperative inventory models with variant permissible delay in payments. **Singh (2008)** assumed optimal ordering policy for decaying items under inflation.

In this paper, supply chain inventory models (i.e. producer, distributor and retailer inventory models) have been developed for perishable items. In these models, demand rate is taken as quadratic function of time and the production rate is taken as demand dependent and variable holding cost which is realistic for newly launched products in the supermarket. Shortages are allowed and partial backlogged

2. ASSUMPATIONS AND NOTATIONS

The mathematical model in this study is developed on the basis of the following assumptions:

1. Single producer, multi-distributors and retailers are assumed.
2. Shortages are allowed and partially backlogged.
3. A single item with a constant rate of deterioration is considered.
4. There is no replacement on repair of deteriorated items.
5. Time horizon is finite.

The following notations are assumed:

- D (t) The demand rate at any time $D(t)=a+bt+ct^2$ here a,b and c are positive constants. a is the initial demand and $0<b<<1$
- P (t) Production rate, $P(t)=mD(t)$ where $m>1$
- I_p (t) The inventory level of producer at time t
- Id (t) The inventory level of distributor at time t
- I_r (t) The inventory level of retailer at time t
- T The finite time horizon at $T=t_1+t_2+t_3+t_4$
- θ $\theta=\alpha\beta t^{\beta-1}$, $0<\alpha<<1, t>0, \beta\geq 1$ Deterioration rate of the on hand inventory
- n_d Integer number of deliveries from the producer to each distributor during of inventory cycle when there is positive inventory
- n_r Integer number of deliveries from each distributor to his retailer during of inventory cycle when there is positive inventory.
- n_{pd} Integer number of distributors supplied by the producer
- n_{dr} Integer number of retailers supplied by his distributor
- Q_p Producer's production lot size
- Q_d Each distributor's lot size
- Q_r Each retailer's lot size
- S Maximum shortage level
- C_p Setup cost for the producer per production cycle
- C_d Ordering cost for each distributor per order
- C_r Ordering cost for each retailer per order
- C_{1p} Inventory carrying cost for the producer per year and per unit
- C_{1r} Inventory carrying cost for reach retailer per year and per unit
- C_{2p} Cost of deteriorated unit for each producer
- C_{2d} Cost of deteriorated unit for reach distributor
- C_{2r} Cost of deteriorated unit for reach retailer
- C_{3p} Shortage cost for the producer
- C_{3d} Shortage cost for the each distributor
- C_{3r} Shortage cost for the each retailer
- C_{4p} Opportunity cost for the producer
- C_{4r} Opportunity cost for each retailer
- η Fixed parameter of transportation change
- ε Variable parameter of transportation charge
- α_1 is positive constant.

In this paper we have considered one producer which produces the item and delivers in fixed period. Each distributor in turn delivers the items in fixed quantities to his retailer.

3. THE PRODUCER INVENTORY PROBLEM

We consider a single item which deteriorates with rate which quadratic in time .The initial inventory of the cycle is zero and production start at the very beginning of the cycle [3]. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. Production stops at time t_1. The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $\{t_1, t_2\}$ shortage starts after t_2 with the concept of partial backlogging and reach to maximum, shortage level at time t_3. Production restarts after t_3 to fulfill the backlog and demand and the cycle ends with zero inventory. The model of this inventory is depicted in figure 1.

For a single echelon producer, the instantaneous inventory level $I_p(t)$ at any time t is governed by the following differential equations.

$$I'_{P1}(t)+\theta I_{P1}(t)=P(t)-D(t) \quad , 0\le t\le t_1 \qquad \dots (1)$$

$$I'_{P2}(t)+\theta I_{P2}(t)=-D(t) \quad , 0\le t\le t_2 \qquad \dots (2)$$

$$I'_{P3}(t)=-\delta D(t) \quad , 0\le t\le t_3 \qquad \dots (3)$$

and $$I^1_{P4}(t)=P(t)-D(t) \quad , 0\le t\le t_4 \qquad \dots (4)$$

With the conditions

$$I_{P1}(0)=0 \ , I_{P2}(t_2)=0, \ I_{P3}(0) \ , I_{P3}(0)=0, I_{P4}(t)=0, I_{P2}(0)=Q_P \qquad \dots (5)$$

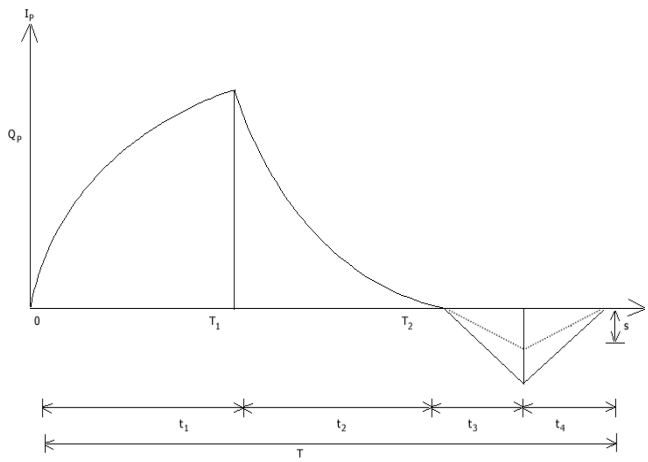


Fig 1: Producer's single-echelon inventory system per production cycle

$$I'_{P1}(t) + \alpha\beta t^{\beta-1} I_{P1}(t) = (m-1)(a + bt + ct^2), 0 \le t \le t_1 \quad \dots (6)$$

$$I'_{P2}(t) + \alpha\beta t^{\beta-1} I_{P2}(t) = -(a + bt + ct^2),\ 0 \le t \le t_2 \quad \dots (7)$$

$$I'_{P3}(t) = -\delta(a + bt + ct^2),\ 0 \le t \le t_3 \quad \dots (8)$$

$$I^1_{P4}(t) = (m-1)(a + bt + ct^2),\ 0 \le t \le t_4 \quad \dots (9)$$

The solution of equation (6), (7), (8) & (9) are:

$$I_{P1}(t) = (m-1)\{at + \frac{bt^2}{2} + \frac{ct^3}{3} + \frac{a\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{(\beta+2)} + \frac{c\alpha t^{\beta+3}}{(\beta+3)}\}e^{-\alpha t^\beta}, 0 \le t \le t_1 \quad \dots (10)$$

$$I_{P2}(t) = \{a(t_2 - t) + \frac{b(t_2^2 - t^2)}{2} + \frac{c(t_2^3 - t^3)}{3} + \frac{a\alpha(t_2^{\beta+1} - t^{\beta+1})}{\beta+1} + \frac{b\alpha(t_2^{\beta+2} - t^{\beta+2})}{(\beta+2)} + \frac{c\alpha(t_2^{\beta+3} - t^{\beta+3})}{(\beta+3)}\}e^{-\alpha t^\beta}, 0 \le t \le t_2 \quad \dots (11)$$

$$I_{P3}(t) = -\delta\{at + \frac{bt^2}{2} + \frac{ct^3}{3}\}, 0 \le t \le t_3 \quad \dots (12)$$

$$I_{P4}(t) = (m-1)\{a(t_4 - t) + \frac{b}{2}(t_4^2 - t^2) + \frac{c}{3}(t_4^3 - t^3)\}, 0 \le t \le t_4 \quad \dots (13)$$

Since $I_{P1}(t_1) = I_{P2}(0) = Q_p$, we can get by (10) and (11)

$$(m-1)\{at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{c\alpha t_1^{\beta+3}}{(\beta+3)}\}e^{-\alpha t^\beta}$$

$$= at_2 + \frac{bt_2^2}{2} + \frac{ct_2^3}{3} + + \frac{a\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2} + \frac{c\alpha t_2^{\beta+3}}{\beta+3}$$

We get $t_2 = f_1(t_1)$ … (14)

Now since $I_{P3}(t_3) = I_{P4}(0) = -S$, we can get from (12) and (13),

$$-\delta\{at_3 + \frac{bt_3^2}{2} + \frac{ct_3^3}{3}\}$$

$$= (m-1)\{at_4 + \frac{b}{2}t_4^2 + \frac{c}{3}t_4^3\}$$

We get $t_4 = f_2(t_3)$ … (15)

The producer's inventory carrying cost

$P_{C.C}$

$$= \{\int_0^{t_1} (C_{1p} + \alpha_1 t) I_{P1}(t)\,dt + \int_0^{t_2} (C_{1p} + \alpha_1 t) I_{P2}(t)\,dt\}$$

$$= \int_0^{t_1} (C_{1p} + \alpha_1 t)(m-1)\{at + \frac{bt^2}{2} + \frac{ct^3}{3} + \frac{a\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{(\beta+2)} + \frac{c\alpha t^{\beta+3}}{(\beta+3)}\}e^{-\alpha t^\beta}\,dt + \int_0^{t_2} (C_{1p} + \alpha_1 t)\{a(t_2 - t) + \frac{b(t_2^2 - t^2)}{2} + \frac{c(t_2^3 - t^3)}{3} + \frac{a\alpha(t_2^{\beta+1} - t^{\beta+1})}{\beta+1} + \frac{b\alpha(t_2^{\beta+2} - t^{\beta+2})}{(\beta+2)} + \frac{c\alpha(t_2^{\beta+3} - t^{\beta+3})}{(\beta+3)}\}e^{-\alpha t^\beta}\,dt$$

$$= C_{1p}[(m-1)\{\frac{at_1^2}{2} + \frac{bt_1^3}{6} + \frac{ct_1^4}{12} + \frac{a\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)} + \frac{c\alpha t_1^{\beta+4}}{(\beta+3)(\beta+4)}\} + \{\frac{at_2^2}{2} + \frac{bt_2^3}{3} + \frac{ct_2^4}{4} + \frac{a\alpha t_2^{\beta+2}}{(\beta+2)} + \frac{b\alpha t_2^{\beta+3}}{(\beta+3)} + \frac{c\alpha t_2^{\beta+4}}{(\beta+4)}\} - \alpha(m-1)\{\frac{at_1^{\beta+2}}{\beta+2} + \frac{bt_1^{\beta+3}}{2(\beta+3)} + \frac{ct_1^4}{3(\beta+4)} + \frac{a\alpha t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_1^{2\beta+3}}{(\beta+2)(2\beta+3)} + \frac{c\alpha t_1^{2\beta+4}}{(\beta+3)(2\beta+4)}\} - \alpha\{\frac{at_2^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{bt_2^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{ct_2^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{a\alpha t_2^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_2^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{c\alpha t_2^{2\beta+4}}{(\beta+1)(2\beta+4)}\}] + \alpha_1[(m-1)\{\frac{at_1^3}{3} + \frac{bt_1^4}{8} + \frac{ct_1^5}{15} + \frac{a\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{b\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\alpha t_1^{\beta+5}}{(\beta+3)(\beta+5)}\} + \{\frac{at_2^3}{6} + \frac{bt_2^4}{8} + \frac{3ct_2^5}{10} + \frac{a\alpha t_2^{\beta+3}}{2(\beta+3)} + \frac{b\alpha t_2^{\beta+4}}{2(\beta+4)} + \frac{c\alpha t_2^{\beta+5}}{2(\beta+5)}\} - \alpha(m-1)\{\frac{at_1^{\beta+3}}{\beta+3} + \frac{bt_1^{\beta+4}}{2(\beta+4)} + \frac{ct_1^{\beta+5}}{3(\beta+5)} + \frac{a\alpha t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{b\alpha t_1^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_1^{2\beta+5}}{(\beta+3)(2\beta+5)}\} - \alpha\{\frac{at_2^{\beta+3}}{(\beta+3)(\beta+2)} + \frac{bt_2^{\beta+4}}{(\beta+4)(\beta+2)} + \frac{ct_2^{\beta+5}}{(\beta+5)(\beta+2)} + \frac{a\alpha t_2^{2\beta+3}}{(\beta+2)(2\beta+3)} + \frac{b\alpha t_2^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_2^{2\beta+5}}{(\beta+2)(2\beta+5)}\}]$$

…(16)

$$= C_{2p}\alpha\beta[(m-1)\{\frac{at_1^{\beta+1}}{(\beta+1)} + \frac{bt_1^{\beta+2}}{2(\beta+2)} + \frac{ct_1^{\beta+3}}{3(\beta+3)} + \frac{a\alpha t_1^{2\beta+1}}{(\beta+1)(2\beta+1)} + \frac{b\alpha t_1^{2\beta+2}}{(\beta+2)(2\beta+2)} + \frac{c\alpha t_1^{2\beta+3}}{(\beta+3)(2\beta+3)}\} -$$

$$\alpha(m-1)\{\frac{at_1^{2\beta+1}}{(2\beta+1)} + \frac{bt_1^{2\beta+2}}{2(2\beta+2)} + \frac{ct_1^{2\beta+3}}{3(2\beta+3)} + \frac{a\alpha t_1^{3\beta+1}}{(\beta+1)(3\beta+1)} + \frac{b\alpha t_1^{3\beta+2}}{(\beta+2)(3\beta+2)} + \frac{c\alpha t_1^{3\beta+3}}{(\beta+3)(3\beta+3)}\} + \{\frac{at_2^{\beta+1}}{\beta(\beta+1)} + \frac{bt_2^{\beta+2}}{\beta(\beta+2)}$$

$$+ \frac{ct_2^{\beta+3}}{\beta(\beta+3)} + \frac{a\alpha t_2^{2\beta+1}}{\beta(2\beta+1)} + \frac{b\alpha t_2^{2\beta+2}}{\beta(2\beta+2)} + \frac{c\alpha t_2^{2\beta+3}}{\beta(2\beta+3)}\} -$$

$$\alpha\{\frac{at_2^{2\beta+1}}{\beta(2\beta+1)} + \frac{bt_2^{2\beta+2}}{\beta(2\beta+2)} + \frac{ct_2^{2\beta+3}}{\beta(2\beta+3)} + \frac{a\alpha t_2^{3\beta+1}}{2\beta(3\beta+1)} + \frac{b\alpha t_2^{3\beta+2}}{2\beta(3\beta+2)} + \frac{c\alpha t_2^{3\beta+3}}{2\beta(3\beta+3)}\}]$$

The producer's deterioration cost

$$P_{D.C} = C_{2p}[\int_0^{t_1} \theta I_{p1}(t)\,dt + \int_0^{t_2} \theta I_{p2}(t)\,dt\ ]$$

$$= C_{2p}[\int_0^{t_1} \alpha\beta t^{\beta-1}(m-1)\{at + \frac{bt^2}{2} + \frac{ct^3}{3} + \frac{a\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{(\beta+2)} + \frac{c\alpha t^{\beta+3}}{(\beta+3)}\}e^{-\alpha t^{\beta}}\,dt + \int_0^{t_2} \alpha\beta t^{\beta-1}\{a(t_2-t) +$$

$$\frac{b(t_2^2-t^2)}{2} + \frac{c(t_2^3-t^3)}{3} + \frac{a\alpha(t_2^{\beta+1}-t^{\beta+1})}{\beta+1} + \frac{b\alpha(t_2^{\beta+2}-t^{\beta+2})}{(\beta+2)} + \frac{c\alpha(t_2^{\beta+3}-t^{\beta+3})}{(\beta+3)}\}e^{-\alpha t^{\beta}}\,dt\ ]$$

…(17)

The producer's shortage cost

$$P_{S.C.} = C_{3p}[\ -\int_0^{t_3} I_{p3}(t)\,dt - \int_0^{t_4} I_{p4}(t)\,dt\ ]$$

$$= C_{3p}[\ -\int_0^{t_3} -\delta\{at + \frac{bt^2}{2} + \frac{ct^3}{3}\}dt - \int_0^{t_4} (m-1)\{a(t_4-t) + \frac{b}{2}(t_4^2-t^2) + \frac{c}{3}(t_4^3-t^3)\}\ ]$$

$$= C_{3p}[\ \delta\{\frac{at_3^2}{2} + \frac{bt_3^3}{3} + \frac{ct_3^4}{4}\} - (m-1)\ \{\frac{at_4^2}{2} + \frac{bt_4^3}{3} + \frac{ct_4^4}{4}\}] \quad \ldots (18)$$

The producer's opportunity cost due to lost sales.

$$P_{O.C} = C_{4p}\ (1-\delta)\int_0^{t_4} (a + bt + ct^2)dt$$

$$= C_{4p}(1-\delta)(at_4 + \frac{bt_4^2}{2} + \frac{ct_4^3}{3}) \quad \ldots (19)$$

Since the total cost function for the producer is the sum of set up cost, carrying cost, deterioration cost, shortage cost and opportunity cost. Therefore the total average cost for producer model is,

$$K_P = \frac{1}{T}[\ C_P + C_{1p}[(m-1)\{\frac{at_1^2}{2} + \frac{bt_1^3}{6} + \frac{ct_1^4}{12} + \frac{a\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)} + \frac{c\alpha t_1^{\beta+4}}{(\beta+3)(\beta+4)}\}$$

$$+ \{\frac{at_2^2}{2} + \frac{bt_2^3}{3} + \frac{ct_2^4}{4} + \frac{a\alpha t_2^{\beta+2}}{(\beta+2)} + \frac{b\alpha t_2^{\beta+3}}{(\beta+3)} + \frac{c\alpha t_2^{\beta+4}}{(\beta+4)}\} - \alpha(m-1)\{\frac{at_1^{\beta+2}}{\beta+2} + \frac{bt_1^{\beta+3}}{2(\beta+3)} + \frac{ct_1^{4}}{3(\beta+4)} +$$

$$\frac{a\alpha t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_1^{2\beta+3}}{(\beta+2)(2\beta+3)} + \frac{c\alpha t_1^{2\beta+4}}{(\beta+3)(2\beta+4)}\} -$$

$$\alpha\{\frac{at_2^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{bt_2^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{ct_2^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{a\alpha t_2^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_2^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{c\alpha t_2^{2\beta+4}}{(\beta+1)(2\beta+4)}\}] +$$

$$\alpha_1[(m-1)\{\frac{at_1^3}{3} + \frac{bt_1^4}{8} + \frac{ct_1^5}{15} + \frac{a\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{b\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\alpha t_1^{\beta+5}}{(\beta+3)(\beta+5)}\} + \{\frac{at_2^3}{6} + \frac{bt_2^4}{8} +$$

$$\frac{3ct_2^5}{10} + \frac{a\alpha t_2^{\beta+3}}{2(\beta+3)} + \frac{b\alpha t_2^{\beta+4}}{2(\beta+4)} + \frac{c\alpha t_2^{\beta+5}}{2(\beta+5)}\} - \alpha(m-1)$$

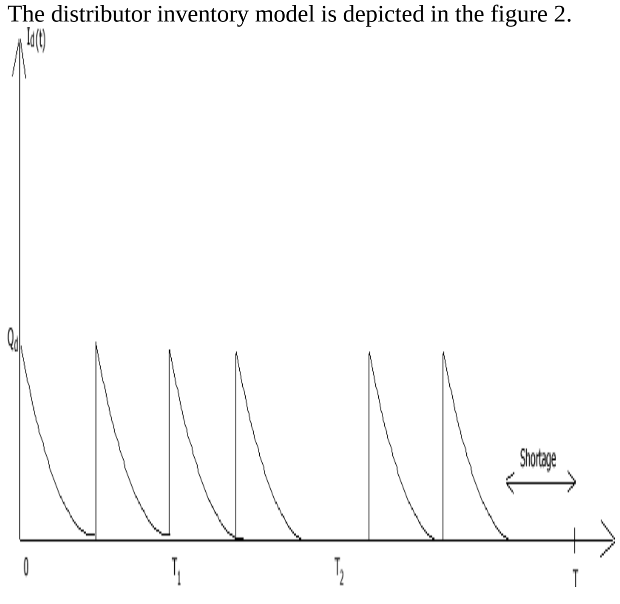
$$\{\frac{at_1^{\beta+3}}{\beta+3} + \frac{bt_1^{\beta+4}}{2(\beta+4)} + \frac{ct_1^{\beta+5}}{3(\beta+5)} + \frac{a\alpha t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{b\alpha t_1^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_1^{2\beta+5}}{(\beta+3)(2\beta+5)}\} - \alpha\ \{\frac{at_2^{\beta+3}}{(\beta+3)(\beta+2)}$$

$$+ \frac{bt_2^{\beta+4}}{(\beta+4)(\beta+2)} + \frac{ct_2^{\beta+5}}{(\beta+5)(\beta+2)} + \frac{a\alpha t_2^{2\beta+3}}{(\beta+2)(2\beta+3)} + \frac{b\alpha t_2^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_2^{2\beta+5}}{(\beta+2)(2\beta+5)}\}]$$

$$+C_{2p}\alpha\beta[(m-1)\{\frac{at_1^{\beta+1}}{(\beta+1)}+\frac{bt_1^{\beta+2}}{2(\beta+2)}+\frac{ct_1^{\beta+3}}{3(\beta+3)}+$$
$$\frac{a\alpha t_1^{2\beta+1}}{(\beta+1)(2\beta+1)}+\frac{b\alpha t_1^{2\beta+2}}{(\beta+2)(2\beta+2)}+\frac{c\alpha t_1^{2\beta+3}}{(\beta+3)(2\beta+3)}\}-$$
$$\alpha(m-1)\{\frac{at_1^{2\beta+1}}{(2\beta+1)}+\frac{bt_1^{2\beta+2}}{2(2\beta+2)}+\frac{ct_1^{2\beta+3}}{3(2\beta+3)}+$$
$$\frac{a\alpha t_1^{3\beta+1}}{(\beta+1)(3\beta+1)}+\frac{b\alpha t_1^{3\beta+2}}{(\beta+2)(3\beta+2)}+\frac{c\alpha t_1^{3\beta+3}}{(\beta+3)(3\beta+3)}\}+$$
$$\{\frac{at_2^{\beta+1}}{\beta(\beta+1)}+\frac{bt_2^{\beta+2}}{\beta(\beta+2)}+\frac{ct_2^{\beta+3}}{\beta(\beta+3)}+\frac{a\alpha t_2^{2\beta+1}}{\beta(2\beta+1)}+$$
$$\frac{b\alpha t_2^{2\beta+2}}{\beta(2\beta+2)}+\frac{c\alpha t_2^{2\beta+3}}{\beta(2\beta+3)}\}-\alpha\{\frac{at_2^{2\beta+1}}{\beta(2\beta+1)}+\frac{bt_2^{2\beta+2}}{\beta(2\beta+2)}+$$
$$\frac{ct_2^{2\beta+3}}{\beta(2\beta+3)}+\frac{a\alpha t_2^{3\beta+1}}{2\beta(3\beta+1)}+\frac{b\alpha t_2^{3\beta+2}}{2\beta(3\beta+2)}+\frac{c\alpha t_2^{3\beta+3}}{2\beta(3\beta+3)}\}]+$$
$$C_{3p}[\delta\{\frac{at_3^2}{2}+\frac{bt_3^3}{3}+\frac{ct_3^4}{4}\}$$
$$-(m-1)\{\frac{at_4^2}{2}+\frac{bt_4^3}{3}+\frac{ct_4^4}{4}\}]+$$
$$C_{4p}(1-\delta)(at_4+\frac{bt_4^2}{2}+\frac{ct_4^3}{3})] \qquad \ldots(20)$$

4. THE DISTRIBUTER INVENTORY MODEL

The distributor inventory model is depicted in the figure 2.



$$\frac{t_1+t_2}{n_d} \quad \frac{2(t_1+t_2)}{n_d} \quad \ldots\ldots\ldots \quad \frac{(n_d-1)(t_1+t_2)}{n_d} \quad (t_1+t_2)$$

Fig 2: Distributor's single-echelon inventory system Per production cycle

For a distributor there are n_d deliveries during the inventory cycle. When there is positive inventory, the distributor inventory level is governed by the following differential equation,

$$I_d'(t)+\theta I_d(t)=-\frac{D(t)}{n_{pd}}, \quad 0\le t\le\frac{t_1+t_2}{n_d} \qquad \ldots(21)$$

$$I_d'(t)+\alpha\beta t^{\beta-1}I_d(t)=-\frac{a+bt+ct^2}{n_{pd}}, \quad 0\le t\le\frac{t_1+t_2}{n_d}$$

Boundary Conditions is, $I_d(\frac{t_1+t_2}{n_d})=0$

$$I_d(t)=e^{-\alpha t^\beta}\int-\frac{a+bt+ct^2}{n_{pd}}e^{\alpha t^\beta}dt, \; 0\le t\le\frac{t_1+t_2}{n_d}$$

$$I_d(t)=\frac{e^{-\alpha(\frac{t_1+t_2}{n_d})^\beta}}{n_{p_d}}\{a(\frac{t_1+t_2}{n_d}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1})+$$
$$b(\frac{(\frac{t_1+t_2}{n_d})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_d})^3}{3}+$$
$$\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}-\frac{e^{-\alpha t^\beta}}{n_{p_d}}\{a(t+\frac{\alpha t^{\beta+1}}{\beta+1})+b(\frac{t^2}{2}+$$
$$\frac{\alpha t^{\beta+2}}{\beta+2})+c(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3})\}, \; 0\le t\le\frac{t_1+t_2}{n_d}$$

… (22)

The maximum inventory level of the distribution is given by

$$Q_d=I_d(0)$$

$$Q_d=\frac{e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}}{n_{p_d}}\{a(\frac{t_1+t_2}{n_d}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1})+$$
$$b(\frac{(\frac{t_1+t_2}{n_d})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_d})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}$$

The transportation charge per shipment is assumed to be linear function of lot size. It is modeled as follows

Transportation charge $=\eta+\varepsilon Q$ … (23)

Where Q is lot size per shipment to each distributor or retailer, η and ε are positive empirical values, η is fixed and ε is variable parameter.

Total transportation cost

$$D_{T.C.}=n_d(\eta+\varepsilon Q_d)$$
$$=n_d[\eta+\varepsilon$$

$$\frac{e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_d}}}{n\,p_d}\{a(\frac{t_1+t_2}{n_d}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1})+ b(\frac{(\frac{t_1+t_2}{n_d})^{2}}{2}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2}) c(\frac{(\frac{t_1+t_2}{n_d})^{3}}{3}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\} \quad \ldots (24)$$

The actual inventory carrying cost per distribution

$$= C_{1d}[n_d\int_0^{\frac{t_1+t_2}{n_d}} I_d(t)dt - n_r n_{d_r}\int_0^{\frac{t_1+t_2}{n_d}} I_r(t)dt]$$

$$= \frac{C_{1d}n_d}{n\,p_d}\int_0^{\frac{t_1+t_2}{n_d}}[e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_d}}\{a(\frac{t_1+t_2}{n_d}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_d})^{2}}{2}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})$$
$$+c(\frac{(\frac{t_1+t_2}{n_d})^{3}}{3}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}-e^{-\alpha t^{\beta}}\{a(t+\frac{\alpha t^{\beta+1}}{\beta+1})+b(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2})+c(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3})\}d\,t]$$
$$-\frac{C_{1d}n_r}{n_{pd}}\int_0^{\frac{t_1+t_2}{n_d}}[e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_r}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^{2}}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^{3}}{3}$$
$$+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-e^{-\alpha t^{\beta}}\{a(t+\frac{\alpha t^{\beta+1}}{\beta+1})+b(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2})+c(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3})\}dt]$$

$$= \frac{C_{1d}n_d}{n\,p_d}[e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_d}}(\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_d}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_d})^{2}}{2}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})+$$
$$c(\frac{(\frac{t_1+t_2}{n_d})^{3}}{3}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}-\{\frac{a}{2}(\frac{t_1+t_2}{n_d})^{2}+$$
$$\frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)})+b\frac{(\frac{t_1+t_2}{n_d})^{3}}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)})$$
$$+c\frac{(\frac{t_1+t_2}{n_d})^{4}}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\}+\alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2}$$
$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)})+b\frac{(\frac{t_1+t_2}{n_d})^{3}}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)})+$$
$$c\frac{(\frac{t_1+t_2}{n_d})^{4}}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]-\frac{C_{1d}n_r}{n_{pd}}[e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_r}}$$
$$(\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^{2}}{2}+$$
$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^{3}}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-$$
$$\{\frac{a}{2}(\frac{t_1+t_2}{n_d})^{2}+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)})+b\frac{(\frac{t_1+t_2}{n_d})^{3}}{6}+$$
$$\frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)})+c\frac{(\frac{t_1+t_2}{n_d})^{4}}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\}+$$
$$\alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2}+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)})+$$
$$b\frac{(\frac{t_1+t_2}{n_d})^{3}}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)})+c\frac{(\frac{t_1+t_2}{n_d})^{4}}{12}$$
$$+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}],$$

…(25)

The actual deterioration cost per distribution

$$D_{D.C.} = C_{2d}[n_d \int_0^{\alpha\beta t} \theta I_d(t)dt - n_r n_{d_r} \int_0^{\frac{t_1+t_2}{n_d}} \theta I_r(t)dt]$$

$$= C_{2\ d}\frac{n_d}{n_{p_d}}\int_0^{\frac{t_1+t_2}{n_d}} \alpha\beta t^{\beta-1}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}\{a(\frac{t_1+t_2}{n_d} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})$$

$$+ c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\} - e^{-\alpha\ t^\beta}\{a(t + \frac{\alpha t^{\beta+1}}{\beta+1}) + b(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}) + c(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3})\}]d\,t -$$

$$\frac{C_{2d}n_r}{n_{pd}}\int_0^{\frac{t_1+t_2}{n_d}} \alpha\beta t^{\beta-1}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) +$$

$$c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} - e^{-\alpha\ t^\beta}\{a(t + \frac{\alpha t^{\beta+1}}{\beta+1}) + b(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}) + c(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3})\}]dt$$

$$= \frac{C_{2\,d}n_d}{n_{p_d}}[\alpha(\frac{t_1+t_2}{n_d})^\beta e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}\{a(\frac{t_1+t_2}{n_d} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})$$

$$+ c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\} - \alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_d})^{\beta+1}$$

$$+ \frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)})$$

$$+ c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\} + \alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1}$$

$$+ \frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)}) +$$

$$c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\ ] - \frac{C_{2d}n_r}{n_{pd}}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}$$

$$\alpha(\frac{t_1+t_2}{n_d})^\beta\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} -$$

$$\alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_d})^{\beta+1} + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)}$$

$$+ \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)}) + c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}$$

$$+ \alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1} + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)}$$

$$+ b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)}) + c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)}$$

$$+ \frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\ ]$$

…(26)

The distributer's shortage cost

$$D_{S.C.} = C_{3d}\Big[-\int_0^{t_3} -\delta\{at + \frac{bt^2}{2} + \frac{ct^3}{3}\}dt - \int_0^{t_4} (m-1) \{a(t_4 - t) + \frac{b}{2}(t_4^2 - t^2) + \frac{c}{3}(t_4^3 - t^3)\}\Big]$$

$$= C_{3d}\,[\,\delta\{\frac{at_3^2}{2} + \frac{bt_3^3}{3} + \frac{ct_3^4}{4}\} - (m-1)\{\frac{at_4^2}{2} + \frac{bt_4^3}{3} + \frac{ct_4^4}{4}\}] \quad \ldots (27)$$

The distributer's opportunity cost due to lost sales.

$$D_{O.C} = C_{4d}\ (1-\delta)\int_0^{t_4}(a + bt + ct^2)dt$$

$$= C_{4d}\,(1-\delta)\,(at_4 + \frac{bt_4^2}{2} + \frac{ct_4^3}{3}) \quad \ldots (28)$$

Since the distributor's total cost is the sum of each distributor's cost which includes ordering cost, carrying cost per distributor, deterioration cost, shortage cots and opportunity cost. Therefore, the total average cost for distributor model is,

$$K_d = \frac{n_d C_d}{T} + \frac{n_d}{T}[\eta + \varepsilon \frac{e^{-\alpha \frac{(t_1+t_2)^\beta}{n_d}}}{n_{p_d}}\{a(\frac{t_1+t_2}{n_d} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2} c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}] +$$

$$\frac{C_{1d}n_d}{n_{p_d}T}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}(\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_d} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}$$

$$-\{\frac{a}{2}(\frac{t_1+t_2}{n_d})^2 + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)})$$

$$+c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\} + \alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2} +$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_d})^4}{12}$$

$$+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}] - \frac{C_{1d}n_r}{n_{pd}T}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}(\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_r} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} - \{\frac{a}{2}(\frac{t_1+t_2}{n_d})^2 + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6}$$

$$+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\} +$$

$$\alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2} + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} +$$

$$\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]$$

$$+\frac{C_{2d}n_d}{n_{p_d}T}[\alpha(\frac{t_1+t_2}{n_d})^\beta e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}\{a(\frac{t_1+t_2}{n_d} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2}$$

$$+c(\frac{(\frac{t_1+t_2}{n_d})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\}-\alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_d})^{\beta+1}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)}+b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)}+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)})+$$

$$c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}+\alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)}+b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)}+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)})+$$

$$c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}]-\frac{C_{2d}n_r}{n_{pd}T}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}$$

$$\alpha(\frac{t_1+t_2}{n_d})^\beta\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-\alpha\beta\{\frac{a}{\beta+1}$$

$$(\frac{t_1+t_2}{n_d})^{\beta+1}+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)}+b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)}+$$

$$\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)})+c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}$$

$$+\alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1}+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)}+b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)}$$

$$+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)})+c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\ ]$$

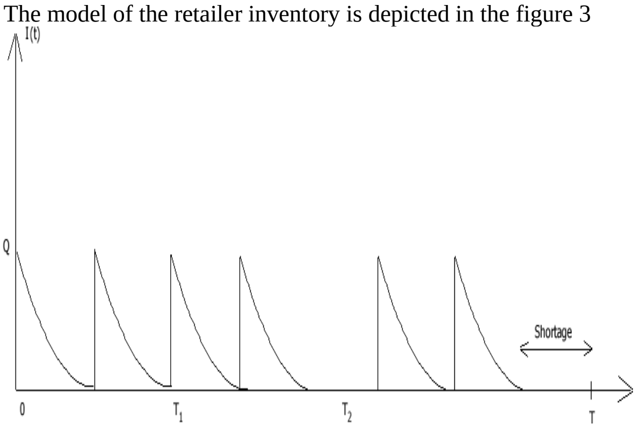
$$+\frac{C_{3d}}{T}[\delta\{\frac{at_3^2}{2}+\frac{bt_3^3}{3}+\frac{ct_3^4}{4}\}$$

$$-\frac{(m-1)}{T}\{\frac{at_4^2}{2}+\frac{bt_4^3}{3}+\frac{ct_4^4}{4}\}]+$$

$$C_{4d}\frac{(1-\delta)}{T}(at_4+\frac{bt_4^2}{2}+\frac{ct_4^3}{3}) \qquad \ldots(29)$$

5. THE RETAILER INVENTORY MODEL

The model of the retailer inventory is depicted in the figure 3



$$\frac{t_1+t_2}{n_r} \quad \frac{2(t_1+t_2)}{n_r} \;\ldots\ldots\; \frac{(n_r-1)(t_1+t_2)}{n_r} \quad (t_1+t_2)$$

Fig 3: Retailer's single-echelon inventory system per production cycle

For the retailer with deliveries during the inventory cycle when there is positive inventory, the retailer inventory level is governed by the following differential equation

$$I_r'(t)+\theta I_r(t)=-\frac{D(t)}{n_{pd}n_{dr}}\quad,\quad 0\le t\le\frac{(t_1+t_2)}{n_r} \qquad \ldots(30)$$

$$I_r'(t)+\alpha\beta t^{\beta-1}I_r(t)=-\frac{a+bt+ct^2}{n_{pd}n_{dr}}\quad,\quad 0\le t\le\frac{(t_1+t_2)}{n_r}$$

$$B.C.\,is,\ I_r(\frac{t_1+t_2}{n_r})=0$$

$$I_r(t)=e^{-\alpha t^\beta}\int-\frac{a+bt+ct^2}{n_{pd}n_{dr}}e^{\alpha t^\beta}dt,\ 0\le t\le\frac{t_1+t_2}{n_r}$$

$$I_r(t)=\frac{1}{n_{pd}n_{dr}}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})$$

$$+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-e^{-\alpha t^\beta}\{a(t+\frac{\alpha t^{\beta+1}}{\beta+1})+b(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2})$$

$$+c(\frac{t^3}{3}+\frac{\alpha t^{\beta+3}}{\beta+3})\}],\ 0\le t\le\frac{t_1+t_2}{n_r}$$

…(31)

The maximum inventory level of the retailer is

$$Q_r = I_r(0) = \frac{e^{-\alpha\left(\frac{t_1+t_2}{n_r}\right)^{\beta}}}{n_{pd}n_{dr}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}$$

…(32)

The transportation charges again per shipment are assumed to be linear function of lot size. It is modeled as follows Transportation charge $= \eta + \varepsilon Q$ Where Q is lot size per shipment to each distributor or retailer, η and ε are positive empirical values, η is fixed and ε is variable parameter.

Total transportation cost for retailers

$$R_{T.C.} = n_r(\eta + \varepsilon Q_r)$$

$$= n_r[\eta + \varepsilon\frac{e^{-\alpha\left(\frac{t_1+t_2}{n_r}\right)^{\beta}}}{n_{pd}n_{dr}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}]$$

…(33)

The retailer's carrying cost,

$$R_{C.C.} = C_{1d}n_r\int_0^{\frac{t_1+t_2}{n_r}} I_r(t)dt$$

$$= \frac{C_{1d}n_r}{n_{pd}n_{dr}}\int_0^{\frac{t_1+t_2}{n_r}}[e^{-\alpha\left(\frac{t_1+t_2}{n_r}\right)^{\beta}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} - e^{-\alpha t^{\beta}}\{a(t + \frac{\alpha t^{\beta+1}}{\beta+1}) + b(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}) + c(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3})\}]dt,\ 0 \le t \le \frac{t_1+t_2}{n_r}dt$$

$$= \frac{C_{1d}n_r}{n_{pd}n_{dr}}[\frac{(t_1+t_2)}{n_r}e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_r}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}$$

$$- \{\frac{a}{2}(\frac{t_1+t_2}{n_r})^2 + \frac{a\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{(\beta+1)(\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_r})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{(\beta+2)(\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_r})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_r})^{\beta+4}}{(\beta+3)(\beta+4)})\} + \alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_r})^{\beta+2} +$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_r})^{2\beta+2}}{(\beta+1)(2\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_r})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_r})^{2\beta+3}}{(\beta+2)(2\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_r})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_r})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}],$$

…(34)

The retailer's actual deterioration cost per distribution

$$R_{D.C.} = C_{2d}n_r\int_0^{\frac{t_1+t_2}{n_r}}\theta I_r(t)dt$$

$$= \frac{C_{2d}n_r}{n_{pd}n_{dr}}[\int_0^{\frac{t_1+t_2}{n_r}}\alpha\beta t^{\beta-1}e^{-\alpha\frac{(t_1+t_2)^{\beta}}{n_r}}\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} - e^{-\alpha t^{\beta}}\alpha\beta t^{\beta-1}\{a(t + \frac{\alpha t^{\beta+1}}{\beta+1}) + b(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2}) + c(\frac{t^3}{3} + \frac{\alpha t^{\beta+3}}{\beta+3})\}]dt$$

…(35)

The retailer's shortage cost

$R_{S.C.}$

$$= C_{3r}[\ -\int_0^{t_3} -\delta\{at + \frac{bt^2}{2} + \frac{ct^3}{3}\}dt - \int_0^{t_4}(m-1)\{a(t_4 - t) + \frac{b}{2}(t_4^2 - t^2) + \frac{c}{3}(t_4^3 - t^3)\}\ ]$$

$$= C_{3r}\,[\,\delta\{\frac{at_3^2}{2}+\frac{bt_3^3}{3}+\frac{ct_3^4}{4}\} - (m-1)\ \{\frac{at_4^2}{2}+\frac{bt_4^3}{3}+\frac{ct_4^4}{4}\}\,] \qquad \ldots (36)$$

The distributer's opportunity cost due to lost sales.

$$R_{O.C} = C_{4r}\,(1-\delta)\int_0^{t_4}(a+bt+ct^2)dt$$

$$= C_{4r}(1-\delta)\,(at_4+\frac{bt_4^2}{2}+\frac{ct_4^3}{3}) \qquad \ldots (37)$$

The total average cost for retailer's model is,

$$K_r = \frac{n_r C_r}{T}+\frac{n_r}{T}[\eta+\varepsilon\frac{e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}}{n_{pd}n_{dr}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+$$

$$c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}]+\frac{C_{1d}n_r}{n_{pd}n_{dr}T}[\frac{(t_1+t_2)}{n_r}$$

$$e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}$$

$$-\{\frac{a}{2}(\frac{t_1+t_2}{n_r})^2+\frac{a\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{(\beta+1)(\beta+2)})+b\frac{(\frac{t_1+t_2}{n_r})^3}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{(\beta+2)(\beta+3)})+c\frac{(\frac{t_1+t_2}{n_r})^4}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{\beta+4}}{(\beta+3)(\beta+4)})\}+$$

$$\alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_r})^{\beta+2}+\frac{a\alpha(\frac{t_1+t_2}{n_r})^{2\beta+2}}{(\beta+1)(2\beta+2)})+b\frac{(\frac{t_1+t_2}{n_r})^3}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{2\beta+3}}{(\beta+2)(2\beta+3)})+c\frac{(\frac{t_1+t_2}{n_r})^4}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]+$$

$$\frac{C_{2d}n_r}{n_{pd}n_{dr}T}[\alpha(\frac{t_1+t_2}{n_r})^\beta e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+$$

$$c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-\alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_r})^{\beta+1}+$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_r})^{2\beta+1}}{(\beta+1)(2\beta+1)}+b\frac{(\frac{t_1+t_2}{n_r})^{\beta+2}}{2(\beta+2)}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{2\beta+2}}{(\beta+2)(2\beta+2)})+$$

$$c\frac{(\frac{t_1+t_2}{n_r})^{\beta+3}}{3(\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}+\alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_r})^{2\beta+1}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_r})^{3\beta+1}}{(\beta+1)(3\beta+1)}+b\frac{(\frac{t_1+t_2}{n_r})^{2\beta+2}}{2(2\beta+2)}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{3\beta+2}}{(\beta+2)(3\beta+2)})+$$

$$c\frac{(\frac{t_1+t_2}{n_r})^{2\beta+3}}{3(2\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\,]$$

$$+C_{3r}\,[\,\frac{\delta}{T}\{\frac{at_3^2}{2}+\frac{bt_3^3}{3}+\frac{ct_3^4}{4}\} - \frac{(m-1)}{T}\ \{\frac{at_4^2}{2}+\frac{bt_4^3}{3}+\frac{ct_4^4}{4}\}\,]+$$

$$C_{4r}\frac{(1-\delta)}{T}\ (at_4+\frac{bt_4^2}{2}+\frac{ct_4^3}{3}) \qquad \ldots (38)$$

6. THE TOTAL AVERAGE COST

The inventory joint total cost is the sum of total cost for retailer's model, distributor model and producer's model. Therefore, the total average cost (K) is the sum of the costs given by (20), (29) & (38) i.e. $K = K_p + K_d + K_r$

$$\frac{1}{T}[(C_P + n_d C_d + n_r C_r) + nC_{1p}[(m-1)\{\frac{at_1^2}{2} +$$

$$\frac{bt_1^3}{6} + \frac{ct_1^4}{12} + \frac{a\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)} +$$

$$\frac{c\alpha t_1^{\beta+4}}{(\beta+3)(\beta+4)}\} + \{\frac{at_2^2}{2} + \frac{bt_2^3}{3} + \frac{ct_2^4}{4} + \frac{a\alpha t_2^{\beta+2}}{(\beta+2)}$$

$$+\frac{b\alpha t_2^{\beta+3}}{(\beta+3)} + \frac{c\alpha t_2^{\beta+4}}{(\beta+4)}\} - \alpha(m-1)\{\frac{at_1^{\beta+2}}{\beta+2} + \frac{bt_1^{\beta+3}}{2(\beta+3)} +$$

$$\frac{ct_1^4}{3(\beta+4)} + \frac{a\alpha t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_1^{2\beta+3}}{(\beta+2)(2\beta+3)} +$$

$$\frac{c\alpha t_1^{2\beta+4}}{(\beta+3)(2\beta+4)}\} - \alpha\{\frac{at_2^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{bt_2^{\beta+3}}{(\beta+1)(\beta+3)} +$$

$$\frac{ct_2^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{a\alpha t_2^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha t_2^{2\beta+3}}{(\beta+1)(2\beta+3)} +$$

$$\frac{c\alpha t_2^{2\beta+4}}{(\beta+1)(2\beta+4)}\}] + \alpha_1[(m-1)\{\frac{at_1^3}{3} + \frac{bt_1^4}{8} + \frac{ct_1^5}{15} +$$

$$\frac{a\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{b\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\alpha t_1^{\beta+5}}{(\beta+3)(\beta+5)}\} +$$

$$\{\frac{at_2^3}{6} + \frac{bt_2^4}{8} + \frac{3ct_2^5}{10} + \frac{a\alpha t_2^{\beta+3}}{2(\beta+3)} + \frac{b\alpha t_2^{\beta+4}}{2(\beta+4)} +$$

$$\frac{c\alpha t_2^{\beta+5}}{2(\beta+5)}\} - \alpha(m-1)\{\frac{at_1^{\beta+3}}{\beta+3} + \frac{bt_1^{\beta+4}}{2(\beta+4)} + \frac{ct_1^{\beta+5}}{3(\beta+5)} +$$

$$\frac{a\alpha t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{b\alpha t_1^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_1^{2\beta+5}}{(\beta+3)(2\beta+5)}\} -$$

$$\alpha\{\frac{at_2^{\beta+3}}{(\beta+3)(\beta+2)} + \frac{bt_2^{\beta+4}}{(\beta+4)(\beta+2)} + \frac{ct_2^{\beta+5}}{(\beta+5)(\beta+2)} +$$

$$\frac{a\alpha t_2^{2\beta+3}}{(\beta+2)(2\beta+3)} + \frac{b\alpha t_2^{2\beta+4}}{(\beta+2)(2\beta+4)} + \frac{c\alpha t_2^{2\beta+5}}{(\beta+2)(2\beta+5)}\}]$$

$$+C_{2p}\alpha\beta[(m-1)\{\frac{at_1^{\beta+1}}{(\beta+1)} + \frac{bt_1^{\beta+2}}{2(\beta+2)} + \frac{ct_1^{\beta+3}}{3(\beta+3)} +$$

$$\frac{a\alpha t_1^{2\beta+1}}{(\beta+1)(2\beta+1)} + \frac{b\alpha t_1^{2\beta+2}}{(\beta+2)(2\beta+2)} + \frac{c\alpha t_1^{2\beta+3}}{(\beta+3)(2\beta+3)}\} -$$

$$\alpha(m-1)\{\frac{at_1^{2\beta+1}}{(2\beta+1)} + \frac{bt_1^{2\beta+2}}{2(2\beta+2)} + \frac{ct_1^{2\beta+3}}{3(2\beta+3)} + \frac{a\alpha t_1^{3\beta+1}}{(\beta+1)(3\beta+1)}$$

$$+\frac{b\alpha t_1^{3\beta+2}}{(\beta+2)(3\beta+2)} + \frac{c\alpha t_1^{3\beta+3}}{(\beta+3)(3\beta+3)}\} + \{\frac{at_2^{\beta+1}}{\beta(\beta+1)} + \frac{bt_2^{\beta+2}}{\beta(\beta+2)}$$

$$+\frac{ct_2^{\beta+3}}{\beta(\beta+3)} + \frac{a\alpha t_2^{2\beta+1}}{\beta(2\beta+1)} + \frac{b\alpha t_2^{2\beta+2}}{\beta(2\beta+2)} + \frac{c\alpha t_2^{2\beta+3}}{\beta(2\beta+3)}\} -$$

$$\alpha\{\frac{at_2^{2\beta+1}}{\beta(2\beta+1)} + \frac{bt_2^{2\beta+2}}{\beta(2\beta+2)} + \frac{ct_2^{2\beta+3}}{\beta(2\beta+3)} + \frac{a\alpha t_2^{3\beta+1}}{2\beta(3\beta+1)}$$

$$+\frac{b\alpha t_2^{3\beta+2}}{2\beta(3\beta+2)} + \frac{c\alpha t_2^{3\beta+3}}{2\beta(3\beta+3)}\}] +$$

$$+\frac{n_d}{T}[\eta + \varepsilon$$

$$\frac{e^{-\alpha\left(\frac{t_1+t_2}{n_d}\right)^{\beta}}}{n p_d}\{a(\frac{t_1+t_2}{n_d} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) +$$

$$b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2}) +$$

$$\frac{C_{1d} n_d}{n_{p_d} T}[e^{-\alpha \frac{(t_1+t_2)^\beta}{n_d}} (\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_d}$$

$$+\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2})$$

$$+c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\} - \{\frac{a}{2}(\frac{t_1+t_2}{n_d})^2$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)})$$

$$+c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\} + \alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)})$$

$$+c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]] - \frac{C_{1d} n_r}{n_{pd} T}$$

$$[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}(\frac{t_1+t_2}{n_d})\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) +$$

$$b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} - \{\frac{a}{2}(\frac{t_1+t_2}{n_d})^2 + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{(\beta+1)(\beta+2)})$$

$$+b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{(\beta+2)(\beta+3)}) + c\frac{(\frac{t_1+t_2}{n_d})^4}{12}$$

$$+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{\beta+4}}{(\beta+3)(\beta+4)})\} + \alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_d})^{\beta+2} +$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+1)(2\beta+2)}) + b\frac{(\frac{t_1+t_2}{n_d})^3}{6} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+2)(2\beta+3)}) +$$

$$c\frac{(\frac{t_1+t_2}{n_d})^4}{12} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]$$

$$+\frac{C_{2d} n_d}{n_{p_d} T}[\alpha(\frac{t_1+t_2}{n_d})^\beta e^{-\alpha\frac{(t_1+t_2)^\beta}{n_d}}\{a(\frac{t_1+t_2}{n_d} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_d})^2}{2} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+2}}{\beta+2}$$

$$+c(\frac{(\frac{t_1+t_2}{n_d})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_d})^{\beta+3}}{\beta+3})\} - \alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_d})^{\beta+1}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)}) +$$

$$c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\} + \alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1}$$

$$+\frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)}) +$$

$$c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\ ] - \frac{C_{2d} n_r}{n_{pd}}[e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}$$

$$\alpha(\frac{t_1+t_2}{n_d})^\beta\{a(\frac{t_1+t_2}{n_r} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1}) + b(\frac{(\frac{t_1+t_2}{n_r})^2}{2} +$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2}) + c(\frac{(\frac{t_1+t_2}{n_r})^3}{3} + \frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\} -$$

$$\alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_d})^{\beta+1} + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{2\beta+1}}{(\beta+1)(2\beta+1)} + b\frac{(\frac{t_1+t_2}{n_d})^{\beta+2}}{2(\beta+2)}$$

$$+\frac{b\alpha(\frac{t_1+t_2}{n_d})^{2\beta+2}}{(\beta+2)(2\beta+2)}) + c\frac{(\frac{t_1+t_2}{n_d})^{\beta+3}}{3(\beta+3)} + \frac{c\alpha(\frac{t_1+t_2}{n_d})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}$$

$$+\alpha^2\beta\{\frac{a}{2\beta+1}(\frac{t_1+t_2}{n_{dr}})^{2\beta+1} + \frac{a\alpha(\frac{t_1+t_2}{n_d})^{3\beta+1}}{(\beta+1)(3\beta+1)}$$

$$+b\frac{(\frac{t_1+t_2}{n_d})^{2\beta+2}}{2(2\beta+2)} + \frac{b\alpha(\frac{t_1+t_2}{n_d})^{3\beta+2}}{(\beta+2)(3\beta+2)}) + c\frac{(\frac{t_1+t_2}{n_d})^{2\beta+3}}{3(2\beta+3)}$$

$$+\frac{c\alpha(\frac{t_1+t_2}{n_d})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}\ ]$$

$$+\frac{n_r}{T}[\eta+\varepsilon\frac{e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}}{n_{pd}n_{dr}}\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})$$

$$+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}]+\frac{C_{1d}n_r}{n_{pd}n_{dr}T}[\frac{(t_1+t_2)}{n_r}e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}$$

$$\{a(\frac{t_1+t_2}{n_r}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})$$

$$+c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-\{\frac{a}{2}(\frac{t_1+t_2}{n_r})^2+$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{(\beta+1)(\beta+2)})+b\frac{(\frac{t_1+t_2}{n_r})^3}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{(\beta+2)(\beta+3)})+$$

$$c\frac{(\frac{t_1+t_2}{n_r})^4}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{\beta+4}}{(\beta+3)(\beta+4)})\}+\alpha\{\frac{a}{\beta+2}(\frac{t_1+t_2}{n_r})^{\beta+2}+$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_r})^{2\beta+2}}{(\beta+1)(2\beta+2)})+b\frac{(\frac{t_1+t_2}{n_r})^3}{6}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{2\beta+3}}{(\beta+2)(2\beta+3)})+$$

$$c\frac{(\frac{t_1+t_2}{n_r})^4}{12}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{2\beta+4}}{(\beta+3)(2\beta+4)})\}]+$$

$$\frac{C_{2d}n_r}{n_{pd}n_{dr}T}[\alpha(\frac{t_1+t_2}{n_r})^\beta e^{-\alpha\frac{(t_1+t_2)^\beta}{n_r}}\{a(\frac{t_1+t_2}{n_r}+$$

$$\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+1}}{\beta+1})+b(\frac{(\frac{t_1+t_2}{n_r})^2}{2}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+2}}{\beta+2})+$$

$$c(\frac{(\frac{t_1+t_2}{n_r})^3}{3}+\frac{\alpha(\frac{t_1+t_2}{n_r})^{\beta+3}}{\beta+3})\}-\alpha\beta\{\frac{a}{\beta+1}(\frac{t_1+t_2}{n_r})^{\beta+1}+$$

$$\frac{a\alpha(\frac{t_1+t_2}{n_r})^{2\beta+1}}{(\beta+1)(2\beta+1)}+b\frac{(\frac{t_1+t_2}{n_r})^{\beta+2}}{2(\beta+2)}+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{2\beta+2}}{(\beta+2)(2\beta+2)})$$

$$+c\frac{(\frac{t_1+t_2}{n_r})^{\beta+3}}{3(\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{2\beta+3}}{(\beta+3)(2\beta+3)}\}+\alpha^2\beta\{\frac{a}{2\beta+1}$$

$$(\frac{t_1+t_2}{n_r})^{2\beta+1}+\frac{a\alpha(\frac{t_1+t_2}{n_r})^{3\beta+1}}{(\beta+1)(3\beta+1)}+b\frac{(\frac{t_1+t_2}{n_r})^{2\beta+2}}{2(2\beta+2)}$$

$$+\frac{b\alpha(\frac{t_1+t_2}{n_r})^{3\beta+2}}{(\beta+2)(3\beta+2)})+c\frac{(\frac{t_1+t_2}{n_r})^{2\beta+3}}{3(2\beta+3)}+\frac{c\alpha(\frac{t_1+t_2}{n_r})^{3\beta+3}}{(\beta+3)(3\beta+3)}\}]$$

$$+\frac{(C_{3p}+C_{3d}+C_{3r})}{T}[\delta\{\frac{at_3^2}{2}+\frac{bt_3^3}{3}+\frac{ct_3^4}{4}\}$$

$$-\frac{(m-1)}{T}\{\frac{at_4^2}{2}+\frac{bt_4^3}{3}+\frac{ct_4^4}{4}\}]+$$

$$(C_{4P}+C_{4d}+C_{4r})\frac{(1-\delta)}{T}(at_4+\frac{bt_4^2}{2}+\frac{ct_4^3}{3}) \quad \ldots(39)$$

The equation (39) contains four variables t_1 and t_2 are dependent variables and similarly t_3 and t_4 are dependent variables .Now for minimization of average cost per unit time the optimal values of t_1 and t_3 can be obtained by solving the following equation simultaneously

$$\frac{\partial k}{\partial t_1}=0 \quad \ldots(40)$$

and $\frac{\partial k}{\partial t_3}=0$ …(41)

provided they satisfying the following conditions

$$\frac{\partial^2 k}{\partial t_1^2}\rangle 0,\frac{\partial^2 k}{\partial t_3^2}\rangle 0, \quad \ldots(42)$$

and $$\left(\frac{\partial^2 k}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 k}{\partial t_3^{\,2}}\right)-\left(\frac{\partial^2 k}{\partial t_1 \partial t_3}\right) \rangle 0 \qquad \ldots(43)$$

7. CONCLUSION

This paper deals with the development of three models for supplying a single product from a single producer to multiple distributors and retailers. Deterioration is Weibull distribution function of two parameters. The occurrence of shortages in inventory is natural phenomenon in real situations. Inventory shortages are allowed with variable holding cost in producer's models and partially backordered. Backlogging rate is considered as constant. Demand is taken as the quadratic increasing function of time and the production rat is taken as demand dependent for producer's inventory model. In this model, we developed three echelon inventory models for weibull deterioration rate with quadratic demand rate. Linear increasing holding cost and finite time horizon is also taken. We considered the joint total cost is considered as sum of producer, retailer and distributer cost. Cost minimization techniques are also used in this paper.

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