International Journal of Computer Applications (0975 – 8887)
International Conference on Communication, Circuits and Systems "iC3S-2012"

Quantitative Feedback Theory based Controller Design of an Unstable System

Chandrima Roy
Department of E.C.E, Assistant Professor
Heritage Institute of Technology, Kolkata, WB

Kalyankumar Datta
Department of E.E, Professor Emeritus (Hon.)
Jadavpur University, Kolkata, WB

Devmalya Banerjee
Department of E.I.E, Assistant Professor
JIS college of Engineering, WB

ABSTRACT
This paper represents the design of a Quantitative Feedback Theory (QFT) based controller for an unstable system with some desired performance specifications [1]. Here a controller and prefilter has been designed so that the given performance specifications like robust stability and robust tracking criterions can be achieved. In order to implement that, different algorithms have been used and their relative advantages are studied and representative performance results are also given.

General Terms
Robust control algorithm, Quantitave Feedback Theory

Keywords
QFT, templates, bounds, loop-shaping, robust stability, robust tracking analysis.

.

1. INTRODUCTION
In the industrial applications plant may be unstable and there are uncertainties in parameters. The controller should be designed to meet the desired system performance in spite of the system parameter uncertainties and solution should be cost effective. QFT technique results in lower degree, easy implementable controllers. The plant chosen here is an unstable plant with structured parametric uncertainties and the overall system stipulates certain performance specifications [1]. A QFT based controller and pre-filter that satisfies all the performance requirements, is proposed for this plant in this paper. QFT was proposed by Isaac Horowitz in 1960 and modified later by Horowitz and Sidi in 1972 [2,3]. A closed-loop control structure is considered in QFT where the amount of feedback is quantitative to the plant uncertainty and hence the name QFT validates [4,5]. The controller is designed through loop-shaping which is executed in the CAD environment [6]. Prior to loop-shaping, there are certain steps like template generation and bound generation which can be executed with a number of different algorithms [7, 8, 9, 10, 11, 12, 13]. This paper proposes the design of a QFT controller and pre-filter for a second order unstable uncertain system. The final design verifications through time responses are also given in the paper.

2. DEFINITION OF DESIGN PROBLEM
The uncertain unstable plant has the transfer function:

$$P(s) = \frac{k}{s^2+as+b} \qquad (1)$$

Where, $k \in [0.5, 3]$, $a \in [-0.6, 3.4]$, and. $b \in [-2, 4]$

These uncertain parameters produce a plant family for which the controller has to be designed. The equivalent problem representation can be such that the controller G(s) and Pre-filter F(s) are to be designed conforming the desired performance specifications.

3. PERFORMANCE SPECIFICATIONS
The parameter uncertainty given in (1) generates a set of plant transfer functions $\mathcal{P}$. It is desired that $\forall P \in \mathcal{P}$, the closed loop system posses the following specifications:

***Tracking Specifications**: -*
(a) Overshoot Nil
(b) Rise time $0.1 \text{ sec.} < t_r < 0.2$ sec
(c) Steady state error Nil ………. (2)

Stability Specifications: -
Gain Margin > 5 dB
Phase Margin> 45 deg (3)

4. CHOICE OF DESIGN PARAMETERS:
The first step in the design process is to select an adequate and finite set of frequencies. This set is determined by the bandwidth of the system and by the frequencies of interest, for which the different desired behavior specifications are defined.

5. DESIGN PROCEDURE
The next step in the design process is to represent as accurately as possible the uncertainty of the system and to convert the closed loop magnitude specifications into magnitude and phase constraints on a nominal open-loop function. This process is carried out by Template and Bound Generation. To take care of the parametric uncertainties, it is required to generate the Templates and Bounds for each of the individual frequencies of the selected set of frequencies.

5.1 Choice of nominal plant
The nominal plant chosen for executing the simulation is: $Po(s) = \frac{1.75}{s^2+1.4s+1}$ (4)

This plant corresponds to the * mark of the templates obtained for different frequencies.

5.2 Template generation:

The templates obtained for the family of plants P(s) and for the set of frequencies ω are as shown in Figure.1 & 2. Each point represents the frequency response of one plant out of the entire plant family and each shape distinguishes the response for each value of the frequency. Template generation is carried out with two algorithms, one is Parametric Gridding method and second one is Kharitonov Segment method out of many other well known methods [7, 8, 9, 10, 11]. Figure.1 & 2 represent the regions of uncertainty of the plant family with respect to the frequency of interest.

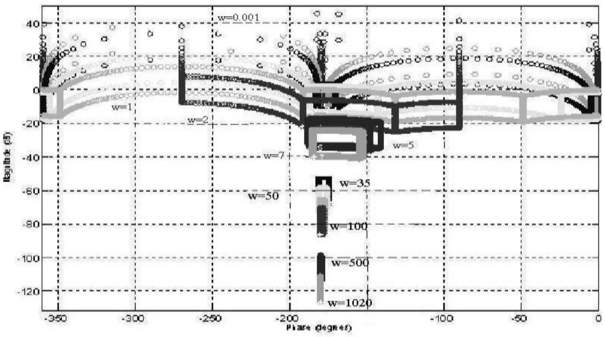

Figure 1: Template using Grid Method

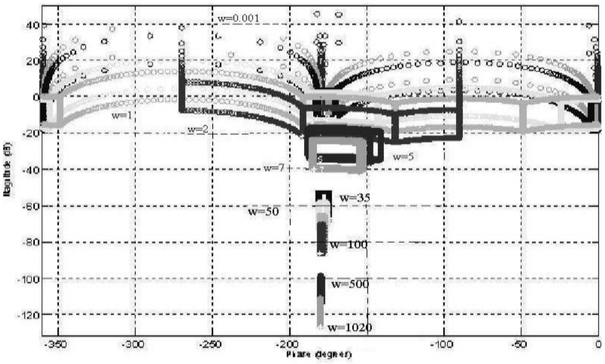

Figure 2: Template using Kharitonov Segment Method

5.3 Bound generation

In the bound generation step of Horowitz's quantitative feedback theory (QFT) procedure [2,3], the plant template design is used to translate the given robustness specifications in the frequency domain using the Nichol chart where the controller gain-phase values are allowed to lie. If the transfer function of the controller is denoted as G(s) and the transfer function of the nominal plant as $P_o(s)$, the bounds are those regions that the open loop frequency response Lo [where, Lo=Po(s)G(s)] must avoid in order to guarantee the fulfillment of the design specifications for the whole set of plants P(s). Conventional method of bound generation is followed here out of many other algorithms [12,13]. Out of various types of bounds, robust margin and tracking bounds are considered here as per the design requirement. The upper &lower bound of tracking can easily be found using time domain specifications. The specification of robust stability is found to be: $W_{s1} = \gamma = 1.316$ (5).

Tracking bounds can be obtained using the tracking specifications. Figure 3 & 4 show the two types of bounds discussed so far.

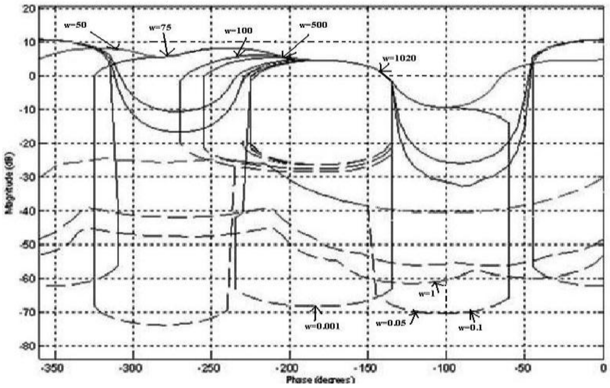

Figure 3: Robust Margin Bounds

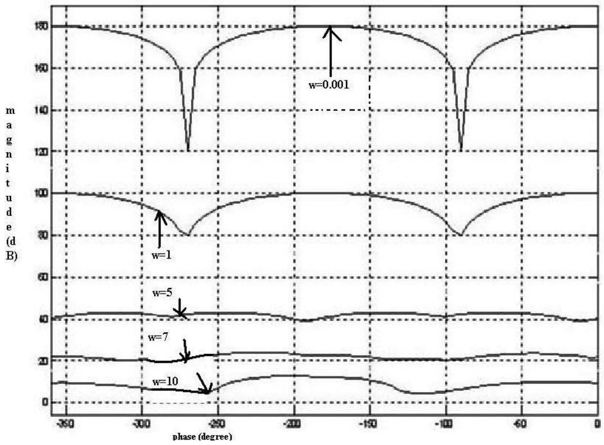

Figure 4: Robust Tracking Bounds

5.4 Design of controller & preflter

The next step in the design process is involved in finding a controller with which all of the desired specifications are to be fulfilled. It is also known as the synthesis or "loop-shaping" phase. The adjustment is made using the Matlab QFT Toolbox [6], shifting the loop curves vertically and horizontally on the Magnitude-Phase plane, until it is situated in such a way as not to violate the bounds and also to have the lowest possible gain. The controller that satisfies all specifications and all bounds is obtained as:

$$G(s) = \frac{900\left(\frac{s}{6}+1\right)}{\frac{s}{5000}+1} \qquad (6)$$

The resultant open loop frequency response with this controller (in Nichols Chart) is illustrated below in Figure 5.The controller design has reduced the variations in the closed loop frequency response to the desired range. A pre-filter is now required to achieve the required shape of the closed loop frequency response and to track the entire response of the plant family to lie with the computed value of the upper and lower bounds. The suitable pre-filter, which satisfies tracking specification perfectly, is:

$$F(s) = \frac{12}{s+12} \quad (7)$$

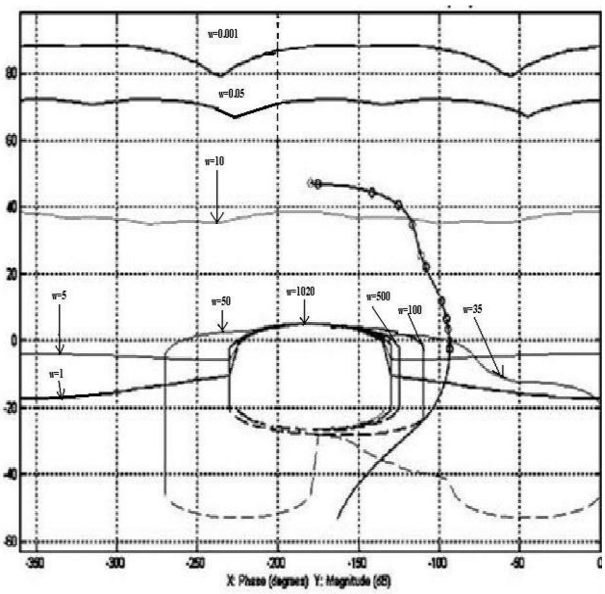

Figure 5: loopshaping

6. DESIGN VALIDATION

The system with Controller & Prefilter satisfies all the required design specifications both in time and frequency domain. Figure 6 & 7 shows that the designed controller maintains the entire plant family response, below robust stability bound and the Prefilter is also successful to retain the system response within the upper and the lower bound. The worst case closed loop response (covering all uncertainty cases) is shown in solid line, together with the design specifications plotted in the dashed line. In Figure.7, the maximum variation of the closed-loop system frequency response is plotted (the solid line) together with the design specifications (the dashed line). Step response of the closed loop uncertain system with Controller G(s) and Prefilter F(s) satisfies the given time domain specifications, as shown in Figure.8.in the next page. This process is called time domain validation. The nominal plant step response of the plant with G(s) and F(s) is shown below. In Figure 7, the third line (solid) from top shows the tracking performance of the nominal plant and the second line from the top (solid) depicts the worst case behavior for the uncertain and unstable system. The vertical arrow both sided shows the deviation between the worst case performance and the nominal plant performance.

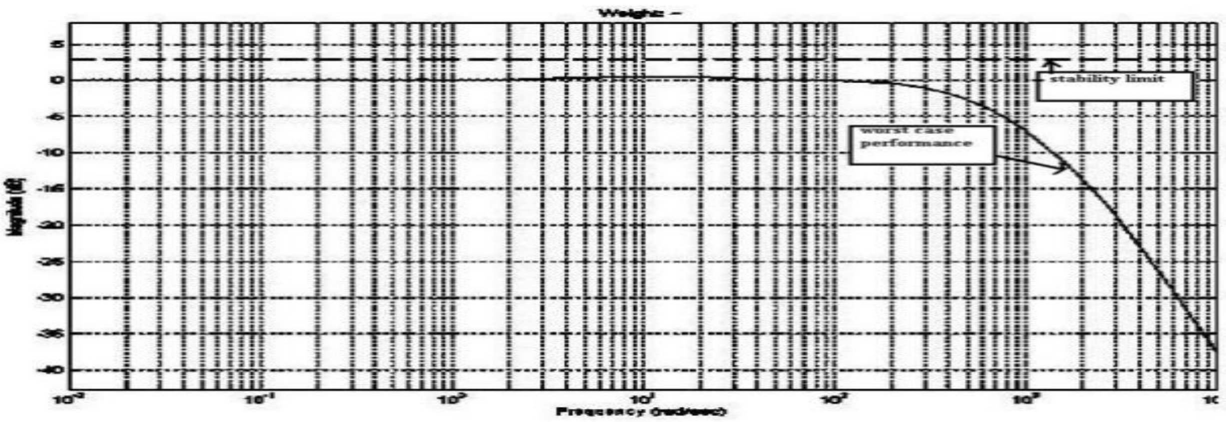

Figure 6: Robust Stability Analysis

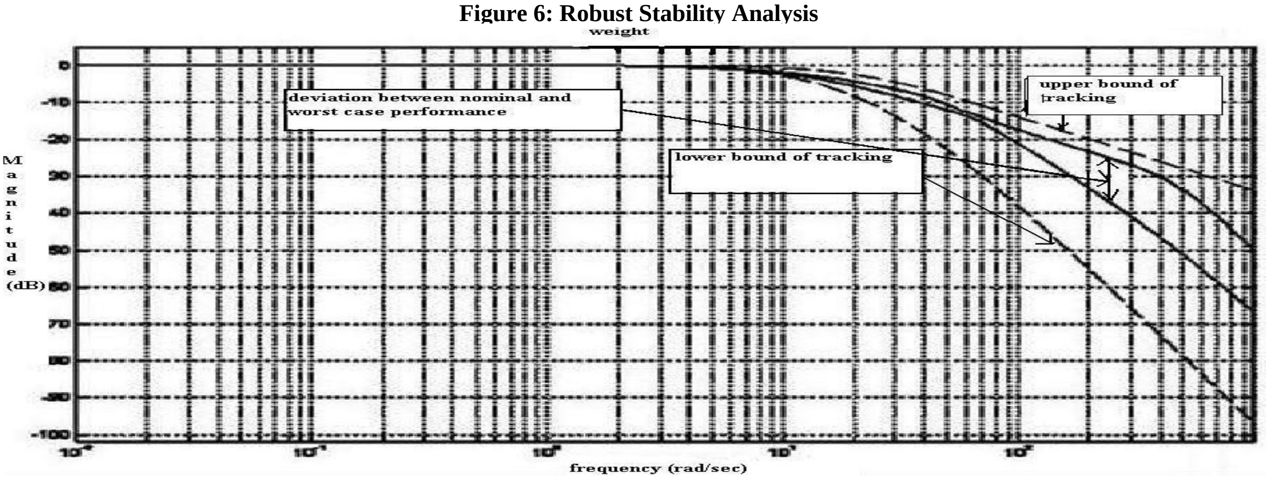

Figure 7: Robust Tracking Analysis

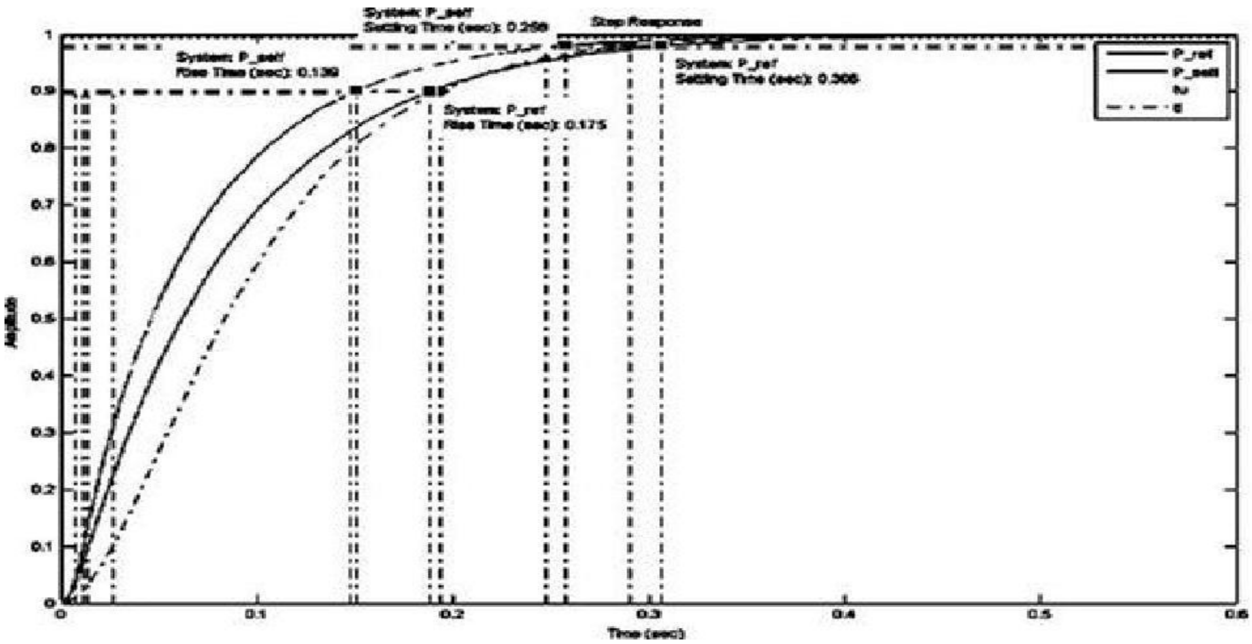

Figure 8: Closed Loop step Response of Nominal Plant with G(S) &F(S)

7. CONCLUSION

In this paper, for an unstable plant the existing QFT methodologies and algorithms for designing a controller have been implemented. In accordance with the specifications given in [1], the necessary computations and calculations are carried out and the results are the compared. Once, the controller and Pre-filter design is completed as an observation, it is realized that the designed Controller and Pre-filter also meet the design specifications given in [1]. This paper shows that Kharitonov Segment Method for generating templates is a better and efficient procedure compared to the other one.

8. ACKNOWLEDGEMENT

It is pleasure to express my gratitude to those people who have made significant contribution to develop the paper. I am thankful to my teachers in every possible way. Without their consistent encouragement and support, this paper would have been a dream still date. I am also thankful to my my parents for inspiring and helping me consistently.

9. REFERENCS

[1] C. Roy, "Design of Robust Controller using Quantitative Feedback Theory," M.E. thesis, Jadavpur University, Electrical Engineering, India, 2011.

[2] I.M. Horowitz and M. Sidi, "Synthesis of Feedback Systems with Large Plant Ignorance for Prescribed Time Domain Tolerances," *Int. Journal of Control*, vol.16, no.2, 1972, pp. 287-309.

[3] I. Horowitz, "Quantitative Feedback Theory," in *IEE Proc*, vol. 129, No. 6, Nov. 1982.

[4] O.Yaniv, *Quantitative Feedback Design of Linear & Non Linear Control Systems*, Boston: Kluwer Academic Publishers, 1999.

[5] C.H. Houpis and S.J. Rasmussen, *Quantitative Feedback Theory – Fundamentals & Applications,* New York, Marcel Dekker Inc., 1999.

[6] C. Borghesani,Y. Chait, O. Yaniv, *Quantitative Feedback Theory Toolbox User's Guide*, The Math Works Inc.,Massachusetts,USA,1994.

[7] D.J. Balance and G. Hughes, "A survey of Templates generation methods for Quantitative Feedback Theory," *UKACC Int. Conf. on Control*, in *Proc. of IEEE*, vol. 1,1996, pp. 172-174.

[8] I.J Fialho, V. Pande and P.S.V Nataraj, "Design of Feedback Systems Using Kharitonov's Segments in

Quantitative Feedback Theory," in *Symposium Fourth Int. Quantitative Feedback Theory*, Ohio, USA, 1992.

[9] W. Chen and D.J. Balance, "Plant Template Generation in Quantitative Feedback Theory," University of Glasgow, Centre for Systems and Control, Technical Report, UK, 1998.

[10] Yang S-F, "An Improvement of QFT Plant Template Generation For Systems With Affinely Dependent

Parametric Uncertainties," *Journal* of the Franklin Institute, vol. 346, issue 7, 1996, pp. 663-675.

[11] Juan José Martín-Romero and Arturo Martín-Romero, "QFT Templates for Plants with a High Number of Uncertain Parameters," in *IEEE Trans. on Automatic Control,* vol. 52, no. 4, April 2007, pp. 754-758.

[12] S.F.Yang, "Generation of QFT Bounds for Robust Tracking Specifications for Plants with Affinely Dependent Uncertainties," *Int. Journal of Robust and Nonlinear Control*, vol. 21, 2011, pp. 237-247.

[13] S.F.Yang, "Efficient Algorithm for Computing QFT Bounds," *Int. Journal of Control*, vol. 83, no.4, Nov.2009, pp. 716 -723.