Auto-Correlation based Spectrum Sensing at Low SNR for Cognitive Radio

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ABSTRACT
A challenging problem in Cognitive radio is that the secondary users in cognitive radios must be able to detect primary users under low signal-to-noise ratio (SNR) and dispersive channel. Spectrum sensing based on auto-correlation of the received signal samples being more prone to correlate under dispersive condition, has been investigated. Simplified theoretical expressions for probability of false alarm and probability of detection of the auto-correlation based algorithm are derived in the presence of multi-path fading channel. Spectrum of an unknown primary signal has been obtained through auto-regressive parametric signal modeling. By the proposed auto-correlation technique the detection probability near unity can be achieved at finite input samples and at a very low SNR for OFDM DVB-T signal and a wireless FM microphone signal in VHF band. It is found that dimension of auto-correlation matrix and signal sample duration is inversely proportional to achieve unity detection probability.

Keywords
Spectrum sensing, autoregressive analysis, OFDM, multi-path fading.

1. INTRODUCTION
Demand for free spectrum, a scare resource for wireless communication is increasing day by day. Cognitive radio (CR) opened the possibility of significant spectrum re-use [1, 2, 3, 4, 5]. In a CR network the secondary users are allowed to utilize the frequency bands of primary users when these bands are not being utilized by the primary. Spectrum sensing is a key function for detecting the presence of primary signals [6]. Furthermore, in order to avoid interference with hidden primary receivers, secondary system must know the presence or absence of very weak primary signals [6, 7] at a low SNR below about -10 dB. Quite a few sensing methods have been proposed [8-11]. A drawback of the proposed sensing methods [8-11] is that the detection depends on the estimated noise power. It is noticed that powerful auto-correlation based sensing has not been explored to its full potential so far. It is important that a detector must have a threshold of detection independent of noise power and thus detector can avoid noise uncertainty problem. Secondly, implementation of the detector should be simple and efficient. However, higher computational complexity is difficult to get away at low SNR regime. But technology trends towards achieving inexpensive large memory together with fast computational devices, the problem of computation complexity will soon be overcome in future.

In this work, spectrum sensing algorithm is derived from auto-correlation of the received samples. It is shown that with proper choice of decision statistic [12], the threshold of detection can be made independent of the noise power. The proposed method is very effective in practical dispersive channel as signal samples are more correlated under this condition. Spectrum of an unknown primary signal is obtained by autoregressive (AR) analysis [13, 14]. Advantages of parametric spectrum estimation method over non-parametric method are manifold [13]. The proposed sensing method is able to detect closely spaced primary signals. From auto-correlation of the received signal samples, an analytical expression of probability of detection for a given probability of false alarm is derived. Finally, the proposed algorithm is employed to detect a low snr VHF OFDM based DTV-T and FM microphone signal and their detection performance is analyzed.

2. SPECTRUM SENSING

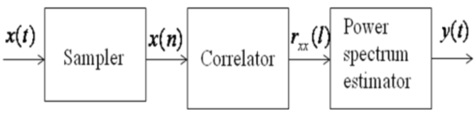


Fig 1: Auto-correlation detector

A simple auto-correlation detector is shown in Fig.1. $x(t)$ is an unknown signal of duration T and carrier frequency f_c. Using a threshold of detection on the sampled correlation matrix R_{xx} the power spectrum of an unknown signal can be estimated. Spectrum estimation model [13] chosen here is an auto regressive all pole process of order p. Let us consider a finite received signal samples $x(n),\ x(n-1),\cdots\cdots x(n-N+1)$ of length N. The correlator output of the input received samples $x(n)'s$ can be written as

$$r_{xx}(l)=\frac{1}{N}\sum_{n=0}^{N-1}x(n)x(n-l),\ l>0 \quad (1)$$

Elements of $r_{xx}(l)$ form a sample correlation matrix R_{xx}

and the corresponding noise correlation matrix is R_{ww}. Notice that $x(n) = s(n) + w(n)$, the sum of clean signal and noise. Dimension of R_{xx} is M x M where M is the maximum value of lag l.

2.1 Parametric spectral estimation of an input signal

From the sample covariance matrix the power spectral density of the input signal can be obtained. Spectrum estimation model chosen here is an auto regressive all pole process of order p. An all pole system model can be described by

$$H(z) = \frac{X(z)}{N(z)} = \frac{1}{1+\sum_{k=1}^{p} a_k z^{-k}} = \frac{1}{A(z)} \qquad (2)$$

where a_k's are the coefficients to be estimated from the input data $x(n)$ .Inverse Z-Transform of Eq.(2) is given by

$$x(n) + \sum_{k=1}^{p} a_k x(n-k) = n(n) \qquad (3)$$

Define $r_{xx}(l) = E\,[x(n)x(n-l)]$, the linear AR equations can be written in matrix form

$$\begin{bmatrix} r_{xx}(0) & r_{xx}(1) & r_{xx}(2) & \cdots & \cdots & r_{xx}(p) \\ r_{xx}(1) & r_{xx}(0) & r_{xx}(1) & \cdots & \cdots & r_{xx}(p-1) \\ \vdots & & & & & \vdots \\ \vdots & & & & & \vdots \\ r_{xx}(p) & r_{xx}(p-1) & r_{xx}(p-2) & \cdots & \cdots & r_{xx}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_n^2 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \qquad (4)$$

The matrix containing elements of $r_{xx}(l)$ is denoted by R_{xx}. In the above equations it is assumed that $a_0 = 1$. Using the time average auto correlations $r_{xx}(l)$ from Eq. (1), estimated AR parameters a_k's can be found out from Eq.(4). Once a_k's is obtained power spectrum can be estimated from Eq. (5) by employing the following relationship.

$$\hat{P}_{xx}(f) = \left|\hat{H}(f)\right|^2 \hat{\sigma}_n^2 = \frac{\hat{\sigma}_n^2}{\left|\hat{A}(f)\right|^2} = \frac{\hat{\sigma}_n^2}{\left|1+\sum_{k=1}^{p} \hat{a}_k\, e^{-j2\pi fk}\right|^2} \qquad (5)$$

Where $$\hat{\sigma}_n^2 = r_{xx}(0)\prod_{k=1}^{p}\left(1-\left|\hat{a}_k\right|^2\right) \qquad (6)$$

2.2 Probability of detection from auto-correlation matrix R_{xx}

Let's denote the sum of all the elements of a correlation matrix R_{xx} by T_X . When the ratio of the sum of the absolute value of signal correlation matrix to the sum of the absolute value of noise correlation matrix elements R_{ww} is greater than a threshold γ_{Th} then the signal is present and otherwise signal is not present. γ_{Th} is a threshold to be determined to meet the required probability of false alarm. The calculation of the mean and variance of the test metric T_W for the noise can be found in [14]. In order to obtain probability of detection P_d from auto correlation detection when signal is present, the two quantities $E(T_x)$ and $\mathrm{var}(T_x)$ are required to be obtained. When $h(n)$ is time dependent channel and amplitude fading 'α' follows Rayleigh distribution with parameter σ_h^2 then it can be shown $E[\alpha] = \sigma_h\sqrt{\pi/2}$ and $E[\alpha^2] = 2\sigma_h^2$. Analytically the mean (μ_{T_X}) and variance ($\sigma_{T_X}^2$) of T_X are evaluated from sample auto-correlation matrix,

which are given by

$$\mu_{T_{Xh}} = M(2\overline{E}_s\sigma_h^2 + \sigma_w^2) + \pi\sigma_h^2 \sum_{l=1}^{M-1}(M-l)\left|\, r_{SS}(l)\right|$$

(7)

and

$$\sigma_{T_{Xh}}^2 = \left(\frac{8M^2}{N}\,\overline{E}_s\,\sigma_h^2\sigma_w^2\right) + \frac{2M(M-1)(2M-1)}{3N}(4\sigma_h^2\overline{E}_s + \sigma_w^2)\sigma_w^2$$

(8) where $$r_{SS}(l) = \frac{1}{N}\sum_{n=0}^{N-1} s(n)s(n-l)$$

and $\overline{E}_s$ is the average signal energy, given by

$\overline{E}_s = \frac{1}{N}\sum_{n=0}^{N-1} s^2(n)$. From Eq. (7) and (8), the probability of detection P_d can be given by

$$P_d = Q\left(\frac{\gamma_{Th}\, M\sigma_w^2 - \mu_{T_X}}{\sigma_{T_X}}\right) \qquad (9)$$

3. COMPUTATIONAL COMP- LEXITY OF THE ESTIMATED SPECTRUM

Total computational complexity for estimating power spectrum $\hat{P}_{xx}(f)$ from Eq.(5) can be calculated from the following expression

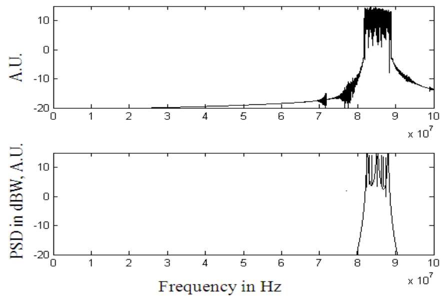
$$NM^2 + M^2\log M + \frac{NM}{2}\log_2 M \qquad (10)$$

The first term is attributed to calculation of sample auto-correlation matrix, the second term contributes to matrix inversion and the third term contributes to frequency response of the parameters. For large samples N the complexity is mainly dominated by the first term. So for $N > 1000$, all practical purposes the order of computational complexity of the algorithm is NM^2.

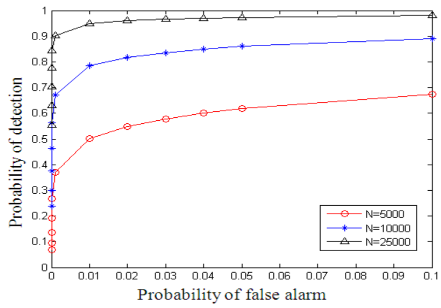
4. RESULTS AND DISCUSSION

4.1 Detection of OFDM DVB-T signal in VHF band channel #6

Performance of an auto-correlation detector is investigated by sensing OFDM based DTV-T signals. DVB-T standard signal is simulated [15] and shown in Fig.2 (a) (top). Following [13.14], power spectral density of OFDM based DVB-T signal is estimated at -20 dB SNR. Result is shown in Fig. 2(a) (bottom). Rayleigh faded channel has been assumed. The probability of



(a)



(b)

Fig 2(a): (Top) Transmitted OFDM DVB-T spectrum in VHF band Ch #6 and (bottom) estimated PSD of an OFDM spectrum at -20 dB SNR. 2(b): Probability of detection versus probability of false alarm for various input samples and SNR -20 dB in a Rayleigh fading channel

detection is calculated from Eq.(9) and the result is shown in Fig. 2(b) for different number of input samples N. Probability of detection is improved with increasing number of samples. At a low SNR -20 dB and N =25,000, the detection probability approaches near unity for probability of false alarm equal to 0.5. Detection performance can be improved by setting higher probability of false alarm.

4.2 Detection of wireless micro- phone signal in VHF band Ch#6

Wireless microphone is regularly employed in TV studios and also for outside broadcast (OB). RF microphone operates on VHF TV channels. Following the documentation in [16] frequency modulated (FM) microphone signal is generated in the VHF band at channel 6. Information signal is a uniform random variable distributed between [-1, 1]. Rayleigh faded channel is assumed. Received signal to noise ratio is -20 dB. Spectrum of the transmitted signal is shown in Fig. 3 (top). Power spectral density of the detected signal is estimated by spectrum estimation methods described in section 2.1. and the result is shown in Fig 3 (bottom). Probability

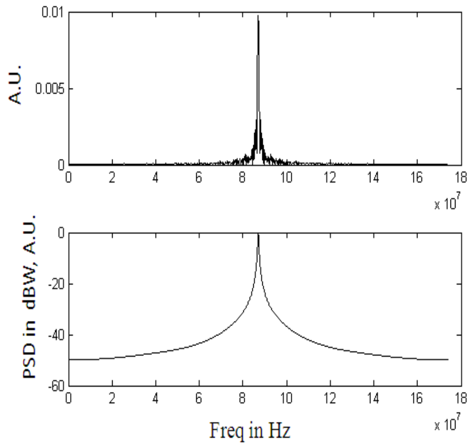


Fig 3: (Top) Transmitted spectrum of FM modulated microphone signal in VHF band channel 6, 82-88 MHz; (bottom) estimated spectrum at -20 dB SNR

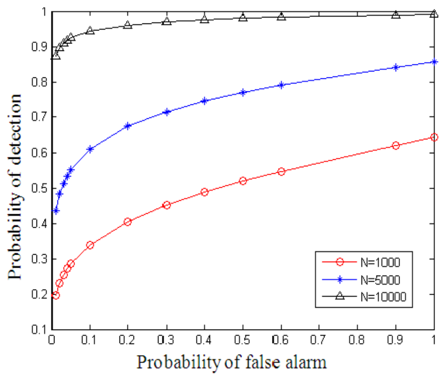


Fig 4: Probability of detection versus probability of false alarm for a microphone signal for various input samples lengths (N) at a SNR = -20 dB in a Rayleigh fading channel

of detection is calculated following the method described in the section 2.2 and the result is shown in Fig. 4 for different number of input samples. Probability of detection is improved with increasing number of samples. For N =10,000 detection probability approaches near unity. The performances of detection can also be improved by setting higher probability of false alarm.

4.3 Detection of two closely spaced spectrums

Our aim is also to estimate power spectral density (PSD) of two closely spaced signals and for this purpose we have assumed two narrow band microphone spectrums [16] in the VHF band 90 MHz and 110 MHz in a low SNR regime. Incoming signals are kept at -10 dB SNR. In order to reduce the sensing time higher order M (=14) has been chosen. It has been found that 800 samples are required to attain unit probability of detection. Detected spectrum is shown in Fig.5. Two spectrums are clearly distinguishable. The choice of N

and M is inversely proportional. This is useful for controlling the sensing time. Sensing time is the product of sample period times the input number

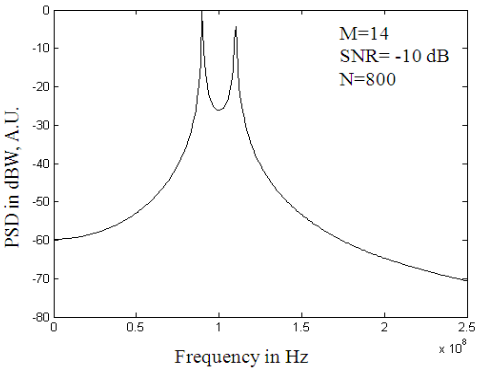


Fig 5: Estimated spectrum of two narrow band signals at 90 MHz and 110 MHz; SNR -10 dB, sample duration $N = 800$ and $M = 14$

of samples. For example, smoothing factor M can be increased to reduce the sensing time. In this example to detect two closely spaced spectrums sensing time is 150 ms at a sampling frequency of 10 KHz.

5. CONCLUSIONS

Performance of auto-correlation detector for spectrum sensing of an unknown signal has been investigated. Analytical expression of the probability of false alarm and detection are obtained under flat fading channel. Spectrum of an unknown primary signal at a low SNR -20 dB is obtained through autoregressive analysis using parametric signal modeling. Detection probability of an OFDM DVB-T signal has been investigated. Detection probability approaches near to unity for N =25,000 samples at SNR=-20 dB. For narrow band FM microphone signal detection probability approaches near to unity for N =10,000 samples when channel fading is present It is found that auto-correlation technique can be a promising method to detect the presence of a low SNR signal without having any noise uncertainty.

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