Subluminal Optical Temporal Solitons in Asymmetric Three Coupled Quantum Wells

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ABSTRACT
We have explored the possibility of generation and propagation of ultraslow bright optical temporal solitons in asymmetric three-coupled quantum well systems. These bright solitons owe their existence to Kerr and quintic nonlinearities which arise due to a probe pulse and two controlling laser beams. We also find numerically that these solitons are stable against weak perturbation.

General Terms
Subluminal, Temporal solitons, Coupled quantum wells.

Keywords
Nonlinear optics,Cubic quintic nonlinearity, optical solitons.

1. INTRODUCTION
In recent years, optical solitons have received tremendous attention due to their potential applications in communication, information processing and optical computing [1-2]. In particular, interest in the studies on sub- and superluminal optical soliton propagation due to quantum optical coherence and interference effects in semiconductor quantum wells (SQWs) [3-8] have increased. Due to small effective electron mass, SQWs possess large electric dipole moments of inter sub-band transitions and high nonlinear optical coefficients. Furthermore, their transition energies, dipole moments and symmetries can be engineered as desired by choosing the materials and structure dimensions in device design. In SQWs, optical solitons can be created at a very low power, of the order of a few mW. The formation of optical solitons in QWs is the manifestation of a delicate balance between group velocity dispersion (GVD) and self phase modulation due to optical nonlinearity induced by the probe pulse and control fields. In this communication we show the existence of bright solitons based on inter subband transition (ISBT) in an asymmetric three-coupled quantum well (ATCQW) structure and investigate their dynamics when the system provides both Kerr and quintic nonlinearities.

2. MATHEMATICAL MODEL
We consider an ATCQW structures having four electronic energy levels that forms the well known cascade configuration. The TCQW sample consists of 40 coupled well periods. Each well period consists of three GaInAs wells of thickness 4.2, 2.0 and 1.8 nm respectively and they are separated by 1.6 nm barriers made of AlInAs. The present structure has energy levels as $\varepsilon_1 = 151$mev, $\varepsilon_2 = 270$mev, $\varepsilon_3 = 386$mev and $\varepsilon_4 = 506$mev. ω_{21}, ω_{32} and ω_{43} respectively represent the energy difference of $|2\rangle \rightarrow |1\rangle$, $|3\rangle \rightarrow |2\rangle$ and $|4\rangle \rightarrow |3\rangle$ transitions. A weak probe optical pulse with angular frequency ω_P, wave vector$k_P = \omega_P/c$, polarization vector $\hat{e}_P$ and amplitude E_P is assumed to propagate in z-direction inside the QW where it interacts with this four level system. The growth direction of the quantum well is along the y-axis and z-axis is parallel to the QW plane. The quantum well system also interacts with two continuous wave (CW) control laser fields E_{C1} and E_{C2}.

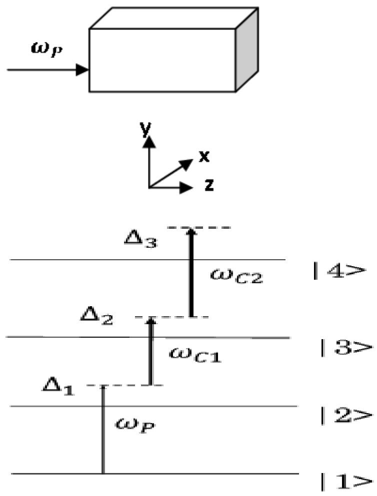

Figure 1: Conduction band energy level diagram for a single period of the three-coupled asymmetric quantum well structure.

In the interaction picture, with the rotating wave approximation and electric dipole approximation, the interaction Hamiltonian can be written respectively as

$$\hat{H}_0 = -\Delta_1|2\rangle\langle 2| - \Delta_2|3\rangle\langle 3| - \Delta_3|4\rangle\langle 4| - \{\Omega_P|2\rangle\langle 1| + \Omega_{C1}|3\rangle\langle 2| + \Omega_{C2}|4\rangle\langle 3| + h.c.\} \quad (1)$$

where Ω_P, Ω_{C1} and Ω_{C2} are the half Rabi frequencies corresponding to the laser driven inter subband transitions $|1\rangle \leftrightarrow |2\rangle$, $|2\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |4\rangle$respectively and the detuning Δ_1, Δ_2 and Δ_3 are defined as $\Delta_1 = \omega_P - (\varepsilon_2 - \varepsilon_1)/\hbar$, $\Delta_2 = \omega_{C1} - (\varepsilon_3 - \varepsilon_2)/\hbar + \Delta_1$ and $\Delta_3 = \omega_{C2} - (\varepsilon_4 - \varepsilon_3)/\hbar + \Delta_2$. The state vector of the system is written as, $|\psi\rangle = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle$ where a_j is the time dependent probability amplitude of finding the electron in the subband $|j\rangle$. Maxwell-

Schrödinger equations for the subband amplitudes and probe field in the interaction picture are obtained as

$$\dot{a_1} = i\Omega_P^* a_2, \quad (2a)$$

$$\dot{a_2} = i\Delta_1 a_2 + i\Omega_P a_1 + i\Omega_{C1}^* a_3 - \gamma_2 a_2 \quad (2b)$$

$$\dot{a_3} = i\Delta_2 a_3 + i\Omega_{C1} a_2 + i\Omega_{C2}^* a_4 - \gamma_3 a_3, \quad (2c)$$

$$\dot{a}_4 = i\Delta_3 a_4 + i\Omega_{C2} a_3 - \gamma_4 a_4, \quad (2d)$$

$$\frac{\partial \Omega_P}{\partial z} + \frac{1}{c}\frac{\partial \Omega_P}{\partial t} = i\kappa a_2 a_1^*. \quad (2e)$$

where $\kappa = N|\mu_{12}|^2 \omega_P/(2\hbar\epsilon_0 c)$, N being the electron density in the well and ω_P is the frequency of the probe field. Decay rates $\gamma_j\,(j = 2,3,4)$ have been added phenomenologically to describe the corresponding total decay rate of the subband $|j\rangle$. We assume that $a_j = \sum_K a_j^{(K)}$, $a_j^{(K)}$ is the Kth order part of a_j in terms of Ω_P. In adiabatic framework, $a_j^{(0)} = \delta_{j1}$ and $a_1^{(1)} = 0$, where δ_{j1} is the Kronecker delta. Equations (2a) through (2e) are solved in the linear regime employing the Fourier transform technique to get the probe field propagation equation

$$\frac{\partial \Lambda_P}{\partial z} - i\beta(\omega)\Lambda_P = 0 \quad (3)$$

where Λ_P is the Fourier transform of Ω_P, ω is the Fourier transform variable, and $\beta(\omega)$ the dispersion function. $\beta(\omega) = \omega/c - \kappa\, D_P(\omega)/D(\omega)$. $D_P(\omega) = (\omega + \Delta_2 + i\gamma_3)(\omega + \Delta_3 + i\gamma_4) - |\Omega_{C2}|^2$, $D(\omega) = (\omega + \Delta_1 + i\gamma_2)(\omega + \Delta_2 + i\gamma_3)(\omega + \Delta_3 + i\gamma 4 - \Omega C22\omega + \Delta 1 + i\gamma 2 - \Omega C12\omega + \Delta 3 + i\gamma 4$ The dispersion function $\beta(\omega)$ can be expanded in Taylor series around the central frequency of the probe field i. e. $\omega = 0$.

$$\beta(\omega) = \beta(0) + \omega\beta'(0) + \frac{\omega^2}{2}\beta''(0) + 0(\omega^3) \quad (4)$$

$\beta(0)$ describes the phase shift and linear absorption, $\beta'(0)$ is related to the group velocity v_g =Re$(1/\beta'(0))$ and $\beta''(0)$ represents group velocity dispersion of the probe field. To investigate nonlinear pulse propagation we need to incorporate the effect of optical nonlinear terms in the pulse dynamics. These nonlinear terms are responsible for self-phase modulation which together with GVD leads to shape preserving solitary wave propagation. Including the nonlinear term, equation (2) reduces to

$$\frac{\partial \Omega_P}{\partial z} + \frac{1}{c}\frac{\partial \Omega_P}{\partial t} = i\kappa a_2^{(1)} + NLT.\, a_2^{(1)}, \quad (5)$$

where $NLT = -i\kappa\left[\left|a_2^{(1)}\right|^2 + \left|a_3^{(1)}\right|^2 + \left|a_4^{(1)}\right|^2 - \left(\left|a_2^{(1)}\right|^2 + a312 + a4122\right.\right.$ Following the method developed by Wu and Deng [9] we obtain the modified cubic quintic nonlinear Schrödinger (MCQNLS) equation in the retarded frame defined by $\xi = z$ and $\eta = t - z\beta'(0)$,

$$i\frac{\partial \tilde{\Omega}_P}{\partial \xi} - \frac{1}{2}\beta''(0)\frac{\partial^2 \tilde{\Omega}_P}{\partial \eta^2} + We^{-\alpha z}\left|\tilde{\Omega}_P\right|^2\tilde{\Omega}_P - Me^{-2\alpha z}\left|\tilde{\Omega}_P\right|^4\tilde{\Omega}_P = 0 \quad (6)$$

where W and M represent the cubic and quintic parts of the nonlinearity provided by the quantum well to the probe field.

$$W = \kappa\frac{D_P(0)}{D(0)}\left\{\frac{|D_P(0)|^2 + |\Omega_{C1}|^2[|\Delta_3 + i\gamma_4|^2 + |\Omega_{C2}|^2]}{|D(0)|^2}\right\}, \quad M = \kappa\frac{D_P(0)}{D(0)}\left\{\frac{|D_P(0)|^2 + |\Omega_{C1}|^2[|\Delta_3 + i\gamma_4|^2 + |\Omega_{C2}|^2]}{|D(0)|^2}\right\}^2$$

and $\alpha = 2\text{Im}[\beta(0)]$

When the absorption co-efficient α is small, in terms of the normalized co- ordinates Z and τ, given by $Z = |W_r|\xi$ and $\tau = \sqrt{|W_r|/\beta''(0)}$, the MCQNLS becomes

$$i\frac{\partial \tilde{\Omega}_P}{\partial Z} - \frac{1}{2}\frac{\partial^2 \tilde{\Omega}_P}{\partial \tau^2} - \left|\tilde{\Omega}_P\right|^2\tilde{\Omega}_P + \delta\left|\tilde{\Omega}_P\right|^4\tilde{\Omega}_P = 0 \quad (7)$$

where $\delta = |M_r|/|W_r|$, subscript r signifies real part. In general, coefficients W and M are complex, for suitable set of system parameters, imaginary parts of these coefficients may be made very small in comparison to their real parts. The parameters taken in our study are as follows: $N = 10^{16}cm^{-3}$, $\mu_{12} = 13eA^0$, $\omega_P = 18.08 \times 10^{13}s^{-1}$ thus $\kappa = 1.4 \times 10^{11}\mu m^{-1}s^{-1}$, decay rates $\gamma_2 = 4.8 \times 10^6 s^{-1}$, $\gamma_3 = 3.8 \times 10^6 s^{-1}$, $\gamma_4 = 4.2 \times 10^{11}s^{-1}$, Rabi frequencies $\Omega_{C1} = 4.0 \times 10^{11}s^{-1}$, $\Omega_{C2} = 4.0 \times 10^{11}s^{-1}$ detunings $\Delta_1 = -1.0 \times 10^{11}s^{-1}$, $\Delta_2 = -2.0 \times 10^{12}s^{-1}$, $\Delta_3 = -4.0 \times 10^{12}s^{-1}$. Under such situations, it is possible to obtain shape preserving soliton solution which propagates over long distance with subluminal group velocity $v_g = 7.72 \times 10^{-6}c$. The robust soliton of equation (5) can be obtained for any arbitrary value of δ which may be written as [9]:

$$\left|\tilde{\Omega}_P(Z,\tau)\right| = \frac{2^{1/2}\Lambda\, exp\left(-i\Lambda^2 Z/2\right)}{\left[1 + \left(1 - \frac{8}{3}\delta\Lambda^2\right)^{1/2} cos\,h(2\Lambda\tau)\right]^{1/2}}, \quad (8)$$

The parameter Λ is related to soliton amplitude and width and thus it determines soliton energy. The expression for soliton energy Q can be obtained using $\int_{-\infty}^{+\infty}\left|\tilde{\Omega}_P(Z,\tau)\right|^2 d\tau$, which comes out to be

$$Q = -\left(\frac{6}{\delta}\right)^{1/2} tanh^{-1}\left\{\frac{6^{1/2}\left[-3 + \left(9 - 24\delta\Lambda^2\right)^{1/2}\right]}{12\Lambda\delta^{1/2}}\right\}$$

To ensure that Q is always positive, we must have

$$\delta\Lambda^2 < 3/8.$$

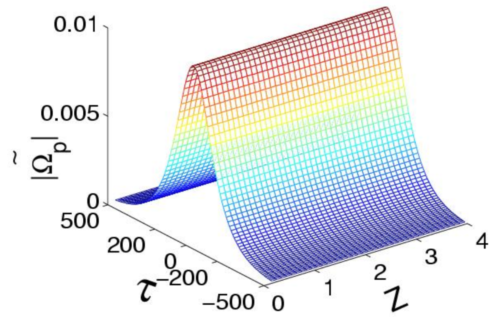

Figure 2: Stable soliton shape of the probe pulse $|\tilde{\Omega}_P(Z,\tau)|$ as a function of Z and τ .

3. CONCLUSION

We have shown the possibility of generation of ultraslow bright optical solitons due to Kerr and quintic nonlinearities in asymmetric three-coupled quantum well systems. These bright solitons arise due to nonlinearities generated by a probe pulse and two controlling laser beams. With the help of numerical simulation of nonlinear Schrödinger equation, we have demonstrated that these solitons are stable during their propagation.

4. ACKNOWLEDGEMENTS

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