Side Lobe Level Reduction of a Linear Array using Chebyshev Polynomial and Particle Swarm Optimization

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ABSTRACT
In this paper a new approach for reducing the side lobe level of amplitude tapered linear array using Chebyshev polynomial and Particle Swarm Optimization (PSO) is presented. The array geometry synthesis is first formulated as an optimization problem with the goal of side lobe level reduction and then solved using PSO algorithm for optimum current excitations. While solving optimization problem using PSO algorithm, the boundary of search space must be available with the algorithm. Conventionally this piece of information is supplied to the algorithm in a random fashion. Instead of selecting the search space boundary in random fashion, it is being proposed to do so by using Chebyshev polynomial. Some designs examples are presented that illustrates the use of Chebyshev polynomial and PSO algorithm to reduce the Side Lobe Level (SLL).

Keywords
Linear array, Array factor, Chebyshev polynomial, PSO, Side Lobe Level.

1. INTRODUCTION
In many communication applications it is desirable to have antennas with very high directive characteristics. One way of achieving high directive characteristics is to form an assembly of radiating elements in certain electrical and geometrical configuration. This nomenclature is termed as an array. The total field of the array is the vector summation of the fields radiated by the individual elements. Geometrical configuration of the overall array, relative displacement between elements, excitation amplitude of the individual elements and relative pattern of the individual elements shapes the radiation pattern of an antenna array[1-2].

One of the primary subjects of investigation in antenna array synthesis is Side Lobe Level (SLL) reduction. Variation in the excitation amplitude of each element of the array can help in achieving a low SLL. This type of variation affects antenna parameters like half power beamwidth (HPBW), directivity etc.. As such a tradeoff between the parameters of interest is required. Dolph in his work reported a way out for obtaining the narrowest possible beamwidth for a given side-lobe level or the smallest side-lobe level for a given beamwidth [3]. Dolph used orthogonal Chebyshev function to design an array with optimum radiation pattern. However, if number of elements of the array increases, this procedure becomes tedious, as it requires matching the array factor expression with an appropriate Chebyshev function. To overcome this difficulty Safaai-Jazi [6] proposed a new methodology for the design of Chebyshev arrays based on solving a system of linear equations. Iterative process was used to obtain the desired pattern [6]. All these methods are based on certain analytical formula which sometimes becomes difficult to analyze. This led to the use of evolutionary algorithms to reduce the side lobe level (SLL) [7-8]. Genetic algorithms (GA) and particle swarm optimization (PSO) are well known evolutionary algorithm techniques. Some studies have been done to compare between GA and PSO [10]. Studies suggest that PSO shows better performance due to its greater implementation simplicity and less computational time.

Presnt work aims at reducing the Side Lobe Level of an amplitude tapered linear array using PSO algorithm, where Chebyshev polynomial is used to define the search space boundary. The paper is organized as follows. In section 2, amplitude tapered linear array is overviewed. A background on use of Chebyshev polynomial to design array is addressed in Section 3. An overview of particle swarm optimization is being presented in Section 4. Illustrative examples and results are given in Section 5. The work ends with Section 6 in which, conclusions are pointed out.

2. LINEAR ARRAY
If elements of the array are placed along a line then such a configuration is termed as linear antenna array. The geometry is as shown in fig.1.In synthesis of linear antenna arrays the elements are considered to be isotopic radiators.

The array factor for an even numbered array can be expressed as [2]:

$$AF(\theta) = 2\sum_{n=1}^{N} a_n \cos\left[\left(\frac{2n-1}{2}\right)kd\cos(\theta)\right] \tag{1}$$

where 2N is the number of elements of the array, k is the wave number, a_n is the excitation amplitude of the nth element, d is the uniform inter-element spacing between each elements and θ represents the angular separation.

In the normalized form, (1) can be written as

$$AF(\theta) = \sum_{n=1}^{N} a_n \cos\left[\left(\frac{2n-1}{2}\right)kd\cos(\theta)\right] \tag{2}$$

(1) and (2) represents broadside case of amplitude tapered linear array.

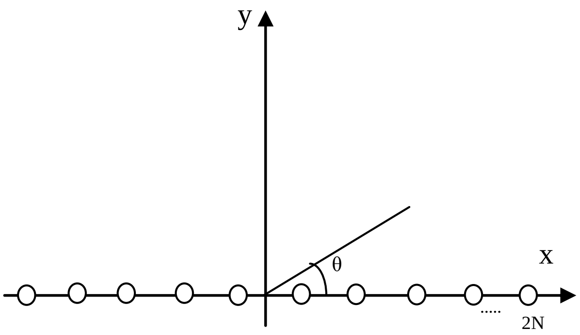

Figure (1) Geometry of the 2N-isotropic element symmetric linear array placed along the x-axis

3. CHEBYSHEV POLYNOMIAL AND LINEAR ARRAY DESIGN

There are different methods to find out the excitation current amplitude for each element of the array. One such method utilizes a set of polynomials referred to as Chebyshev polynomials, after the Russian mathematician Pafnuti Chebyshev (1821-1894).

The method was originally introduced by Dolph [3], investigated by others [4-5]. The method is as explained:

Referring to (2), the array factor of even numbered array with symmetric excitation amplitude (a_n) is equal to the summation of N cosine terms. Each cosine term, whose argument is an integer times a fundamental frequency, can be rewritten as a series of cosine functions with fundamental frequency as argument.

The recursion formula for Chebyshev polynomial is

$$T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z) \quad (3)$$

Each polynomial can also be calculated as

$$T_m(z) = \cos[m\cos^{-1}(z)],\ -1 \le z \le 1 \quad (4a)$$

$$T_m(z) = \cosh[m\cosh^{-1}(z)],\ z<-1,\ z>1 \quad (4b)$$

Here m defines the order of the polynomial which should be one less than the total number of elements.

The array factor is a summation of cosine terms whose form is same as the Chebyshev polynomials; the unknown coefficients of the array factor can be determined by equating the series of the array factor to the appropriate Chebyshev polynomial. Values of these coefficients define the range of excitation amplitude for a particular array configuration.

4. PARTICLE SWARM OPTIMIZATION (PSO) ALGORITHM

PSO is a stochastic optimization technique that has been effectively used to solve multidimensional discontinuous optimization problems in a variety of fields [9-12]. The algorithm is as explained:

Consider swarm of bees that encounter in their travel an open

field with flowers. The natural instinct of the swarm is to find the most suitable place where there is maximum number of flowers. The swarm has no previous knowledge of the field, so each bee spread out and begins their search in random locations. During the search process each bee updates its position and velocity based on two pieces of information. The first is its ability to remember the previous location where it has found the most flowers (particle best (p_{best})). The second information is all about the location of the most flowers found by all the bees of the swarm (global best (g_{best})) at the present instant of time. This process of continuous updating of velocity and position continues until one of the bees finds the location of highest density of flowers in the search space. Ultimately all the bees will be drawn to this location since they will not be able to find any other better location. The process is pictorially represented in fig.2[13].

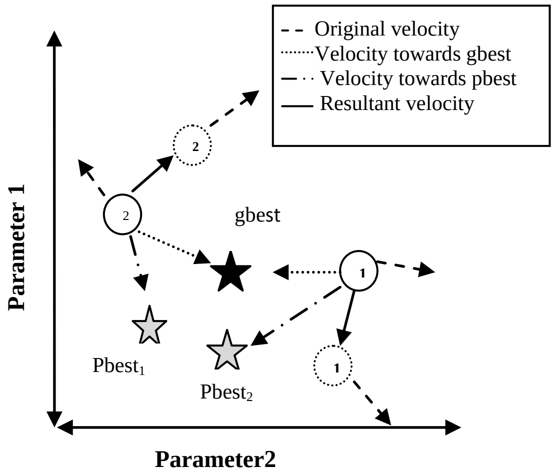

Figure 2 Individual particles (1and 2) are accelerated toward the location of the best solution g_{best}, and the location of their own personal best p_{best}, in a 2-dimensional parameter space.

In present application of PSO, particle's velocity is manipulated according to the following equation:

$$v_{n+1} = wv_n + c_1 rand()[p_{best,n} - x_n] + c_2 rand()[g_{best} - x_n] \quad (5)$$

where v_n is the velocity of the particle in the n^{th} dimension and x_n is the particle's coordinate in the n^{th} dimension. The parameter w is the inertial weight that specifies the weight by which the particle's current velocity depends on the previous velocity. rand() is a random function generating a number in the range [0,1]. In this work c_1 is decreased from 2.5 to 0.5 whereas c_2 is increased from 0.5 to 2.5 [12]. This facilitates the global search over the entire space during the early part of the optimization. This also encourages the particles to converge to global optima at the end of the search. The inertia weight w decreases linearly from w = 0.9 to 0.4. After the time step, the new position of the particle is given by:

$$x(t+1) = x(t) + v(t+1) \quad (6)$$

The termination criterion depends upon whether the number of iteration equals the predefined maximum number of iteration or the value of g_{best} is close to the desired value. If none of the conditions satisfies the algorithm continues to update the position and velocity. Fig.3 shows the flowchart of the algorithm that has been used in present work.

To achieve the lowest peak SLL, the fitness function is defined as [13-14]:

$$f = \max\left\{20\log\left|\frac{(AF(\theta))}{AF_{\max}(\theta)}\right|\right\} \quad (7)$$

in the side lobe region.

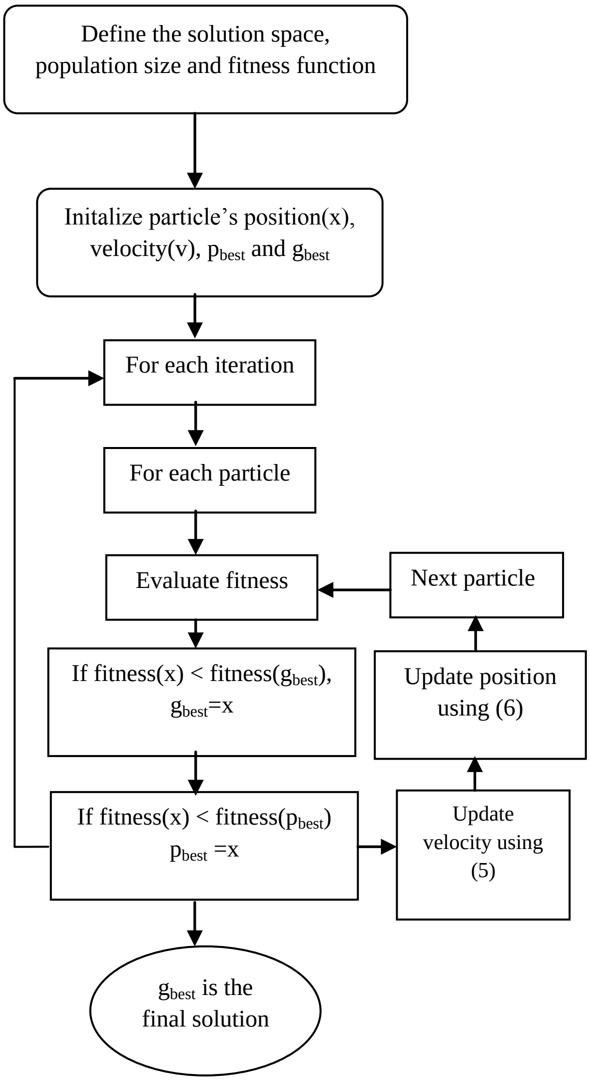

Figure (3) Flow chart depicting the main steps of PSO algorithm.

The optimization problem is modeled in to min-max problem. In every iteration, algorithm tries to find current excitation that leads to minimum value of fitness function. This finally leads to side lobe level reduction.

5. ILLUSTRATIVE EXAMPLES

In this section, the capabilities of new methodology are being demonstrated by some examples. First of all Chebyshev polynomial defines the search space boundary for -20 dB Side lobe level. The range obtained for 12 elements is 1.1602 to 2.0981. This information is used by PSO that returned optimum excitation amplitude resulting in almost equi-ripple array factor. Fig.4 shows the radiation pattern of a 12 element array. The newly defined range became 0.9348 to 2.0162.The Side Lobe Level (SLL) gets reduced to -23.31 dB using the same search space. Also the pattern is almost equi-ripple in nature. The corresponding optimization curve is as shown in fig.5. The inter-element spacing is fixed at λ/2 for this case.

The convergence occurs at 896th iteration. Total number of particles used for optimization is 25. The optimum values of current excitation and other parameters obtained for this array along with other array configurations are tabulated in Table 1. There are deviations in different parameters of the optimized array with respect to Chebyshev array. Table2 and table3 shows parametric comparison with some of the methods available in open literature [14-15]. Though there is little deviation in different parameters of the arrays a higher amount of SLL reduction is observed.

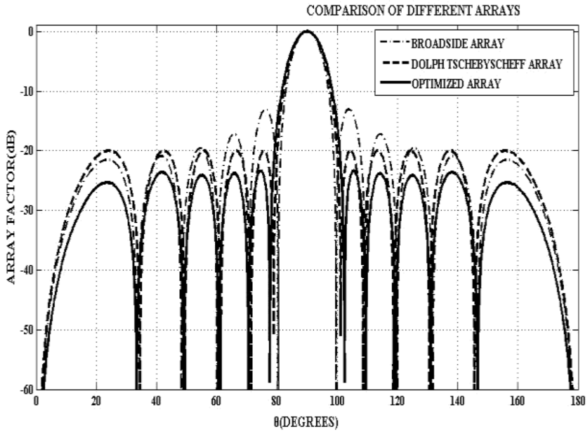

Figure (4) Radiation pattern of 12 element Conventional Broadside Array, Chebyshev and Optimized array

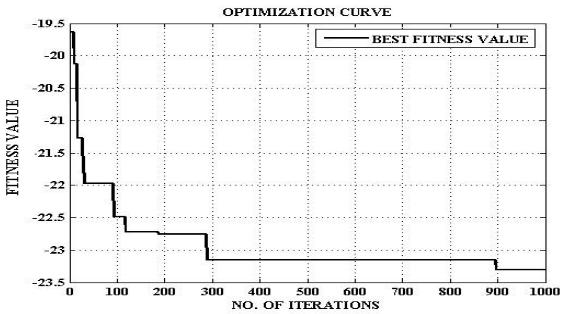

Figure (5) Convergence curve showing optimum Side Lobe Level for 12 element Optimized array.

Table 1 Magnitude of current excitations and other parameters for different array configurations with λ/2 Spacing.

Array Size	Parameters	Chebyshev Array	Optimized Array (Amplitude tapering)
4	SLL (dB)	-20	-24.7854
	Excitation amplitudes	6.3447, 3.6553	5.0710, 2.4652
	Directivity (dB)	5.7173dB	5.5301
	FNBW	76.8degrees	85.4degrees
	HPBW	30.0812 degrees	31.475 degrees
12	SLL(dB)	-20	-23.3122
	Excitation amplitudes	2.0981, 1.9854, 1.7735, 1.4876, 1.1602, 1.4953	2.0162, 1.8562, 1.6625, 1.3027, 0.9971, 0.9348

	Directivity (dB)	10.6339	10.5339
	FNBW	22.4degrees	24.8degrees
	HPBW	9.2012 degrees	9.8418 degrees
16	SLL(dB)	-40	-41.5059
	Excitation amplitudes	21.9893 20.5675 17.9507 14.5418 10.8320 7.2993 4.3179 2.5015	20.634 19.261 16.611 13.248 9.763 6.347 3.6122 1.8412
	Directivity (dB)	10.8731	10.7594
	FNBW	26.6 degrees	27.8 degrees
	HPBW	8.9924 degrees	9.2256 degrees

Table 2 Parametric comparison-1

Array Size	Parameters	Referred work [14]	Optimized Array (Amplitude tapering)
12	SLL (dB)	-28.5	-30.6663
	FNBW (degrees)	28degrees	30.2degrees
16	SLL (dB)	-22.4	-24.6536
	FNBW (degrees)	17.6	19.4
20	SLL (dB)	-21.15	-24.3606
	FNBW (degrees)	11.1	15
24	SLL (dB)	-21.05	-23.9608
	FNBW (degrees)	11.1	12.4

Table 3 Parametric comparison-2

Array Size	Parameters	Referred work [15]	Optimized Array (Amplitude tapering)
8	SLL (dB)	-40	-42.0753
	HPBW (degrees)	13.8degrees	18.4871 degrees
12	SLL (dB)	-45	-45.2024
	HPBW (degrees)	9.4	12.7619

6. CONCLUSION

Application of Chebyshev polynomial to define the search space boundary for PSO algorithm to reduce the side lobe level (SLL) of linear array has been demonstrated in this paper. This removes the requirement of having the knowledge of upper and lower limit of search space at the time of developing the optimization algorithm. As such the required side lobe level gets defined. Results shows higher amount of side lobe level reduction than the defined value. In comparison to other methods, the proposed method achieves greater side lobe level reduction. The method needs further investigation with beamwidth narrowing maintaining same side lobe level value.

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