Applications of Fuzzy Logic in Decision Making Theory

P.Bharathi
Asst.Prof of Mathematics
Sri Sarada Niketan College For Women
Amaravathipudur, Sivagangai district.

ABSTRACT
Most important decisions in organizations are finalized by group of experts. Human judgments including preferences are often vague and cannot be estimated in exact numerical values. This paper proposes a user-friendly fuzzy approach under the linguistic frame work to obtain optimal solution for Multi Criteria Decision Making problems. To accomplish this, an aggregate-deviation method based on fuzzy numbers is proposed. A fuzzy decision matrix plays an important role in our research problem .The purpose of this method is to enhance group agreement on the group decision making outcomes.

GENERAL TERMS
Multi Criteria Decision Making, Fuzzy numbers, Linguistic variables.

KEYWORDS
Fuzzy decision matrix, Aggregate- deviation method

1. INTRODUCTION
1.1 Decision Making
Decision making can be defined as a process of specifying a problem, identifying and evaluating criteria or alternatives and selecting a preferred alternative among possible ones (Chen, 2005).

1.2 Multi-criteria Decision Making (MCDM
Multi-criteria decision making (MCDM) method**)** is a technique where alternatives or options are assessed based on a set of criteria and it is one of the most widely used methods in decision making (Hwang & Yoon, 1981). MCDM methods have been employed in many areas such as engineering, agricultural, banking, energy, forestry, health services and education. General form of MCDM problem with *m* alternatives and *n* criteria can be illustrated in matrix format as follows:

Alternatives/Criteria	*C1*	*……Cn*
A1	—	—
.	.	.
.	.	.
Am	—	—

1.3 Fuzzy MCDM
In real life, decision makers often make evaluation based on a set of criteria which are normally vague and imprecise. Due to this, fuzzy set was introduced particularly in representing the vague information or criteria. Fuzzy set theory was first utilized in solving decision making problem by Bellman and Zadeh in 1970. The key concept of fuzzy set theory is that its elements have a varying grade of membership, ranging from 0 to 1. The boundaries of these fuzzy sets are not sharp or imprecise. The individual membership in a fuzzy set is represented by the *degree of compatibility* (Klir et.al, 1997) and fuzzy sets are used to describe linguistic values for example "very good," "good," "fair," "poor," and "very poor". Instead of using exact numbers as input values, fuzzy numbers were utilized in representing these linguistic terms.
The introduction of fuzzy set theory also motivates many researchers in integrating the theory with some of the classical MCDM methods. Pioneer work in incorporating fuzzy element into decision making was done (Baas and Kwakernaak, 1977) by introducing an algorithm for rating and ranking multiple aspects of alternatives using fuzzy sets. Decision makers' opinions can be expressed in terms of linguistic variables.

2 . PRELIMINARIES
2.1 Fuzzy set:
A fuzzy set X is a function f: X→[0,1].

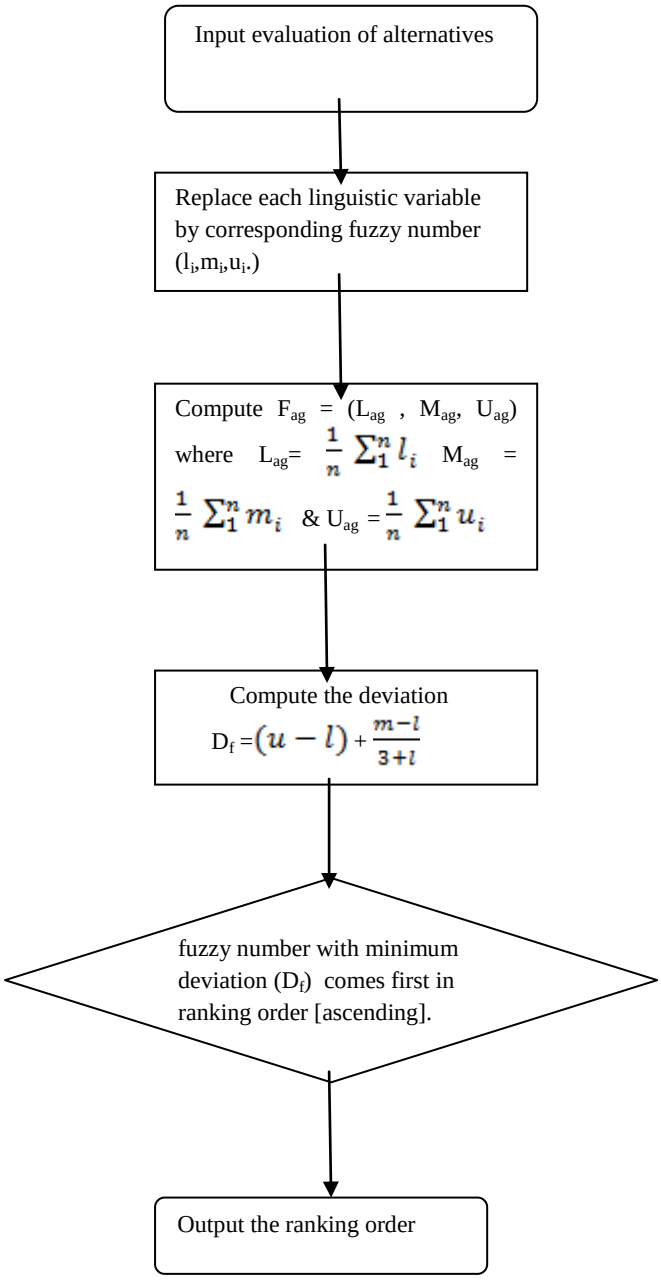
2.2 Fuzzy number:
Fuzzy numbers are uncertain numbers for which, in addition to knowing a range of possible values. A fuzzy number A_f is a normalized and convex fuzzy set.

Examples
Triangular fuzzy numbers, e.g., [1,2,3]
Trapezoidal fuzzy numbers, e.g., [1,2,3,4]

2.3 Linguistic Variable:
A linguistic variable is a variable whose values are words or sentences in a natural or artificial language. These linguistic variables can be expressed in positive triangular fuzzy numbers.

3. PROPOSED METHOD
- Collect the evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables and we can form a decision matrix.
- Replace each linguistic variable by corresponding fuzzy number.
- Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2 ,……..
- Aggregate the fuzzy numbers column wise based on decision makers P_1, P_2 ,…..
- Find the deviation in triangular fuzzy number.
- The fuzzy number with minimum deviation comes first in ranking order [ascending].

4. FIGURE



5. COMPUTATIONAL ASPECTS

Suppose group of expert decision makers want to select a most suitable candidate from several alternatives based on some criteria.

STEP 1

Evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables.

Linguistic frame work **[1].**

Very Poor	VP	(0,0,1)
Poor	P	(0,1,3)
Medium Poor	MP	(1,3,5)
Fair	F	(3,5,7)
Medium Good	MG	(5,7,9)
Good	G	(7,9,10)
Very Good	VG	(9,10,10)

Evaluation table

		C_1			C_2			……			…C_m		
P 1	**A 1**	**L 1**	**L 2**	**L 3**	…	. .	…	. .	. .	. .	. .	. .	. .
	. .												
	A n												
P 2	**A 1**											. .	. .
.	. .												
P k	**A n**												

Here C_1, C_2,…. are the criteria. A_1, A_2, A_3…. are the alternatives. P_1,P_2,…. are the decision makers.L_1, L_2,… are the linguistic variables.

STEP 2

To construct a fuzzy decision matrix replace each linguistic variable by corresponding fuzzy number.

STEP 3

Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2,…….. by using the formula F_{ag} = (L_{ag} , M_{ag}, U_{ag})

where $L_{ag}= \frac{1}{n}\sum_1^n l_i$, $M_{ag} = \frac{1}{n}\sum_1^n m_i$ & $U_{ag} = \frac{1}{n}\sum_1^n u_i$ for all fuzzy numbers (l_i,m_i,u_i).

We get,

	P_1	P_2……..	…..P_k
A_1	$F_{ag\ 1}$	$F_{ag\ 2}$……	
. . .	. . .		
A_m	……..		……

Where $F_{ag\ 1}, F_{ag\ 2}$…… are fuzzy numbers.

STEP 4

Aggregate fuzzy numbers column wise using the same formula based on decision makers P1, P2,…..

We get

A_1	$F_{ag\ 1}$
A_2	$F_{ag\ 2}$
.	.
A_m	$F_{ag\ m}$

STEP 5

Find the deviation in triangular fuzzy number by using the formula. $D_f = (u-l) + \frac{m-l}{3+l}$ where (l,m,u) is triangular fuzzy number.

STEP 6 [CONCLUSION]

The fuzzy number with minimum deviation (D_f) comes first in ranking order [ascending].

6. NUMERICAL EXAMPLE

Suppose 3 expert decision makers want to select a most suitable candidate from 3 alternatives based on 3 criteria which are attitude, communication skills, hardworking .

STEP 1

Collect the evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables and we can form a decision matrix.

	C1			**C2**			**C3**			**C4**			**C5**		
	A 1	**A 2**	**A 3**	**A 1**	**A 2**	**A 3**	**A 1**	**A 2**	**A 3**	**A 1**	**A 2**	**A 3**	**A 1**	**A 2**	**A 3**
P 1	MG	G	VG	G	VG	MG	F	VG	G	VG	VG	VG	F	VG	G
P 2	G	G	G	MG	VG	G	G	VG	MG	G	VG	VG	F	MG	G
P 3	MG	MG	F	F	VG	VG	G	G	VG	VG	VG	MG	F	G	MG

STEP 2

Replace each linguistic variable by corresponding fuzzy number.

FUZZY DECISION MATRIX						
		C1	**C1**	**C3**	**C4**	**C5**
P 1	**A_1**	(5,7,9)	(7,9,10)	(3,5,7)	(9,10,10)	(3,5,7)
	A_2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)
	A_3	9,10,10	(5,7,9)	(7,9,10)	(7,9,10)	(7,9,10)
P 2	**A_1**	(7,9,10)	(5,7,9)	(7,9,10)	(7,9,10)	(3,5,7)
	A_2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(5,7,9)
	A_3	(7,9,10)	(7,9,10)	(5,7,9)	(9,10,10)	(7,9,10)
P 3	**A_1**	(5,7,9)	(3,5,7)	(7,9,10)	(9,10,10)	(3,5,7)
	A_2	(5,7,9)	(9,10,10)	(7,9,10)	(9,10,10)	(7,9,10)
	A_3	(3,5,7)	(9,10,10)	(9,10,10)	(5,7,9)	(5,7,9)

STEP 3

Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2 ,……..

		C_1	C_2	C_3	C_4	C_5	Aggregation
P_1	A_1	(5,7,9)	(7,9,10)	(3,5,7)	(9,10,10)	(3,5,7)	(5.4,7.2,8.6)
	A_2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)	(8.6, 9.8, 10)
	A_3	9,10,10	(5,7,9)	(7,9,10)	(7,9,10)	(7,9,10)	(7,8.8,9.8)
P_2	A_1	(7,9,10)	(5,7,9)	(7,9,10)	(7,9,10)	(3,5,7)	(5.8,7.8,9.2)
	A_2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(5,7,9)	(7.8,9.2,9.8)
	A_3	(7,9,10)	(7,9,10)	(5,7,9)	(9,10,10)	(7,9,10)	(7,8.8,9.8)
P3	A_1	(5,7,9)	(3,5,7)	(7,9,10)	(9,10,10)	(3,5,7)	(5.4,7.2,8.6)
	A_2	(5,7,9)	(9,10,10)	(7,9,10)	(9,10,10)	(7,9,10)	(7.4,9,9.8)
	A_3	**(3,5,7)**	**(9,10,10)**	**(9,10,10)**	**(5,7,9)**	**(5,7,9)**	**(6.2,7.8,9)**

We get

	P_1	P_2	P_3
A_1	(5.4,7.2,8.6)	(5.8,7.8,9.2)	(5.4,7.2,8.6)
A_2	(8.6, 9.8, 10)	(7.8,9.2,9.8)	(7.4,9,9.8)
A_3	(7,8.8,9.8)	(7,8.8,9.8)	(6.2,7.8,9)

STEP 4

Aggregate the fuzzy numbers column wise based on decision makers P_1, P_2 ,…..

	Total evaluation
A_1	(5.53,7.4,8.8)
A_2	(7.93,9.33,9.86)
A_3	(6.73, 8.46, 9.53)

STEP 5

Find the deviation in triangular fuzzy number by using $D_f = (u - l) + \frac{m-l}{3+l}$.

For A_1 (l=5.53,m=7.4,u=8.8) **D_f=3.489**

For A_2 (l=7.93,m=9.33,u=9.86) **D_f=2.058**

For A_3 (l=6.73, m=8.46, u=9,53) **D_f=2.977**

STEP 6

The fuzzy number A_2 with minimum deviation **D_f=2.058** comes first in ranking order [ascending]. The final ranking order is A_2,A_3,A_1.

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