Design and Implementation of Fractional Order Controller for CSTR Process

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ABSTRACT
A Fractional Order (FO) Proportional- Integral- Derivative (PID) controller has been proposed in this paper which works on the closed loop error and its fractional derivative and fractional integrator. FOPID is a PID controller whose derivative and integral orders are of fractional rather than integer. The extension of derivative and integral order from integer to fractional order provides more flexibility in design of the controller, thereby controlling wide range of dynamics of a system. Frequency domain specifications are used as the performance criteria to be optimizing the FOPID controller parameters: Proportional (Kp), Integral (Ki), Derivative (Kd) gains, integral order (λ), and the derivative order (μ).

General Terms
Modelling of CSTR, Control of CSTR

Keywords
Fractional order controller, CSTR process Tuning, ZN method, Astrom method

1. INTRODUCTION
The important requirement for a closed loop control system including the controller is to maintain the stability and robustness through the rejection of the disturbance and elimination of noise. The most popular controllers are the PID controllers which has parameters to be tuned to get a well specification for the system both in time domain and frequency domain.

The most important advantages of the $PI^{\lambda}D^{\mu}$ controller is that it provides very good control on dynamical systems and it is affected much lesser for variations in control system parameters. The first time, calculus generation to fractional, was proposed Leibniz and Hopital after words, the systematic studies in this field carried out by many researchers such as Liouville(1832), Holmgren (1864) and Riemann (1953).Due to widespread usage of PID controller in industries and product manufactures so researchers always motivated to look for abettor and suitable design method or alternative controller.

For example, the fractional order algorithm for the control of dynamic systems has been introduced by utilization of CRONE (French abbreviation for Command Robusted'Ordre Non Entier), over the PID controller, which has been demonstrated by Oustaloup. PID controller generalization has been proposed by Podlubny as $PI^{\lambda}D^{\mu}$ controller which is known as fractional order PID controller, where λ is the non-integer order of integrator and μ is the non-integer order of the differentiator term. He also demonstrated that the response of the $PI^{\lambda}D^{\mu}$ controller is good on comparing with classical PID controller. Frequency domain approaches of $PI^{\lambda}D^{\mu}$ controller are studied and for the implementation of fractional order controllers also carried out. Crucial importance of tuning of the controllers cannot be underestimated. Thus, many tuning techniques for obtaining the parameters of the controllers were introduced during last few decades. Tuning methods of $PI^{\lambda}D^{\mu}$ controllers are recent research subject. Most of the researchers oriented to the classical optimization and intelligent methods.

Some tuning rules for robustness to plant uncertainty for $PI^{\lambda}D^{\mu}$ controller are given in literature [9-14]. However in order to achieve better results, there are still needs for new methods to obtain the parameters of $PI^{\lambda}D^{\mu}$ controllers. In this paper, optimization is used to obtain the parameters of fractional $PI^{\lambda}D^{\mu}$ controller.

2. FRACTIONAL ORDER $PI^{\lambda}D^{\mu}$ CONTROLLER
The most common form of a fractional order PID controller is the $PI^{\lambda}D^{\mu}$ controller involving an integrator of order λ and a differentiator of order μ where λ and μ can be any real numbers. The transfer function of such a controller has the form

$$Gc(s) = \frac{U(S)}{E(S)} = k_p + k_I \frac{1}{S^{\lambda}} k_p + k_D S^{\lambda}, (\lambda, \mu > 0) \quad (1)$$

Where $Gc(s)$ is the transfer function of the controller, $E(s)$ is an error, and $U(s)$ is controller's output. The integrator term is 1 s^{λ}, that is to say, on a semi-logarithmic plane, there is a line having slope -20^{λ}dB/decade. The control signal u(t) can then be expressed in the time domain as

$$u(t) = k_p e(t) + k_I D^{-\lambda} e(t) + k_D D^{\mu} e(t) \quad (2)$$

Fig. 1 shows the block-diagram configuration of FOPID. Clearly, selecting $\lambda = 1$ and $\mu = 1$, a classical PID controller can be recovered. The selections of $\lambda = 1$, $\mu = 0$, and $\lambda = 0$, $\mu = 1$ respectively corresponds conventional PI & PD controllers

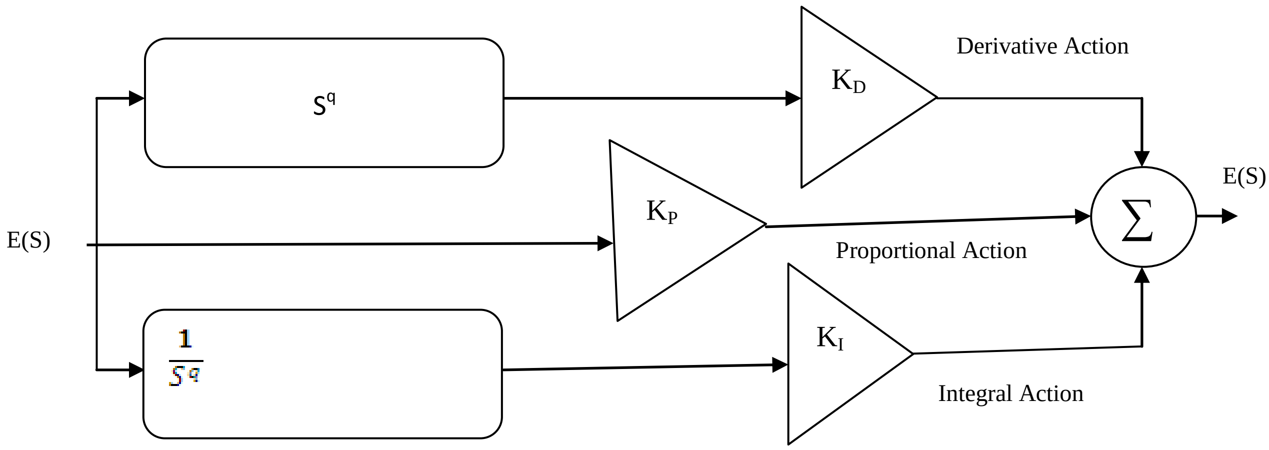

Fig 1: Block diagram configuration of Fractional Order PID controller

It can be expected that the $PI^{\lambda}D^{\mu}$ controller may enhance the systems control performance. One of the most important advantages of the $PI^{\lambda}D^{\mu}$ controller is that it provides very good control on dynamical systems and it is affected much lesser for variations in control system parameter.

3. MODELING OF THE CONTINUOUS STIRRED TANK REACTOR SYSTEM

The concentration of the outlet flow of two chemical reactors will be forced to have a specified response in this section [7]. Figure 2 shows the simple concentration process control. It is assumed that the overflow tanks are well-mixed isothermal reactors, and the density is the same in both tanks. Due to the assumptions for the overflow tanks, the volumes in the two tanks can be taken to be constant, and all flows are constant and equal. It is assumed that the inlet flow is constant. Figure 3 shows the block diagram of two tanks of chemical reactor.

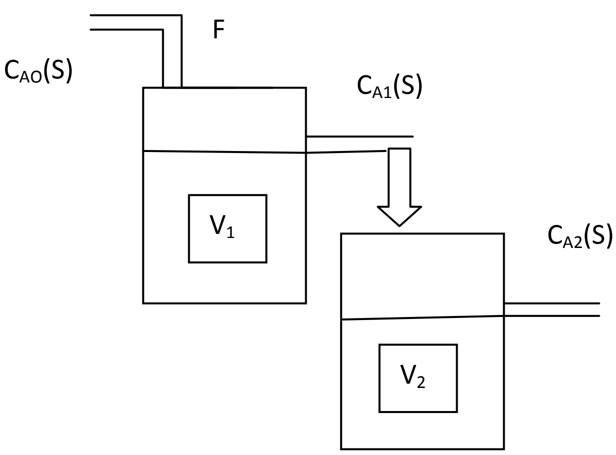

Fig 2 : The simple concentration Process control

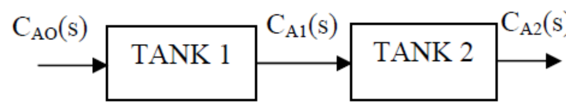

Fig 3: The block diagram of the two tank system

The value of the concentration in the second tank is desired, but it depends on the concentration in the first tank. Therefore the component balances in both tanks are formulated. The transfer function of the first tank can be obtained as;

$$V_1 \frac{dC_{A2}}{dt} = FC_{A0} - FC_{A1} - V_1 \mathbf{K} C_{A1}$$
(3)

Where V_1 is the volume of the first tank, F is the flow, C_{A0} is the inlet concentration of the first tank, C_{A1} is the outlet concentration of the second tank and K is the reaction rate. Equation 3 can be rearranged to be

$$\frac{dC_{A2}}{dt} + \frac{1}{\tau_2} C_{A1} - FC_{A1} = \frac{F}{V_1} C_{A0}$$
(4)

Where $\tau_1 = \frac{V_1}{F + V_1}$ is the time constant of the first tank.

By taking Laplace Transform and rearranging equation 4, the transfer function of the first tank can be expressed as

$$\frac{C_{A1}(s)}{C_{A0}(s)} = \frac{k_{p1}}{\tau_1 s + 1}$$
(5)

Where $k_{p1} = \frac{F}{F + KV_1}$ is the gain of the transfer function of the first tank. The transfer Function of the second tank can be derived.

$$V_2 \frac{dC_{A2}}{dt} = FC_{A1} - FC_{A2} - V_2 K C_{A2}$$
(6)

Where V_2 and C_{A2} are the volume and the inlet concentration of the second tank respectively. Equation (6) can be rearranged to be

$$\frac{dC_{A2}}{dt} + \frac{1}{\tau_2} C_{A2} = \frac{F}{V_2} C_{A2}$$
(7)

Where $\tau_2 = \frac{V_2}{F + KV_2}$ is the time constant for the second tank.

By taking Laplace Transform and rearranging equation (7), the transfer function of the second tank can be obtained.

$$\frac{C_{A2}(s)}{C_{A1}(s)} = \frac{k_{p2}}{\tau_2 s + 1}$$
(8)

Where $k_{p2} = \frac{F}{F + KV_2}$ is the gain of the transfer function of the second tank. The transfer function of the whole system can be obtained according to the following assumptions of parameters.

1. The flow rate is constant for the whole system F= 0.085 m^3/min.

2. The volume of the two tanks is the same V_1=V_2=V=1.05 m^3.

3. Reaction Rate K= 0.05min^{-1} .

Since the time constants and the gains are equal for both tanks, they can be completed as follows:

$$\tau = \frac{V}{F + KV} = 8.25\ min$$

$$K = \frac{V}{F + KV} = 0.0669$$

The transfer function of the combined two tanks with the assumed parameters can be obtained

$$G(s) = \frac{C_{A2}(s)}{C_{A0}(s)} = \frac{{k_p}^2}{(\tau_2 s + 1)^2}$$
(9)

4. MATHEMATICAL MODELING AND ANALYSIS OF FOPID

To obtain the K_p (proportional gain), a constant of integral term (K_i), the constant of derivative term K_d, the fractional order of differentiator μ and the fractional order of integrator λ . The method uses classical Zeigler – Nichols tuning rule [1]to obtain K_p and K_i. To obtain initial value of K_d, then some fine tuning has been done by using Astrom Hagglund method described earlier[3,4]. The fractional order λ and μ are obtained to achieve specified phase margin.

Let ϕ_{pm} be the required phase margin and $j\omega_{cp}$ be the frequency of the critical point on the Nyquist curve of G(s) at ($G(j\omega_{cp})$ = -180) then the gain margin defined as ,

$$g_m = \frac{1}{|G(j\omega cp)|} = k_c$$

In order to make the phase margin of the system equal to ϕ_{pm}

And $|C(j\omega_{cp})\ G(j\omega_{cp}) = 1|$, the following equation must be satisfied.

$$G(j\omega cp)\,|\,)\, e^{j\phi_{pm}} = k_c \cos\phi_{pm} + j\, k_c \sin\phi_{pm}$$ (10)

Then we write $C(j\omega_{cp})$ using equation

$$C(j\omega cp) = k_p + k_i \omega_{cp}^{-\lambda} \cos\left(\frac{\pi}{2}\lambda\right) + k_d \omega_{cp}^{\mu} \cos\left(\frac{\pi}{2}\mu\right) + \left[- K_i \omega_{cp}^{-\lambda} \sin\left(\frac{\pi}{2}\lambda\right) + k_d \omega_{cp}^{\mu} sin\left(\frac{\pi}{2}\mu\right)\right]$$
(11)

Considering equation 10 and 11 we can write

$$f_1(\lambda,\mu) = k_p + k_i \omega_{cp}^{-\lambda} \cos\left(\frac{\pi}{2}\lambda\right) + k_d \omega_{cp}^{\mu} \cos\left(\frac{\pi}{2}\mu\right) - k_c\ (\cos][\phi_{p}$$
(12)

$$f_2(\lambda,\mu) = - k_i \omega_{cp}^{-\lambda} \sin\left(\frac{\pi}{2}\lambda\right) + k_d \omega_{cp}^{\mu} \sin\left(\frac{\pi}{2}\mu\right) - k_c\ (\sin][\phi_{pm}) =$$
(13)

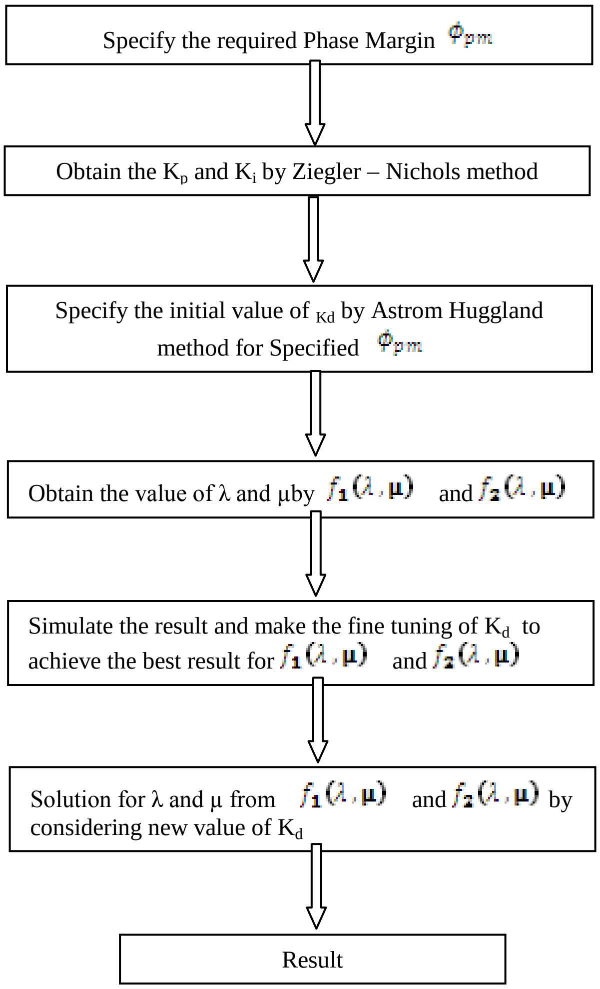

Figure 4: Flowchart for tuning of $PI^{\lambda}D^{\mu}$ Controller

5. SIMULATION RESULTS

0A method for tuning of PID using conventional method and fractional order PID controller using phase margin specifications has been designed. The presented method is based on idea of using Zeigler-Nichols for K_p and K_i while Astrom Hagglund method is used for determining K_d for the conventional PID. Similarly K_p and K_i parameter for fractional order PID controller have been computed from Zeigler and Nichols method and the remaining parameter K_d, λ and μ have been found from Astrom – Hagglund method.

The three controllers successfully designed were compared. The response of each controller was plotted in one window as illustrated in Figures5 and 6.

First capacity of designed controllers to follow the set point changes is tested. Initially set point of one is given at time zero and set point two at time T=200and set point of 0.5 at time T=700 and set point of 1 at t=850 were given. The response of the process is shown in figure 5 for three controllers.

The designed controllers also tested for load disturbances. Load disturbance of 0.5 is given at time T=400 and disturbance of 0.25 at T=750and load of 0.5 at time T=850 were given. The response of the process is shown in figure6 for three controllers. The simulation results show that the FOPID controller has the best performance because it has zero steady-state error at lowest time. It takes short time to reach the steady state. Furthermore, it has less overshoot. Hence, it can be concluded that FOPID controller is the best controller for the continuous stirred tank reactor system (CSTR)

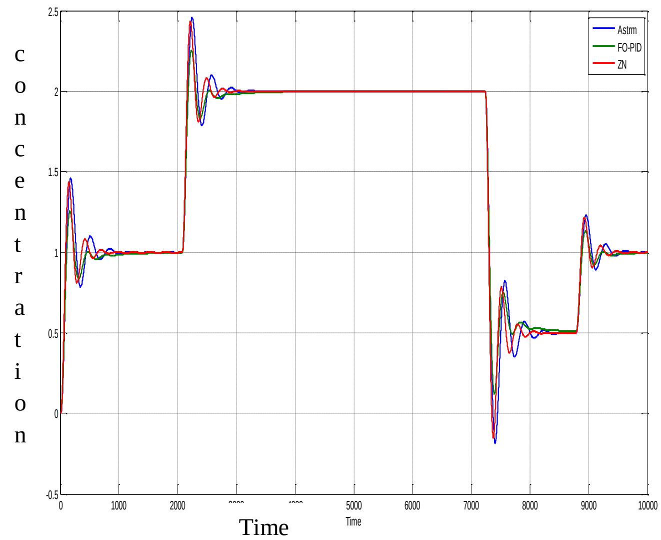

Figure 5: Servo Response of the system

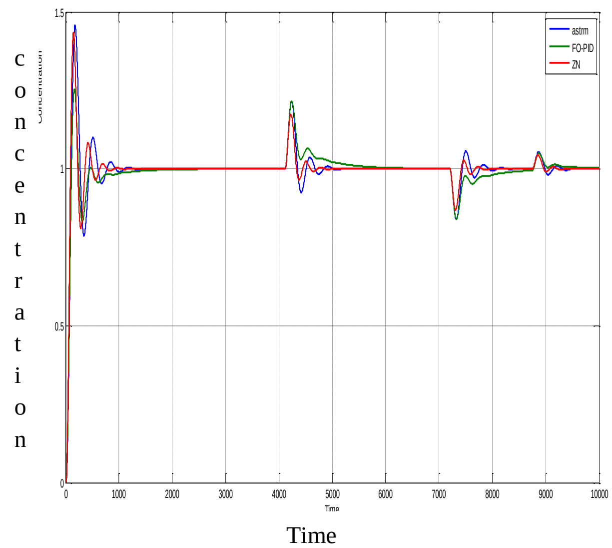

Figure 6: Regulatory Response of the system

Table:1 Comparison of PID and FOPID controller

Step response Specification	Maximum overshoot Mp %	ISE servo	ISE load	Settling time Ts
Z-N PID	1.76	7	29.13	105
A-H PID	1.43	5.98	22.75	102
FO PID	1.13	5.35	21.78	84

From the table 1, the proposed FOPID method gives much better performance with respect to Z-N method and Astrom-Hagglund method especially for Maximum overshoot (Mp %), ISE and settling time(Ts).

6. CONCLUSION

In this paper three controllers have been designed for a CSTR process. A model for a CSTR system is designed and developed successfully. Among the various types of controllers the best controller must be determined. In this paper some criterion like small settling time and rise time, has no steady-state error and overshoot considered to select the best controller. The best controllers never achieve all these criterions at same time, so it is necessary to decide which criterion we want the most. For CSTR system, the most required criterion is that the system has a no overshoot and zero steady-state error. Between these controllers, a comparison has been done to see which controller can meet the criterion. It is observed from the results the FOPID controller gives best results.

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