Application of Rough Finite State Automata in Decision Making

Swati Gupta
Student
Dept. of Computer
Science and Engineering
Thapar University,
Punjab, India

SunitaGarhwal
Assistant Professor
Dept. of Computer
Science and Engineering
Thapar University,
Punjab, India

ABSTRACT
RST is a formal scientific tool presented by shine researcher Pawlak [5] as in the early 1980s that oversees powerfully the instability which emerges from incomplete, noisy or inexact data. The rough set hypothesis is an essential method for data mining which incorporates extracting knowledge from a lot of information, finding new patterns, and anticipating the future trends. As of late, Basu [1] outlined a numerical model, named rough finite state automata, which perceives such rough sets and is believed to end up being of awesome significance to the researchers in the field of data analysis in near future. The aim of the paper is to design a RFSA for a rough dataset taken from the UCI machine repository.

Keywords
Rough finite state automata; Rough finite state semi automata; Rough set; Rough set theory; Decision making

1 INTRODUCTION
In 1982 Z. Pawlak[5] presented a philosophy as an expansion of the conventional crisp hypothesis in order to deal with the vague and uncertain information while analyzing data. It is a mathematical methodology to handle imperfection and imprecision in knowledge systems, i.e., imprecision, uncertainty and incompleteness. RST is the current research focus for knowledge discovery. It plays a great role in decision making as well. It is found to be a great formal tool in processing real world data sets. The main idea remains to obtain same knowledge as was obtained before reducing the attributes which corresponded to roughness. RST is a popular tool because of the ability to found knowledge from the data without any additional set of information. Recently, RST has pulled in extraordinary consideration from scientists, and numerous uses of RST have been effectively proposed, particularly in the machine-learning space.

Rough set hypothesis is advantageous to data analysis. The main advantage being that it extracts information based on the original data and no external or additional information is required. Assumptions on the other hand may or may not be required in most cases. Both quantitative and qualitative features can be analyzed. Also the results of the same are easy to understand and interpret. The rough set theory also provide a formal framework for data mining.

Since RST has proved a long way in its approach to deal with uncertainty and vagueness, it has captured the eyes of many mathematicians and computer scientists in the area of cognitive sciences, decision making and artificial intelligence.Following the advent of RST, Basu [1] lately presented the idea of a rough finite state semi-automaton and rough finite state automata,after an input set is provided, by permitting a state to "transit" to a rough set of the state set unquestionably and broadened the thought further by planning a recognizer that acknowledges uncertain articulations [3].

The rough finite state automaton is a mathematical model that recognizes rough sets. Its input conduct ends up being a set of words which are uncertain in in arriving to a decision as for an appropriate equivalence relation on the input word set. Such rough finite-state machine will demonstrate an incredible arrangement to help the specialists in artificial intelligence field.

In the field of uncertainty and imprecision, RST has played a major role. It has been used since decades and has been a hot research topic in data analysis and decision making with successful application in many domains. It has been effectively connected and applied in number of real life fields like saving money, building, budgetary, drug, pharmacology, market examination and others. Banking applications involve analysis a bankruptcy risk and statistical surveying.

Additionally, it is a powerful methodology for the uncertainty of information widely used in the fields of cognitive science and artificial intelligence, especially in the areas of [7] decision making, machine learning, data mining, statistics, inductive reasoning, knowledge discovery from databases, expert systems and pattern recognition. Sooner rather than later exceptionally encouraging new zones of utilization of the rough set idea will rise. They incorporate rough control, rough databases, rough data recovery, rough neural system, and others.

It appears of specific significance to decision support networks and data mining. The main advantage of using RST being that no preliminary information about data is required like in Dempster Shafer Theory or Fuzzy Set Theory.

Rough-set inspired methodology for intelligent decision support. With each object of the given universe we relate some data and items portrayed by comparable data are indiscernible, is the suspicion in the establishment of the rough set hypothesis. Approximations [6] are two essential operations in RST. There are two sorts of approximations: the lower approximation, for specific information and the upper approximation, for uncertain or plausible information. Rough sets can be connected for affecting decision rules from information. Every choiceis approximated by equivalence classes of the indiscernibility connection which is resolved on the instance set by the feature set framing a reduct. The affected decision standards are classified into definite and conceivable.

2 PRELIMINARIES
2.1 Rough Set
Over the past three decades, RST has resulted to be a topic of great interest for research and has been applied to many domains.The framework ofPawlak's [5] rough sets deals with

imprecision and uncertainty in data analysis. RST is a way to deal with decision making within the sight of uncertainty. It arranges uncertain, deficient or imprecise data communicated regarding information obtained from experience. Rough set hypothesis is by all accounts appropriate as a scientific model of uncertainty and vagueness [9].

2.1.1 Basic Concepts

The fundamental ideas are presented by the accompanying definitions:

2.1.1.1 Information or Decision Systems

A decision system, as a fundamental idea in rough set hypothesis, gives a helpful structure to the representation of features or instances as far as their attribute values. Information around an arrangement of instances in term of a predefined set of features is contained in decision framework. A decision system is defined as an ordered pair S = (U, A = C ∪{d}), where U is a definite non-empty set of instances called the universe and A=C ∪{d} is a definite non-empty set of features which is a union of C and {d}.C is a definite and non-empty set of conditional features and d is a decision feature. C and {d} are disjoint sets with nothing in common.

2.1.1.2 IndiscernibilityRelation

Indiscernibility relation Ind(B) on U is described as, provided two objects, x, y ϵ U, they are indiscernible due to the non-empty set of attributes B ⊆ A, provided that a(x) = a(y) for each a ϵ B [10].

2.1.1.3 Set Approximations

The RST portrays a crisp subset of a universe by two perceptible subsets called lower and upper approximations. By utilizing these, the learning covered up in decision frameworks can be found and communicated as decision rules.

Let B be the attributesetin A i.e. B⊆A, X be the objectsetin U i.e. X⊆U,

- the lower approximation of X can be defined as the union of all the elementary sets which are certainly contained in X. That is, $\underline{XB} = \cup\{x \in U \mid [x]_B \subseteq X\}$ [2].
- the upper approximation of X is defined as the union of the elementary sets which have a non-empty intersection with X. That is, $\overline{XB} = \cup\{x \in U \mid [x]_B \cap X \neq \emptyset\}$ [2].

The upper approximation incorporates all instances that conceivably have a place with the idea at the same time whenthe lower approximation incorporates all the instances which most likely have a place with the idea.

2.1.1.4 Rough Membership

The proportion of the number of elements in the lower approximation and the number of elements in the upper approximation is characterized as the exactness of computation, which is the roughness evaluated. It is introduced as [4]

$$R_B(X) = 1 - \frac{|\underline{XB}|}{|\overline{XB}|}$$

The order pair $(\underline{RX}, \overline{RX})$ is called a rough set of X regarding R.

2.2 Rough Finite State Automata

RFSA introduced by Basu[1] is a tool which can perform analysis of uncertain data and recognize rough languages. It is a new concept, with not much research done, but we feel will definitely prove to be useful in the long run.

2.2.1 Basic Concepts

The fundamental ideas are presented by the accompanying definitions:

2.2.1.1 Rough Finite State Semi-Automata

Let there be a rough finite-state semi automaton (RFSSA) A=(Q,R, M,X) [1] which is defined as a semi-automata which accepts rough languages where Q denotes a definite set of interior states, R denotes a given equivalence relation on Q, X denotes the set of input patterns, D describes the class of all definable sets in <Q,R>, and M denotes the transition function such that $M: Q \times X \rightarrow D \times D$ is so that $M(q,\sigma) = (D_1, D_2)$, for $\forall q \in Q$, $\forall \sigma \in X$,such that D_1 is the lower approximation and D_2 is the upper approximation of the rough set, where $D_1, D_2 \in D$ such that $D_1 \subseteq D_2$ in correspondence to the equivalence relation R, that is,

$D_1 = \underline{M(q,\sigma)}$, $D_2 = \overline{M(q,\sigma)}$

M(q, σ) can also be composed as $q\sigma^M$, such that $q\sigma^M = (\underline{q\sigma M}, \overline{q\sigma M})$

Now, rough finite state automata may be obtained by adjoining a set of concluding states and an initial set to the definition of rough finite state semi automaton.

2.2.1.2 Rough Finite State Automata

Let A = (Q,R,M, X, I,H) be a rough finite state automata [1] then it is defined as a RFSSA with configuration (Q,R,M, X), I being a determinable set in <Q,R> known as the initial configuration, and H⊆ Q denoted as the set of accepting or final states of A.

3 EXAMPLE RFSA

Amid a specific season in Mumbai, specialists observe that patients experiencing migraine and high or very high temperature certainly experience the ill effects of influenza, those having no migraine yet high temperature or no migraine however high temperature or migraine yet ordinary temperature could possibly experience the ill effects of influenza, and patients experiencing no migraine and ordinary temperature don't experience the ill effects of influenza.

From the prior perceptions clearly the infection influenza might be described by a rough set of side effects whose lower approximation is {(migraine, high temperature), (migraine, very high temperature)} and upper approximation is {migraine, high temperature, very high temperature, no migraine, ordinary temperature, migraine and high temperature, migraine and very high temperature, no migraine yet high temperature, no migraine yet very high temperature, migraine yet ordinary temperature}.Let A = (Q, R, M, X, I, H) be the required RFSA that recognizes the disease influenza:

Table 1. Decision Table for detecting influenza

Patient	Migraine	Temperature	Influenza
P1	√	N	×
P2	√	N	√
P3	√	H	√
P4	√	VH	√
P5	×	N	×
P6	×	H	×
P7	×	VH	√
P8	×	H	√
P9	×	VH	×

√ Yes; × No; N Normal; H High; VH Very High

Q={q1, q2, q3, q4, q5, q6, q7, q8, q9}

R={$[q_0]$, $[q_1, q_2]$, $[q_3]$, $[q_4]$, $[q_5]$, $[q_6, q_7]$, $[q_8, q_9]$}

I={$[q_0]$}

H={q2, q3, q4, q7, q9}

X={σ_1, σ_2, σ_3, σ_4,σ_5, σ_6}.

The inputs for the same are described as follows:

σ_1 = (Yes, Normal)

σ_2 = (Yes, High)

σ_3 = (Yes, Very High)

σ_4 = (No, Normal)

σ_5 = (No, High)

σ_6 = (No, Very High).

M is described by table 2, where

βA= ($\underline{\beta A}$, $\overline{\beta A}$) where

$\underline{\beta A}$={σ_2, σ_3}, $\overline{\beta A}$ = {σ_1, σ_2, σ_3, σ_5, σ_6}

Table 2. State Transition Table for detecting influenza

Q	**σ_1**	**σ_2**	**σ_3**
q_0	($[q_1, q_2] \cup [q_5]$)	($[q_3]$, $[q_3]$)	($[q_4]$, $[q_4]$)
	σ_4	**σ_5**	**σ_6**
q_0	($[q_5]$, $[q_5]$)	($[q_6,q_7] \cup [q5]$)	($[q_8,q_9] \cup [q_5]$)

4 ROUGH FINITE STATE AUTOMATA IN DECISION MAKING

Rough Finite State Automata (RFSA) is helpful in decision making process. The uncertainty observed in the datasets while making decision making can be resolved with the help of RFSA. It proves to be a helpful tool while making any sorts of decisions on the datasets where imprecision and uncertainty is involved.

One such RFSA is made for the rough dataset (Hayes-Roth) taken from UCI Machine Repository. The rough dataset contains 80 objects and 4 attributes. The dataset has been classified into 2 precise classes: class 1 and class 2, which will act as the lower approximation for the RFSA, and an imprecise classification group i.e. class 3, whose objects classify to be a part of either class 1 or class 2. This class acts as rough class in decision makingand is included in the upper bound of the RFSA. Another classification for class 4 is made, which includes objects that don't belong to either of the class 1 or class 2. These objects act as non-final states in the RFSA.

Two RFSAs are created which will be further helpful in decision making on the same data for class 1 and class 2 respectively.

The RFSA A_1 = (Q, R, M, X, I, H) for class 1 is defined as follows:

Q = {q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{17}, q_{18}, q_{19}, q_{20}, q_{21}, q_{22}, q_{23}, q_{24}, q_{25}, q_{26}, q_{27}, q_{28}, q_{29}, q_{30}, q_{31}, q_{32}, q_{33}, q_{34}, q_{35}, q_{36}, q_{37}, q_{38}, q_{39}, q_{40}, q_{41}, q_{42}, q_{43}, q_{44}}

R = {$[q_0]$, $[q_1, q_2]$, $[q_3, q_4]$, $[q_5, q_6]$, $[q_7, q_8]$, $[q_9, q_{10}]$, $[q_{11}, q_{12}]$, $[q_{13}, q_{14}]$, $[q_{15}, q_{16}]$, $[q_{17}, q_{18}]$, $[q_{19}, q_{20}]$, $[q_{21}, q_{22}]$, $[q_{23}, q_{24}]$, $[q_{25}, q_{26}]$, $[q_{27}, q_{28}]$, $[q_{29}, q_{30}]$, $[q_{31}, q_{32}]$, $[q_{33}, q_{34}]$, $[q_{35}, q_{36}]$, $[q_{39}, q_{40}]$, $[q_{41}, q_{42}]$, $[q_{43}, q_{44}]$}

I = {$[q_0]$}

H = {q_{17}, q_{18}, q_{35}, q_{36}}

X = {1, 2, 3, 4}

M is described by the table3, where

βA= ($\underline{\beta A}$, $\overline{\beta A}$) where

$\underline{\beta A}$={1111, 1112, 1113, 1131, 1132, 1133, 1121, 1211, 1311, 2111, 3111, 1123, 1231, 2311, 1213, 2131, 2113, 3112, 3121, 3211, 1333, 3331, 3133, 3313}

$\overline{\beta A}$ = {3333, 222, 223, 232, 322, 233, 323, 332, 122, 212, 221, 1111, 1112, 1113, 1131, 1132, 1133, 1121, 1211, 1311, 2111, 3111, 1123, 1231, 2311, 1213, 2131, 2113, 3112, 3121, 3211, 1333, 3331, 3133, 3313}

Table 3. State Transition Table for Class 1

Q	1	2	3	4
q_0	$\{[q_1,q_2], [q_1,q_2]\}$	$\{[q_3,q_4], [q_3,q_4]\}$	$\{[q_5,q_6], [q_5,q_6]\}$	$\{Ø, [q_7,q_8]\}$
q_1	$\{[q_9,q_{10}], [q_9,q_{10}]\}$	$\{[q_{11},q_{12}], [q_{11},q_{12}]\}$	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{Ø, [q_7,q_8]\}$
q_3	$\{[q_{11},q_{12}], [q_{11},q_{12}]\}$	$\{[q_{21},q_{22}], [q_{21},q_{22}]\}$	$\{[q_{27},q_{28}], [q_{27},q_{28}]\}$	$\{Ø, [q_7,q_8]\}$
q_5	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{27},q_{28}], [q_{27},q_{28}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{Ø, [q_7,q_8]\}$
q_7	$\{Ø, [q_{43},q_{44}]\}$	$\{Ø, [q_{43},q_{44}]\}$	$\{Ø, [q_{43},q_{44}]\}$	$\{Ø, [q_7,q_8]\}$
q_9	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{[q_{15},q_{16}], [q_{15},q_{16}]\}$	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{Ø, [q_7,q_8]\}$
q_{11}	$\{[q_{15},q_{16}], [q_{15},q_{16}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{29},q_{30}], [q_{29},q_{30}]\}$	$\{Ø, [q_7,q_8]\}$
q_{13}	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{[q_{29},q_{30}], [q_{29},q_{30}]\}$	$\{[q_{31},q_{32}], [q_{31},q_{32}]\}$	$\{Ø, [q_7,q_8]\}$
q_{15}	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{Ø, [q_7,q_8]\}$
q_{17}	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{Ø, [q_7,q_8]\}$
q_{21}	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{Ø, [q_7,q_8]\}$
q_{23}	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{25},q_{26}], [q_{25},q_{26}]\}$	$\{[q_{25},q_{26}], [q_{25},q_{26}]\}$	$\{Ø, [q_7,q_8]\}$
q_{27}	$\{[q_{29},q_{30}], [q_{29},q_{30}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{26}]\}$	$\{[q_{39},q_{40}], [q_{39},q_{40}]\}$	$\{Ø, [q_7,q_8]\}$
q_{29}	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{33},q_{34}], [q_{33},q_{34}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{Ø, [q_7,q_8]\}$
q_{31}	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{Ø, [q_7,q_8]\}$
q_{39}	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{41},q_{42}], [q_{41},q_{42}]\}$	$\{[q_{41},q_{42}], [q_{41},q_{42}]\}$	$\{Ø, [q_7,q_8]\}$

The RFSA A_2 = (Q, R, X, M, I, H) for class 2 is defined as follows:

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{17}, q_{18}, q_{19}, q_{20}, q_{21}, q_{22}, q_{23}, q_{24}, q_{25}, q_{26}, q_{27}, q_{28}, q_{29}, q_{30}, q_{31}, q_{32}, q_{33}, q_{34}, q_{35}, q_{36}, q_{37}, q_{38}, q_{39}, q_{40}, q_{41}, q_{42}, q_{43}, q_{44}, q_{45}, q_{46}\}$

$R = \{[q_0], [q_1, q_2], [q_3, q_4], [q_5, q_6], [q_7, q_8], [q_9, q_{10}], [q_{11}, q_{12}], [q_{13}, q_{14}], [q_{15}, q_{16}], [q_{17}, q_{18}], [q_{19}, q_{20}], [q_{21}, q_{22}], [q_{23}, q_{24}], [q_{25}, q_{26}], [q_{27}, q_{28}], [q_{29}, q_{30}], [q_{31}, q_{32}], [q_{33}, q_{34}], [q_{35}, q_{36}], [q_{39}, q_{40}], [q_{41}, q_{42}], [q_{43}, q_{44}], [q_{44}, q_{45}]\}$

$I = \{[q_0]\}$

$H = \{q_{13}, q_{14}, q_{23}, q_{24}\}$

X = {1, 2, 3, 4}

M is described by the table 4, where

βA= ($\underline{\beta A}, \overline{\beta A}$) where

$\underline{\beta A}$={2222, 2221, 2223, 2232, 2231, 2233, 2212, 2122, 2322, 1222, 3222, 2213, 2132, 1322, 2123, 1232, 1223, 3221, 3212, 3122, 2333, 3332, 3233, 3323}

$\overline{\beta A}$ = {3333, 111, 113, 131, 311, 133, 313, 331, 211, 121, 112, 2222, 2221, 2223, 2232, 2231, 2233, 2212, 2122, 2322, 1222, 3222, 2213, 2132, 1322, 2123, 1232, 1223, 3221, 3212, 3122, 2333, 3332, 3233, 3323}

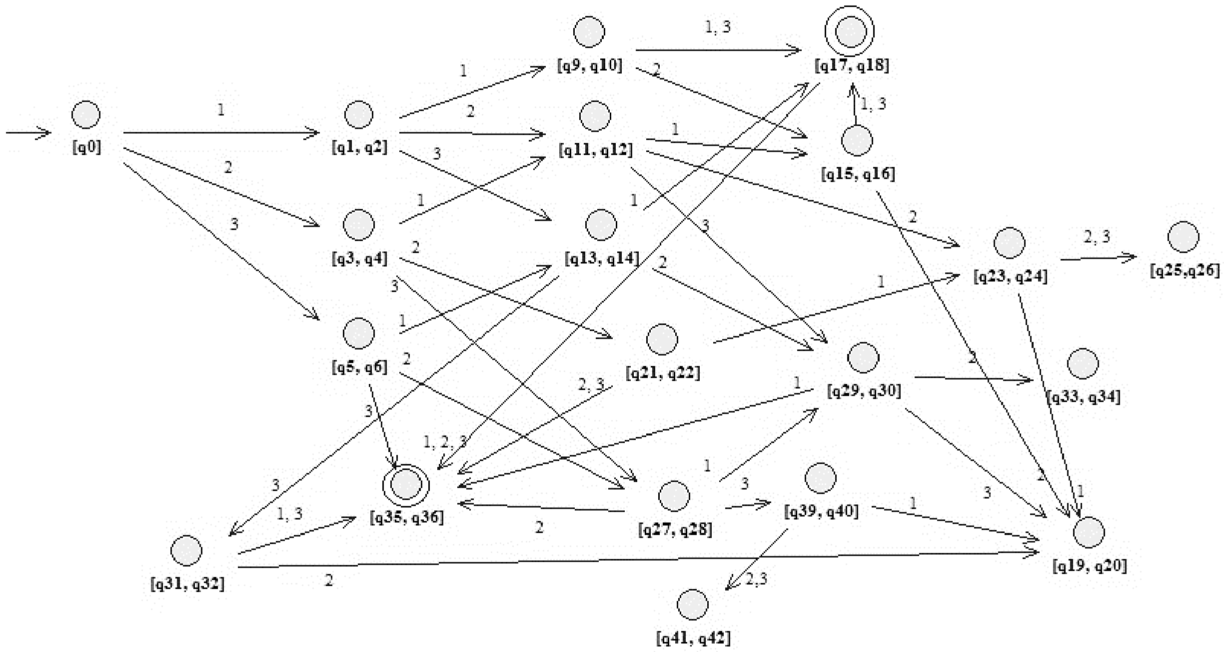

Fig 1: The transition state diagram for the rough finite state automata of class 1

Table 4. State Transition Table for Class 2

Q	1	2	3	4
q_0	$\{[q_1,q_2], [q_1,q_2]\}$	$\{[q_3,q_4], [q_3,q_4]\}$	$\{[q_5,q_6], [q_5,q_6]\}$	$\{ Ø, [q_7,q_8]\}$
q_1	$\{[q_9,q_{10}], [q_9,q_{10}]\}$	$\{[q_{15},q_{16}], [q_{15},q_{16}]\}$	$\{[q_{31},q_{32}], [q_{31},q_{32}]\}$	$\{ Ø, [q_7,q_8]\}$
q_3	$\{[q_{15},q_{16}], [q_{15},q_{16}]\}$	$\{[q_{25},q_{26}], [q_{25},q_{26}]\}$	$\{[q_{29},q_{30}], [q_{29},q_{30}]\}$	$\{ Ø, [q_7,q_8]\}$
q_5	$\{[q_{31},q_{32}], [q_{31},q_{32}]\}$	$\{[q_{29},q_{30}], [q_{29},q_{30}]\}$	$\{[q_{37},q_{38}], [q_{37},q_{38}]\}$	$\{ Ø, [q_7,q_8]\}$
q_7	$\{ Ø, [q_{45},q_{46}]\}$	$\{ Ø, [q_{45},q_{46}]\}$	$\{ Ø, [q_{45},q_{46}]\}$	$\{ Ø, [q_{45},q_{46}]\}$
q_9	$\{[q_{11},q_{12}], [q_{11},q_{12}]\}$	$\{[q_{21},q_{22}], [q_{21},q_{22}]\}$	$\{[q_{11},q_{12}], [q_{11},q_{12}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{11}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{15}	$\{[q_{21},q_{22}], [q_{21},q_{22}]\}$	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{17}	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{24}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{25}	$\{[q_{17},q_{18}], [q_{17},q_{18}]\}$	$\{[q_{27},q_{28}], [q_{27},q_{28}]\}$	$\{[q_{33},q_{34}], [q_{33},q_{34}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{27}	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{29}	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{33},q_{34}], [q_{33},q_{34}]\}$	$\{[q_{39},q_{40}], [q_{39},q_{40}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{31}	$\{[q_{11},q_{12}], [q_{11},q_{12}]\}$	$\{[q_{35},q_{36}], [q_{35},q_{36}]\}$	$\{[q_{41},q_{42}], [q_{41},q_{42}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{33}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{35}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{37}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{39},q_{40}], [q_{39},q_{40}]\}$	$\{[q_{43},q_{44}], [q_{43},q_{44}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{39}	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{41}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{19},q_{20}], [q_{19},q_{20}]\}$	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{ Ø, [q_7,q_8]\}$
q_{43}	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{[q_{23},q_{24}], [q_{23},q_{24}]\}$	$\{[q_{13},q_{14}], [q_{13},q_{14}]\}$	$\{ Ø, [q_7,q_8]\}$

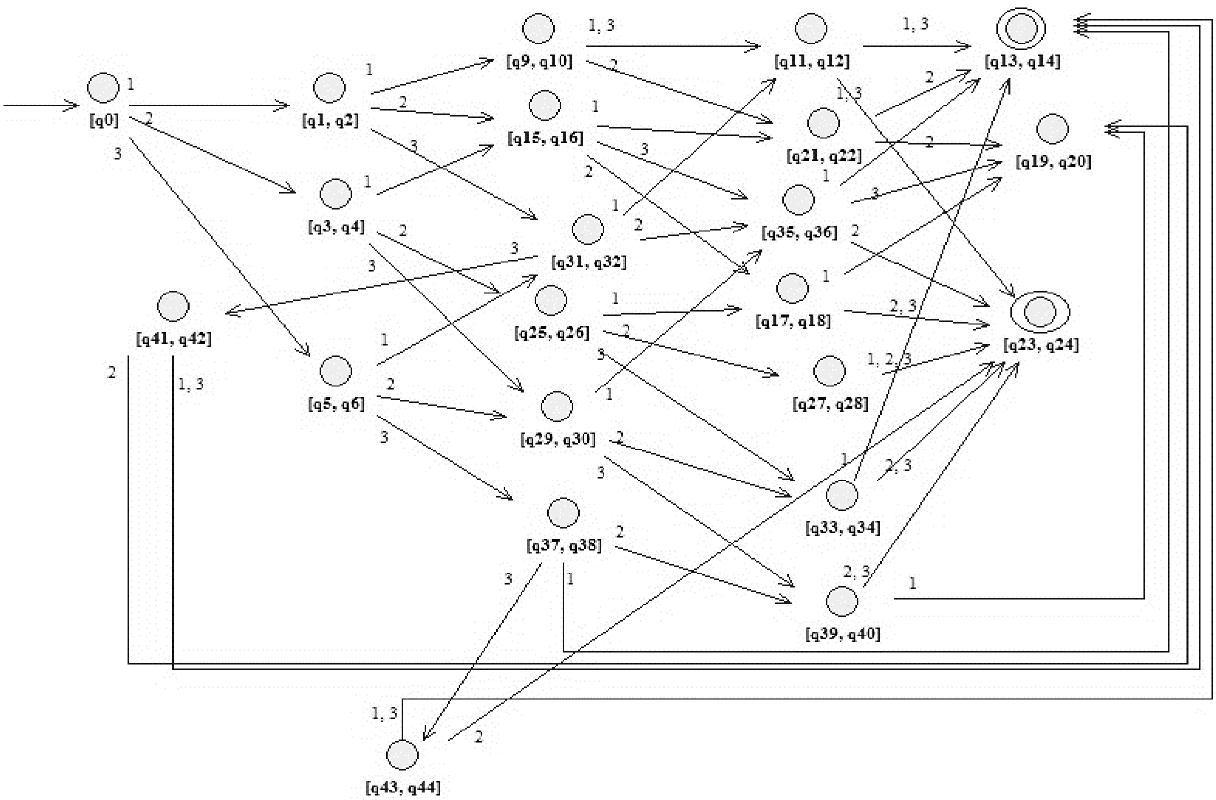

Fig 2: The transition state diagram for the rough finite state automata of class 2

5 REFERENCES

[1] S. Basu, "Rough Finite-State Automata", Cybernetics and Systems: An InternationalJournal, vol. 36, pp. 107-124, 2005.

[2] A. K. Bag, B. Tudu, N. Bhattacharyya and R. Bandyopadhyay, "Dealing With Redundant Features and Inconsistent Training Data in Electronic Nose: A Rough Set Based Approach", IEEE Sensors Journal, vol. 14(3), March 2014.

[3] S. Sharan, A. K. Srivastava andS. P. Tiwari, "Characterizations of rough finite state automata", International Journal of Machine Learning and Cybernetics, pp. 1-10, May 2015.

[4] D. Parmar, T. Wu and J. Blackhurst, "MMR: An algorithm for clustering categorical data using Rough Set Theory", Data & Knowledge Engineering, vol. 63, pp. 879–893, 2007.

[5] Z. Pawlak, "Rough Sets", International Journal of Computer & Information Sciences,vol. 11(5), pp. 341-356, October 1982.

[6] A. G. Jackson, Z. Pawlak and S. R. LeClair, "Rough sets applied to the discovery of materials knowledge", Journal of Alloys and Compounds, vol. 279(1), pp. 14–21, September 1998.

[7] P. Zhu, "Covering rough sets based on neighborhoods: An approach without using neighborhoods", International Journal of Approximate Reasoning, vol. 52(3), pp. 461–472, March 2011.

[8] A. J. Krener, "Reduced Order Modeling of Nonlinear Control Systems", Analysis and Design of Nonlinear Control Systems, pp. 41-62.

[9] Q. Zhang, J. Wang, G. Wang and H. Yu, "The approximation set of a vague set in rough approximation space", Information Sciences, vol. 300, pp. 1–19, April 2015.

[10] P. Kumar and B.K. Tripathy, "MMeR: an algorithm for clustering heterogeneous data using rough set theory", International Journal of Rapid Manufacturing (IJRAPIDM), vol. 1(2), 2009.