Reduced-Size FFT Correlation Techniques for GPS Signal Acquisition

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ABSTRACT

Signal acquisition is the primary step in a GPS receiver. In this paper, frequency-domain methods of GPS L1 signal acquisition are proposed that use reduced number of FFT points to perform circular correlation for faster as well as reliable acquisition. The possibility of reduction has been considered by analyzing the spectrum of C/A code alone. Simulation of various FFT sizes are carried out on MATLAB and the results are verified by those obtained from the GPS Signal Tap Receiver. Besides, the performances of reduced-size FFT correlation techniques are compared and evaluated using different lengths of noncoherent pre-integration period.

General Terms

GPS, Signal acquisition, Navigation.

Keywords

GPS, Signal acquisition, C/A code, FFT, Circular correlation, MATLAB.

1. INTRODUCTION

Global Positioning System (GPS) is a satellite-based 3-D navigational system developed and launched by the US Department of Defense (US DoD) in 1978 under the name NAVSTAR (NAVigation Satellite Timing and Ranging) to allow the uers to accurately determine their 3-D PVT (Position, Velocity and Time information) anytime, anywhere in the world. The 32 GPS satellites are differentiated by 32 PRN sequences known as the Coarse Acquisition (C/A) code for the GPS signal. The C/A code spreads the navigation message sent at 50 bps, and is broadcasted on a common Link1 (L1) carrier frequency of 1575.42 MHz using BPSK (Binary Phase-Shift Keying) modulation format and DSSS (Direct Sequence Spread Spectrum) techniques [1].

Acquisition and tracking of GPS satellites are the key processes involved in a GPS receiver. Software GPS receivers capture the RF modulated signals at L1 frequency, downconvert them to an intermediate frequency (IF), digitize them, and perform signal processing to extract the position information from the navigation message. Compared to conventional hardware GPS receivers which use application-specific integrated circuits (ASICs), software GPS receivers provide flexibility and low cost for algorithms redesign using field-programmable gate arrays (FPGA) and digital signal processing (DSP) chips.

This paper is organized as follows: Section 2 describes the generation and properties of C/A code. Section 3 discusses the GPS signal acquisition using FFT-based parallel search method. Thereafter, the spectrum of C/A code is analyzed, based on which the reduced-size FFT correlation methods are developed subsequently. After acquisition, the process of tracking the code and carrier of the visible satellites is explained in brief in Section 4. Section 5 presents the required experimental setup for MATLAB simulation, followed by the graphical results and comparative analysis of the proposed acquisition techniques. Finally, the conclusion of the paper is discussed in Section 6.

2. GPS C/A CODE

C/A code is a pseudorandom sequence of 0s and 1s based on Gold codes and is unique for every satellite. The frequency of the C/A code is 1.023 MHz. Two sets of gold codes with different phase tappings are used to generate unique C/A code for every satellite. The C/A code for a particular satellite is created with an algorithm that includes the identification number of the GPS satellite, thus creating a unique code for each satellite.

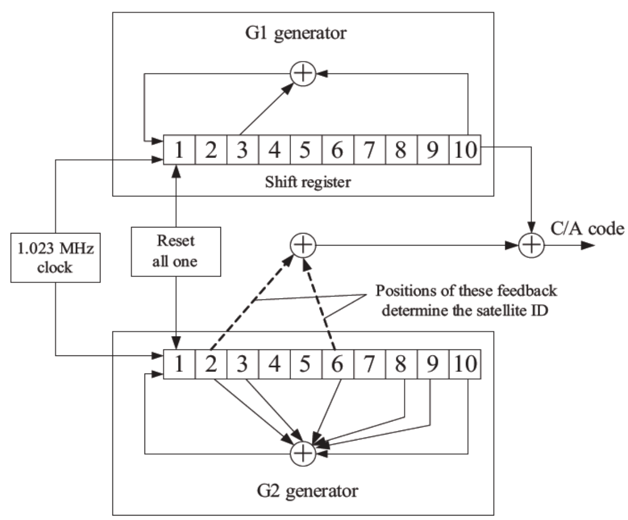

Fig. 1: C/A Code Generator

As shown in Figure 1, two 10-bit linear feedback shift registers (LFSRs), *G1* and *G2* generate maximum-length pseudorandom codes with a length of $2^{10} – 1 = 1023$ bits. A modulo-2 adder is used to generate the C/A code, which uses the outputs from *G1* and *G2* as inputs. Initially, both *G1* and *G2* are all set to 1's since the all-zero state is illegal. The *G1* and *G2* LFSR's feedback taps are defined by the generator polynomials [3]:

$$G1 = 1 + X^3 + X^{10} \quad (1)$$
$$G2 = 1 + X^2 + X^3 + X^6 + X^8 + X^9 + X^{10} \quad (2)$$
$$G_i = G1(10) \oplus G2(s_{i1}) \oplus G2(s_{i2}) \quad (3)$$

where G_i : Gold code sequence for *i*th satellite vehicle (SV_i)
s_{i1}, s_{i2} : signal tap values predefined for *i*th satellite

The satellite identification is determined by the two output positions of the *G2* generator. There are 37 unique output positions, of which 32 are utilized for the C/A codes of 32 satellites. As only 24 satellites are in orbit, the other 5 outputs are reserved for other applications such as ground transmission.

High autocorrelation peak and low cross-correlation peaks are the characteristics of the C/A codes. In order to detect a weak signal in the presence of strong signals, the autocorrelation peak of the weak signal must be greater than the cross-correlation peaks from the stronger signals [2]. As will be shown later, the spectrum of C/A code will be exploited to reduce the number of FFT points required to perform circular correlation.

3. GPS SIGNAL ACQUISITION

3.1 FFT-based Parallel Search Acquisition

The main purpose of acquisition is to identify the visible satellites in the incoming data and then find the code phase (beginning point of the C/A code) by correlating the incoming signal with the locally-generated C/A code (de-spreading) and finding the carrier frequency, including the Doppler shift, by correlating the incoming signal with the receiver generated signal (demodulation). After correlation, if the two signals match, we find a very high correlation peak (this is the characteristics of C/A codes). If this peak is greater than a certain threshold value, the satellite is said to be acquired, else not. The well-known acquisition method used in software GPS receivers is the FFT-based parallel search method as shown in Figure 2.

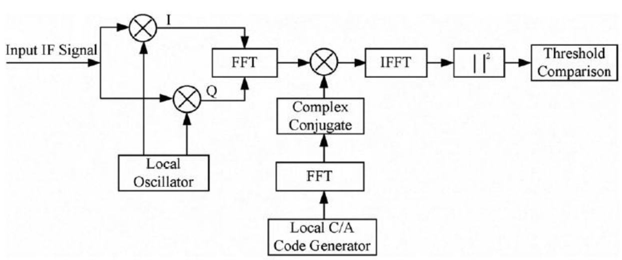

Fig. 2: Implementation of FFT-based parallel search acquisition method

The incoming signal is multiplied by a locally generated carrier signal. The locally generated carrier f_i is varied in steps of 500 Hz with Doppler shift of ±5 kHz. Multiplication with the signal generates the *I* signal, and multiplication with a 90° phase-shifted version of the signal generates the *Q* signal.

$$I = d(n)\cos(2\pi f_i n T_s) \quad (4)$$
$$Q = d(n)\sin(2\pi f_i n T_s) \quad (5)$$

where $d(n)$: Received and stored data vector

$f_i = f_c \pm k*500, \quad k = 0,1,2,3,\ldots$
f_c : Center frequency
T_s : Sampling time period

The *I* and *Q* signals are combined to form a complex input signal:

$$x(n) = I(n) + jQ(n) \quad (6)$$

to the FFT function. After combining the two and taking its FFT as in (7), it is multiplied with the complex conjugate FFT, denoted by *, of the C/A code locally generated using (1), (2) and (3), for the *i*th satellite to be searched.

$$X(k) = FFT[x(n)] \quad (7)$$
$$CA_i(k) = FFT[ca_i(n)] \quad (8)$$
$$Y(k) = X(k)\, CA_i^*(k) \quad (9)$$

Since the sampling frequency of the signal in this thesis work is 20 MHz, there are 20000 points (chips) in 1 C/A code period of duration 1 ms. In order to perform acquisition, we generate C/A code of 1 ms period and resample it to 20 MHz so that we will have 20000 points. The carrier wave is generated for a frequency range of 5 kHz above and below the centre frequency with a frequency step of 1000 Hz or 500 Hz. If we use 500 Hz search frequency step, we will have a total of 21 Doppler frequency steps to cover a search space of 5 kHz above and below the center frequency. FFT on 20000 points with 21 frequency components will generate altogether 20000 x 21 point outputs.

The IFFT of the result as in (10) is squared, summed up to noncoherent integration period and compared with a pre-defined threshold value.

$$y(n) = IFFT[Y(k)] \quad (10)$$
$$c(f_i, n) = [y(n)]^2 \quad (11)$$

If the correlation is above this threshold, the satellite is said to be acquired, else not. The code phase is the value of phase for which the correlation has a peak value.

3.2 Analysis of C/A Code Spectrum

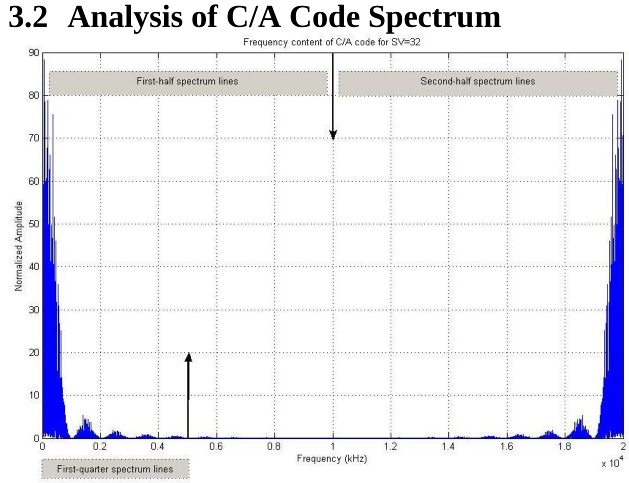

Fig. 3: Normalized Amplitude Spectrum of C/A code

Figure 3 shows the normalized amplitude spectrum of C/A code for SV32. Upon analysis, the power spectrum of the locally generated C/A code is found symmetrical, that is the first-half of the spectrum is a mirror image of the other half. Since the signal is digitized at 20 MHz, 1 ms of data contains 20000 data points, so a 20000-point FFT contains 20000 frequency components. However, only the first 10000 points of the FFT contains useful information. The second half of the frequency components is the complex conjugate of the first half [3].

A closer look on the first-half of the spectrum reveals that the signal power is concentrated in the first quarter of the spectrum. This is illustrated in Figure 4. More concentration of power is found at very low frequencies.

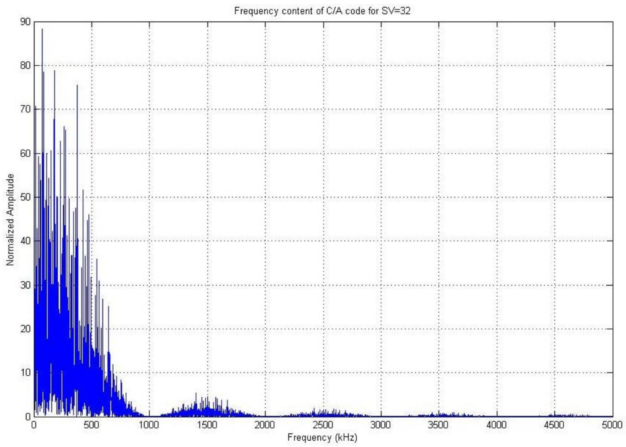

Fig. 4: Normalized Amplitude Spectrum of C/A code in the first quarter

3.3 Reduced size FFT Correlation Methods

References [2] and [4] used half the spectrum of the signal, the product of C/A code and the locally generated carrier, to speed up the acquisition process. This paper studies the spectrum of the C/A code alone to suggest faster acquisition methods by reducing the number of FFT points used for correlation. Equations (7), (8) and (9) give the frequency-domain operation of acquisition. The improved methods in this paper are based on modifying equation (9). There are as many points in Y as in X and CA_i. Hence, number of IFFT points in y will be equal to those in Y as calculated in (10).

Half-size circular correlation (HSCC) method [4] is based on the full size circular correlation (FSCC) method. From Figure 3, the symmetric redundancy of the spectrum lines can be effectively used to reduce the number of FFT points used to perform correlation, to half the original samples taken to perform correlation as calculated in (12):

$$Y(k/2) = X(k/2)\, CA_i^*(k/2) \qquad (12)$$

It performs correlation using only half the spectrum size. For example, the sampling frequency of 20 MHz, as used in this paper, results in 20000 sampled points, hence 20000 FFT points in frequency-domain. Due to symmetry of the C/A code spectrum, only half i.e. 10000 FFT points are used to perform circular correlation with the incoming signal. This reduction in number of points obviously tends to reduce the computation time and hardware complexity.

Referring to Figure 4, it is further seen that the information is mainly contained in the first-quarter of the spectrum lines. The second-quarter of the spectrum lines contains very little information. So only the first-quarter FFT points can be used in circular correlation algorithm which can be termed as quarter-size circular correlation (QSCC).

$$Y(k/4) = X(k/4)\, CA_i^*(k/4) \qquad (13)$$

The 10000 FFT points are further cut to 5000 points to perform circular correlation yielding a successful acquisition.

To check for the maximum limit to which this reduction is possible, the points can further be factorized as follows:

$$Y(k/5) = X(k/5)\, CA_i^*(k/5) \qquad (14)$$

Reducing the correlating points to one-fifth size i.e. 4000 correlating points lends the same results as above. Similarly, one-eighth size (2500-point) circular correlation can be written as:

$$Y(k/8) = X(k/8)\, CA_i^*(k/8) \qquad (15)$$

These methods result in a tremendous decrease in number of multiplications involved during correlation and hence saves acquisition time. However, it is to be noted that such a reduction in size of correlation points is possible only if the sampling rate is much higher than the chipping rate and the small degradation in the precision of the correlation peaks imposes a limit to which such reduction is possible. In the present case, the sampling rate is taken as 20 MHz which is much greater than the chipping rate of C/A code is 1.023 MHz, and reduction is possible upto one-eighth size beyond which the peaks are found ambiguous.

An additional improvement has been achieved by reducing the noncoherent integration period from a maximum of 10 ms to a minimum of 1 ms.

4. TRACKING

DLL is used to track the C/A code (de-spread) and PLL is used to track the frequency of the incoming signal that is related with Doppler frequency. Figure 5 shows the algorithm details of tracking loop. DLL consists of early, prompt and late code generators, filters and discriminators. The early and late codes are prompt code that is time shifted by half a chip or less. The early and late codes correlate with incoming C/A codes to produce two outputs. These outputs are filtered, squared and compared using an E-L envelope discriminator. Based on discriminator output, a control signal can be generated to adjust the rate of the locally generated C/A code to match the C/A code of the incoming signal. The locally generated prompt signal is used to de-spread the incoming signal [DM].

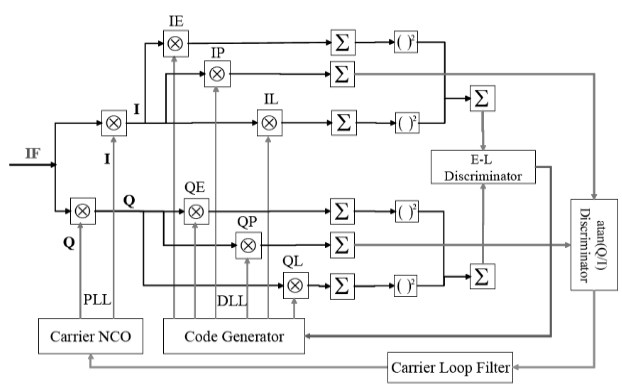

Fig. 5: Signal tracking loop containing DLL and PLL

The PLL consists of NCO (Numerically Controlled Oscillator), carrier loop filter and a discriminator. PLL receives signal that is only modulated by navigation message. The NCO generates a carrier frequency based on the Doppler frequency computed during the acquisition process. The signal generated is divided into I and Q components and each is correlated with the input signal. The outputs of the correlators are filtered and the phase is analyzed using an arc tangent discriminator that is insensitive to the phase transition. Detailed equations for tracking are given in reference [2] and [3].

After, demodulation and de-spreading, we get only the navigation message and noise.

5. SIMULATION AND RESULTS

5.1 Experimental Setup

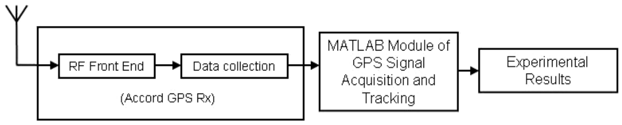

Fig. 6.: Block Diagram of Experimental Setup

Real GPS data has been collected by GPS Signal Tap from Accord Software and Systems Pvt. Ltd. and the acquisition and tracking algorithms were implemented using MATLAB® 7.0 as shown in Figure 6. Figure 7 illustrates the setup for collecting data from the GPS Signal Tap for further processing [5].

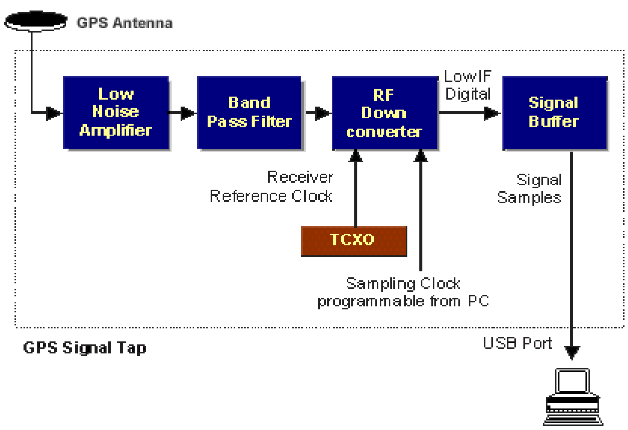

Fig. 7: Setup for collection of downsampled data from GPS Signal Tap

The data used in this paper consists of 1-bit quantized samples of the GPS L1 signal at 1575.42 MHz down converted to an IF = 15.42 MHz and sampled at 20 MHz, thereby stored at a center frequency of 4.58 MHz on PC's hard disk for offline processing later. Due to sampling rate of 20 MHz, 20000 samples are correlated at a time corresponding to 1 ms in case of FSCC. The acquisition program searches the frequency range of 4.58 MHz ± 5 kHz in 500 Hz steps, hence a total of 21 frequency bins are to be searched.

In the tracking loop, integration period is 1 ms. Carrier loop updates every 1ms and code loop updates every 20 ms by averaging over the integration output. The carrier loop filter parameters include: noise bandwidth = 50 Hz, damping factor = 0.707, loop gain = 400π.

5.2 Simulation Results in MATLAB

Figures 8—12 are the acquisition results for the satellite vehicle SV32 with reduced size circular correlation for a non-coherent pre-integration period of 10 ms.

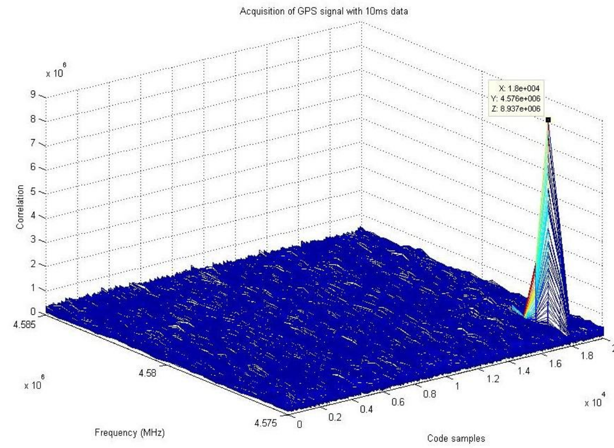

Fig. 8: Acquisition of SV32 with FSCC

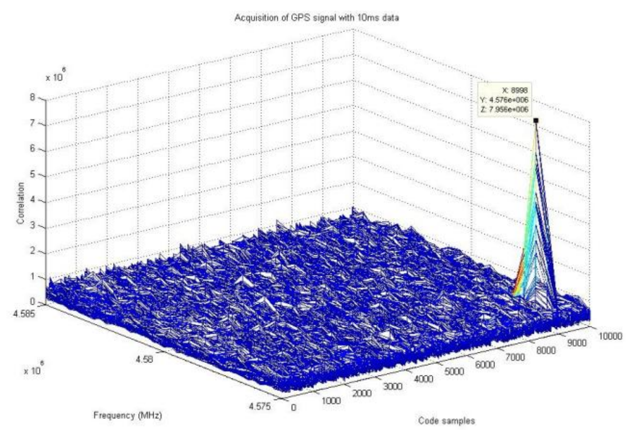

Fig. 9: Acquisition of SV32 with HSCC

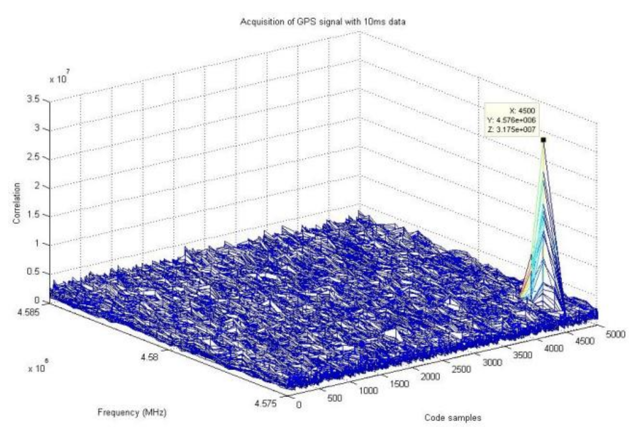

Fig. 10: Acquisition of SV32 with QSCC

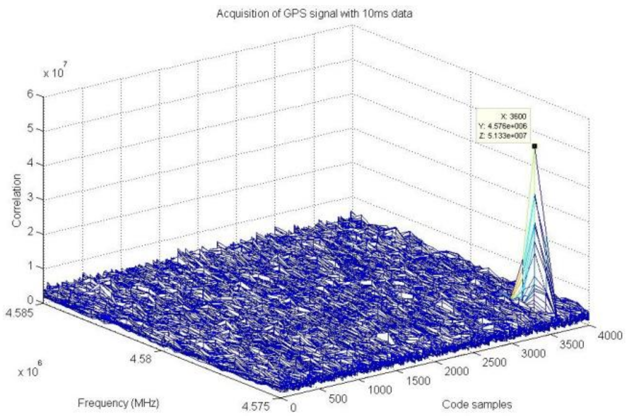

Fig. 11: Acquisition of SV32 with one-fifth size circular correlation

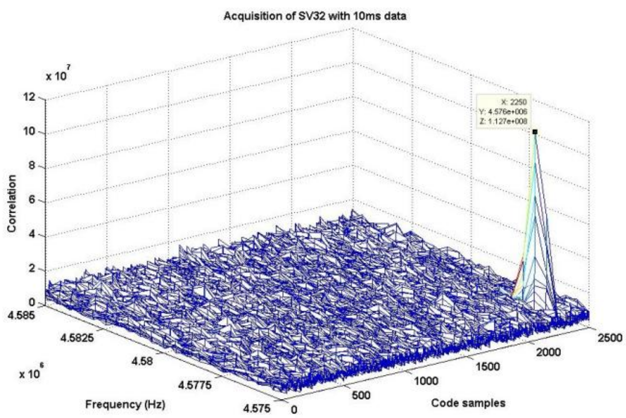

Fig. 12: Acquisition of SV32 with one-eighth size circular correlation

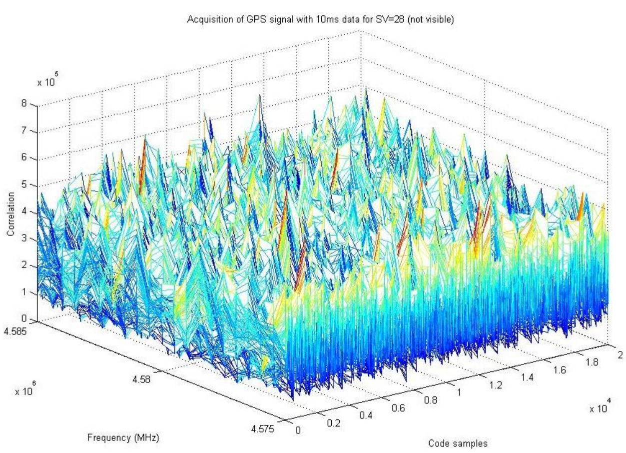

Fig. 13: Acquisition result of an invisible satellite

Figure 13 shows the result when the acquisition algorithm tries to acquire an invisible satellite, giving multiple ambiguous correlation peaks.

Figure 14 gives the I-channel and Q-channel outputs and the corresponding navigation data bits according to the I-channel output. I-channel output contains the navigation data information while Q-channel contains only noise. There is one navigation data bit for every 20 ms of data.

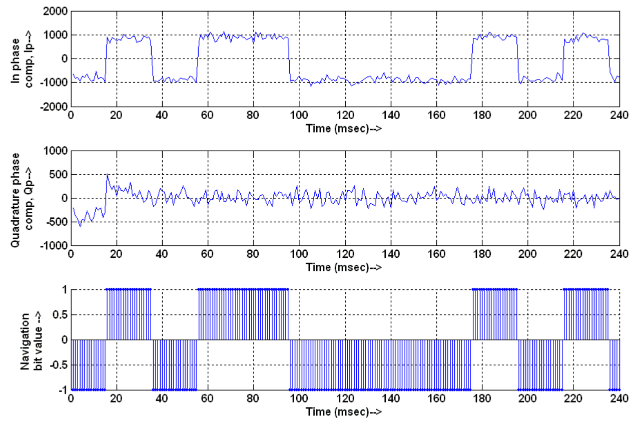

Fig. 14: I (top) and Q (center) channel outputs of SV32 from tracking loop. Coding of navigation data (bottom) from I-channel

5.3 Comparison of Acquisition Methods

Correlation peak values for SV32 for different non-coherent integration periods and the improved acquisition methods have been compared in Table 1.

Table 1. Correlation Peak Values for SV32

Non-coherent Integr-ation Period	10 ms	5 ms	2 ms	1 ms
FSCC	8.937e+6	4.216e+6	1.409e+6	8.998e+5
HSCC	7.956e+6	3.048e+6	9.629e+5	Peak not detected
QSCC	3.175e+7	1.202e+7	3.621e+6	2.142e+6
One-fifth size CC	5.133e+7	1.951e+7	6.067e+6	3.561e+6
One-eighth size CC	1.127e+8	4.291e+7	1.323e+7	7.534e+6

From the above table, it is examined that for any given integration period, HSCC provides lowest correlation peak values among the five techniques used. The correlation peak values decrease with decrease in the integration period. HSCC gives such low peak values for 1 ms integration period that the peak goes undetected, and the satellite cannot be detected. Hence, HSCC cannot detect weak signals. However, on reducing the size of FFT points, used for circular correlation, to one-quarter, one-fifth and one-eighth of its full-size, the correlation peak values increase for a given integration period.

SV numbers 11, 16, 20, 23, 31 and 32 have been successfully acquired using each of the above methods. Tables 2 and 3 compare the Doppler offsets and code phase offsets, respectively, for the visible SVs using various acquisition methods in MATLAB with those obtained from the Accord GPS Signal Tap Receiver. The Doppler offset values obtained from various acquisition methods can be found to closely agree with those obtained from the GPS Signal Tap Receiver.

Table 2. Doppler Offset Values (in Hz) after acquisition

SV No.	Accord Rx Data	FSCC	HSCC	QSCC	One-fifth size CC	One-eighth size CC
11	1484	1480	1480	1480	1490	1480
16	2760	2770	2770	2775	2760	2775
20	4837	4850	4850	4845	4845	4845
23	3779	3780	3775	3770	3780	3770
31	–458	–455	–455	–455	–455	–455
32	4047	4050	4050	4045	4045	4045

Table 3. Code Phase Offset (in Code Samples) after acquisition

SV No.	Accord Rx Data	FSCC	HSCC	QSCC	One-fifth size CC	One-eighth size CC
11	1088	1088	544	273	218	137
16	8804	8804	4402	2202	1761	1101
20	1955	1954	977	488	391	244
23	6605	6605	3303	1652	1322	827
31	10995	10995	5498	2749	2200	1375
32	17996	17996	8998	4500	3600	2250

It is observed that the code phase obtained from HSCC is half, and those obtained from QSCC is approximately one-fourth the value obtained from FSCC, and so on. This is obvious from the fact that only half spectrum is used in performing HSCC, one-fourth in QSCC, and so on. Hence, a trade-off study is done to seek a good balance between the acquisition accuracy and the processing time.

6. CONCLUSIONS

Faster and reliable acquisition methods have been proposed by analyzing the C/A code spectrum alone. The symmetric redundancy and concentration of more power at lower frequencies leads to reduction in FFT points used to perform circular correlation during acquisition process. This reduces the computational load and speeds up the acquisition process. However, such improvement is possible only if the sampling frequency is much higher than the chipping rate. There is also a limit to which the correlating points can be reduced as precision of correlation peaks is to be compromised with reduced sizes.

Simulation results of the reduced-size correlation acquisition methods show a reduction up to one-eighth the total FFT size is possible for successful acquisition of satellites. Also, reducing the noncoherent integration period to a minimum of 1 ms before threshold detection shows significant increase in processing speed.

Correlation with less than half the C/A spectrum has produced better results than those with HSCC. The Doppler offset values and code phase offset values obtained from the proposed methods closely agree with those obtained from Accord's GPS Signal Tap Receiver. Hence, the reduced-size correlation methods perform reliable acquisition, and pave way for further development of new algorithms to enhance GPS receiver performance.

7. REFERENCES

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