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Variants of Koch curve: A Review

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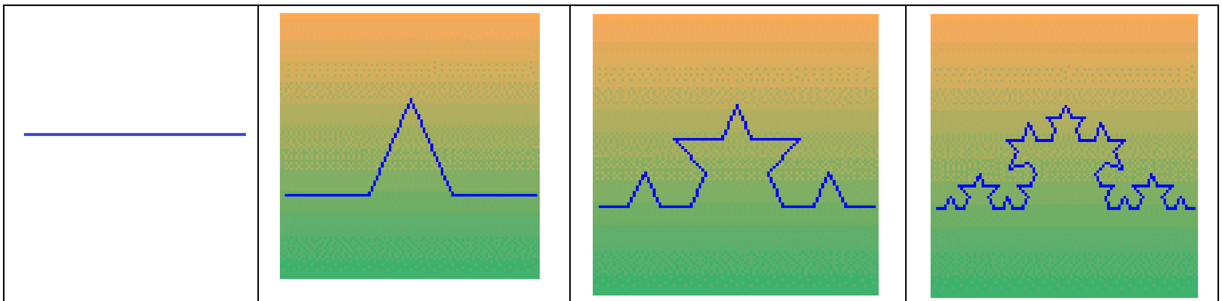
ABSTRACT
The von Koch snowflake curve is a mathematical curve, and is being used as antenna in wireless communications. There are various variants of the Koch curve scattered in literature. This paper attempts to present a critical review of the variants of Koch curve.

1. INTRODUCTION
Benoit B. Mandelbrot is a French and American mathematician, and the best known as the founder of fractal geometry, which impacts mathematics [2, 5], diverse sciences [1, 11, 13, 22], arts [6] and architecture [cf. 15], [23]. In fractals, there is always some kind of pattern that repeats itself and repetition is a seldom-failing source of self-similarity [24, p. 45]. Iteration is one of the richest sources of self-similarity [24, p. 49]. Most of the fractal-literature have been created using one-step feedback process (via Peano-Picard iterative approach). A few fractals were generated using two-step feedback process in 2002 (via superior iterations) [16] and since then many fractals have been added into this category [25].
The von Koch curve is a classical example of fractals. The Koch snowflake (also known as the Koch star and Koch island) is a mathematical curve, which is continuous everywhere but differentiable nowhere. It is a bounded curve of infinite length [24, p.13], [7, p. xxiii]. It is based on the Koch curve, which appeared in 1904 in a paper titled "On a continuous curve without tangents, constructible from elementary geometry" (original French title: "Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire") by the Swedish mathematician Helge von Koch [28].
The von Koch curve has proved its usefulness as antenna in wireless communication. Many variants of the Koch curve have been given in the literature. The purpose of this paper is to present a review of variants of Koch curve.

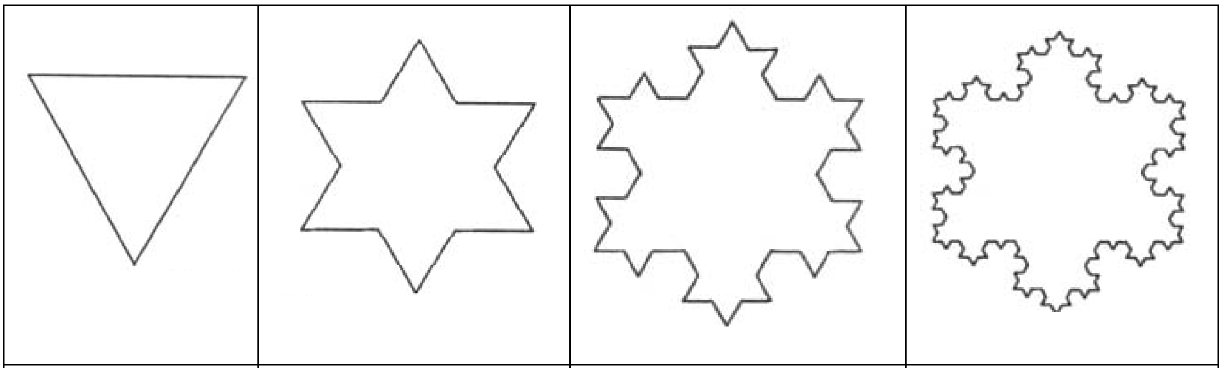
2. THE VON KOCH CURVE
Swedish mathematician Helge von Koch introduced what is now called Koch curve. Initiator of the Koch curve is a straight line it will be partitioned into three equal parts. Middle part will be replaced by an equilateral triangle. This is the basic step and the reduced figure will have four equal lines joined with each other. This new figure is known as a generator. The same operation will be repeated be on each of the four lines [7]. Iterative construction of the Koch curve is given in Figure 1(a-d)



(a) Step 0 | (b) Step 1 | (c) Step 2 | (d) Step 3

Fig. 1: Iterative Construction of Koch Curve (Shroeder, 1991)

When initiator is a triangle and the same procedure is applied, Koch snowflake curve is obtained. Iterative construction of the Koch snowflake curve is shown in Figure 2(a-d) [7, 12, 24].



(a) Step 0	(b) Step 1	(c) Step 2	(d) Step 3

Fig. 2: Iterative Construction of Koch Snowflake Curve (Shroeder, 1991)

Self-Similarity Dimension: This is generally used in calculating dimension of self-similar figures. There is a nice power relation between the number of pieces *n* of an object and the reduction factor *s* as shown in Eq. 1 [12].

$$n = \frac{1}{s^D} \qquad \ldots(1)$$

where *n* is the number of pieces of an object,
D is the dimension of the object and
s is the scaling factor.

Fractal dimension of the von Koch curve is ≈ 1.262. The beauty of the Koch snowflake is that its perimeter is infinite but area is compact and equal to $\frac{6\sqrt{3}}{5} r^2$, where r denotes the radius of the circle which accommodates the Koch snowflake. This geometrical property makes the Koch fractal as better antenna in place of circular antenna

3. VARIANTS OF KOCH CURVES

In 1984, Barcellos [3] gave variants of Koch curve by dividing the initiator into 4 equal parts (see Fig. 3). These curves have a fixed dimension. Further, there was no suggestion to obtain the curves of lesser dimension.

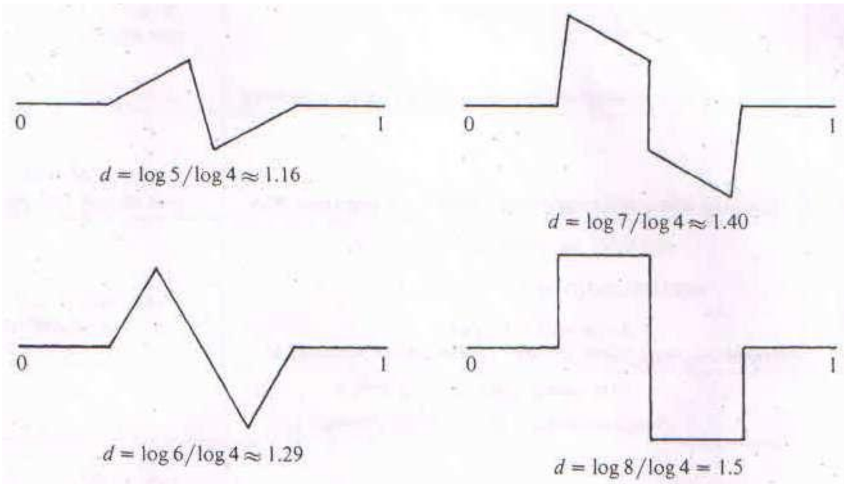


Fig. 3: Koch curves obtained by dividing the initiator into four equal parts (Barcello, 1984)

In 2002, Vinoy, Jose and Vardan [26] generated new shapes of Koch antenna by varying its indentation angle. Further, they gave formula (Eq. 2) to calculate fractal dimensions of these curves.

$$D = \frac{log4}{\log[2(1 + cos\theta)]} \qquad \ldots(2)$$

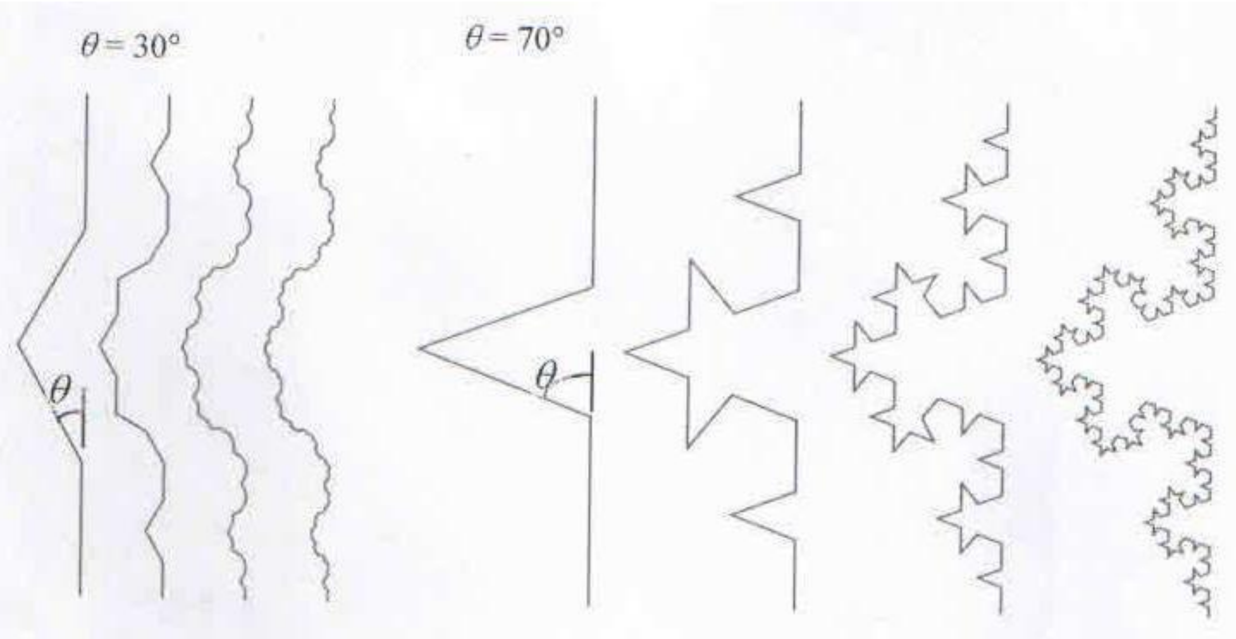
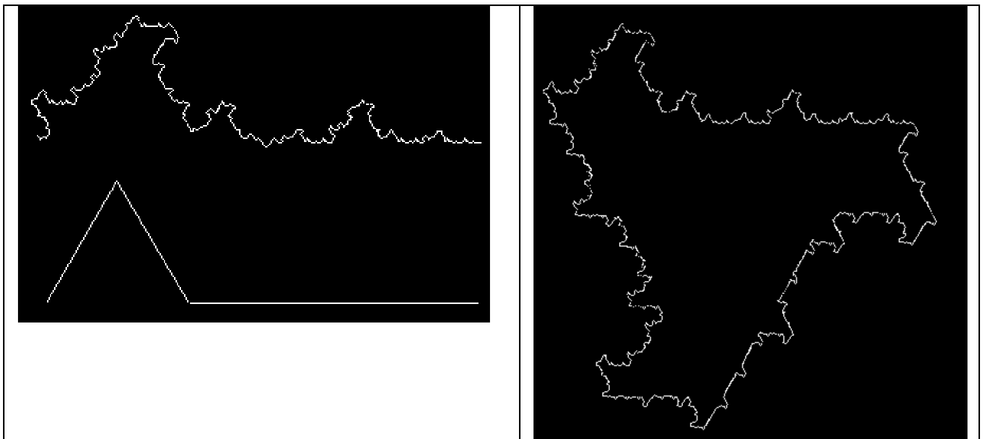


Fig. 4 Generalized Koch curves of first four iterations with two different indentation angles. The length of subsections for a given iteration is a function of angle of indentation.

All these variants have lesser dimension than the conventional Koch curve. The curves given by Vinoy et al. occupy more area than the conventional Koch curve.

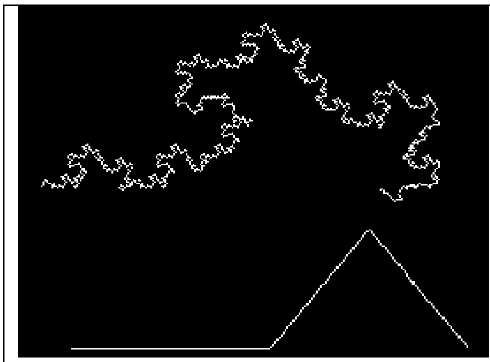
From the Koch curves given by Barcellos [3] and Vinoy et. al. [26], Haq [8] concluded that there was need to propose Koch shapes of lesser dimension and compact size. Such types of Koch shapes will performance better as antenna with compact size as Vinoy et. al. [27] had shown that fractals with lesser dimension are better antennas. Beside this one should have flexibility in designing of antenna for a certain performance. Also, there is a curiosity left in the literature whether the two different shapes having same dimension will have same performance as antenna or not.

For the above requirements, Haq, Rani and Sulaiman [9, 10, 18, 19] looked into gallery of superior fractals. They found that by dividing the initiator into unequal parts fractal plants [4], fractal carpets [15, 17, 20] and Cantor sets have been generated [21]. Fractals generated by dividing the initiator into unequal parts are part of another gallery named as superior fractals. For a detailed study of superior fractals, one may refer to [25]. Seeing the success of superior fractals, Haq, Rani and Sulaiman proposed to generate new Koch models of lesser dimension and smaller size (compared to Koch models of Vinoy et. al. [26]) by dividing the initiator into unequal parts. See Fig. 5-9 [9, 10, 19]. Geometrical properties of these curves have been discussed by them [18]. Koch left one-third curve (Fig. 5a) has same dimension as conventional Koch curve. Koch right-half (Fig. 6a) and Koch middle-half curves (Fig. 7a) have larger dimensions than the conventional Koch curve. Koch middle-one fifth curve (Fig. 8a) and Koch middle one-sixth curve (Fig. 9a) has dimensions 1.113 and 1.086 respectively that are smaller than the conventional Koch curve

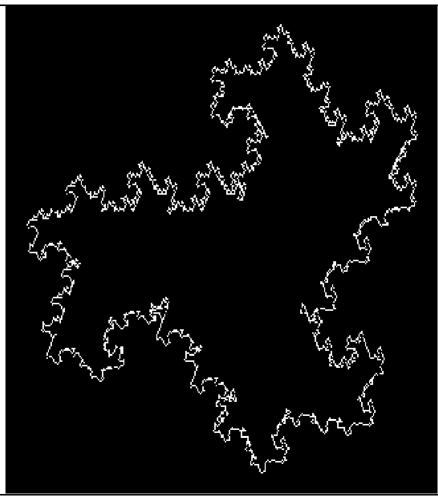


(a) Koch left one-third curve with Generator | **(b) Koch left one-third Snowflake**

Fig. 5: Koch Left One-Third Curve and Koch Snowflake for (r1, r2, r3) = (1/3,1/3,1/3)

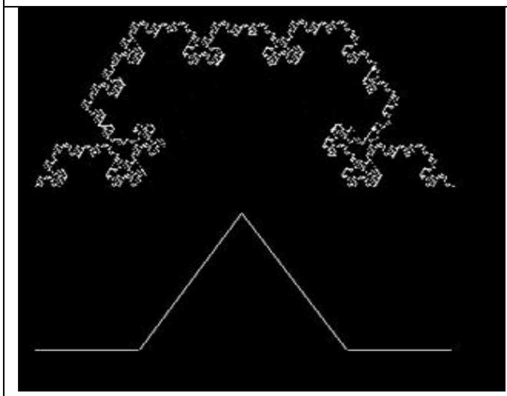


(a) Koch right half curve with generator

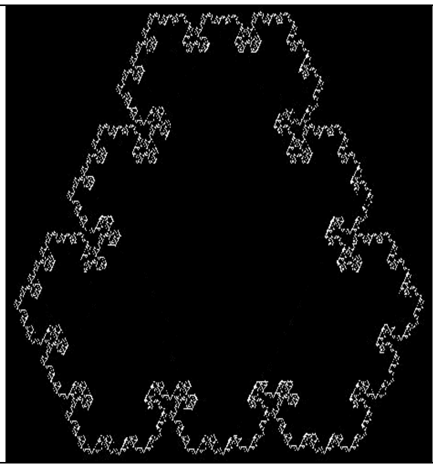


(b) Koch right half snowflake

Fig. 6: Koch right half curve and snowflake for (r1, r2, r3) = (1/4, 1/4, 1/2)

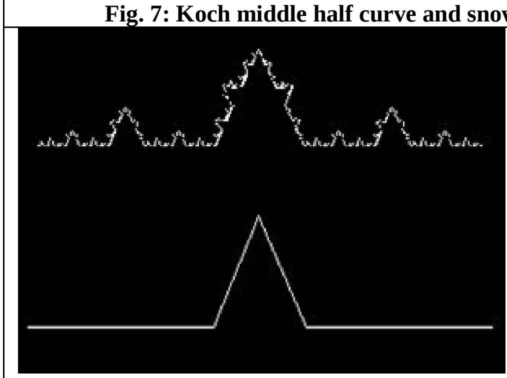


(a) Koch middle half curve with its generator

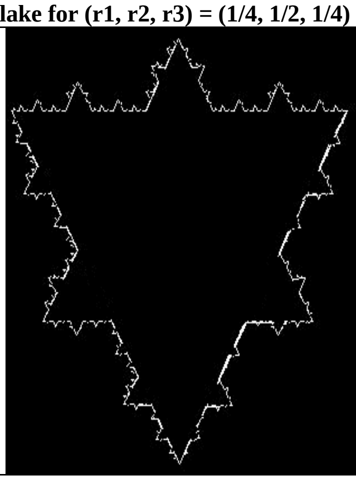


(b) Koch middle half snowflake

Fig. 7: Koch middle half curve and snowflake for (r1, r2, r3) = (1/4, 1/2, 1/4)

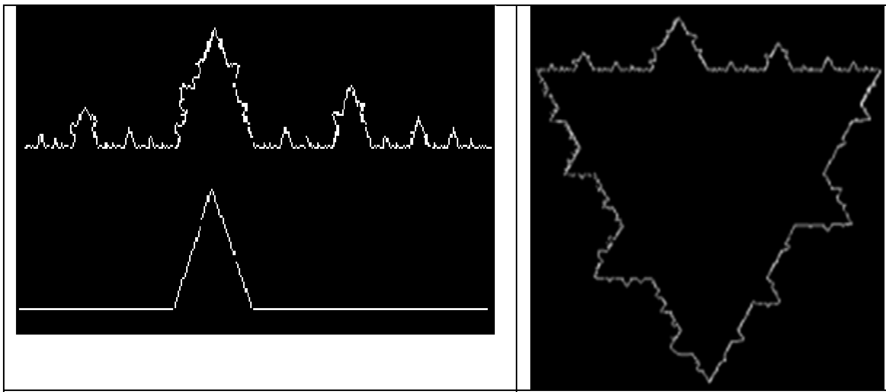


(a) Koch middle one-fifth curve with its initiator



(b) Koch middle one-fifth snowflake

Fig. 8: Koch Middle one-fifth curve and snowflake for (r1, r2, r3) = (2/5, 1/5, 2/5)

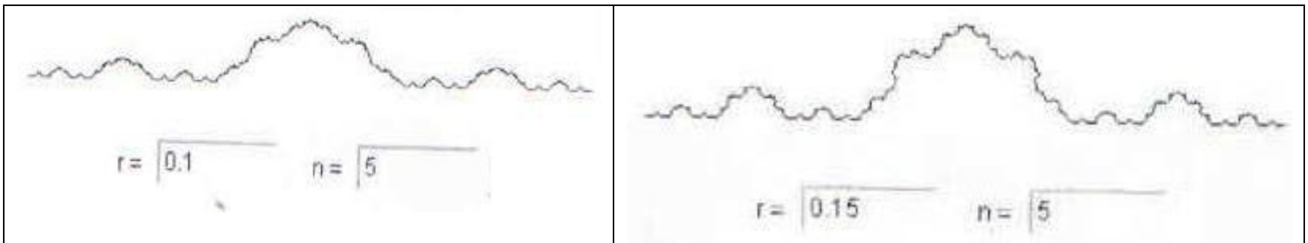


(a) Koch middle one-sixth curve with its generator | **(b) Koch middle one-sixth snowflake curve**

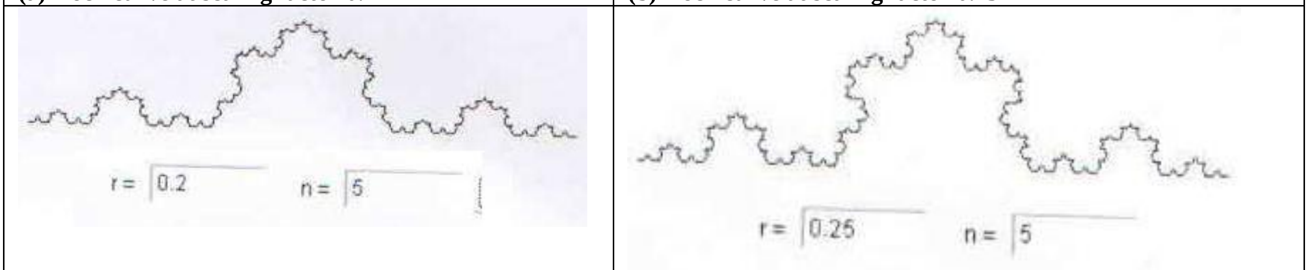
Fig. 9: Koch middle one sixth curve and snowflake for (r1, r2, r3) = (1/3, 1/6, 1/2)

With the increased number of Koch curves in the literature, Haq, Rani and Sulaiman felt the need to systematize them into different categories to understand the methodology of their generation. They proposed classification based on the method of division of initiator. Different variants of Koch curve have been classified into two categories: generation by equal division and generation by unequal division. All the variants discussed here may be made fit into these two categories.

In 2011, Prasad gave a few more variants of Koch curves (Fig. 10 (a-d)). The shapes given by Prasad in Fig. 10 do not seem correct. If there, is scaling factor 0.1 (Fig. 10a), then either there will 5 equal divisions of the initiator or one of the part should be of 0.1 length and rest of unequal length is division is only into three parts. Fig. 10c and Fig. 10d will have 5 and 4 equal parts as their scaling factors are 0.5 and 0.25 respectively. Fig. 10b must show unequal division in its figure, which can not be seen.



(a) Koch curve at scaling factor 0.1 | **(b) Koch curve at scaling factor 0.15**



(c) Koch curve at scaling factor 0.2 | **(d) Koch curve at scaling factor 0.25**

Fig. 10

4. CONCLUSION

Koch curve is a classical fractal model. The paper presented a critical review of variants of Koch curves, and their comparison

5. REFERENCES

[1] Michael F. Barnsley, and Hawley Rising, Fractals Everywhere (2nd ed.), Academic Press Professional, Boston, 1993.

[2] John Briggs, Fractals: The Patterns of Chaos (2nd ed.), Thames and Hudson, London, 1992.

[3] Anthony Barcellos, The fractal geometry of Mandelbrot, The College Mathematics Journal, 15(2), 1984, 98-114.

[4] M. Chandra, and M. Rani, Categorization of fractal plants. Chaos, Solitons, Fractals, 41(3), 2009 1442-1447.

[5] Robert L. Devaney, A First Course in Chaotic Dynamical Systems: Theory and Experiment, Westview Press, CO, 1992.

[6] Ron Eglash, African Fractals: Modern Computing and Indigenous Design, Rutgers University Press, New Brunswick, 1999.

[7] K. Falconer, Fractal Geometry: Mathematical foundations and applications, Second edition, John Wiley & Sons, NJ, 2003.

[8] Riaz Ul Haq, Design and Simulation of Variants of Koch Models for Fractal Antenna, MS thesis, Faculty of Computer Systems & Software Engineering, University Malaysia Pahang, Kuantan, Malaysia, 2011.

[9] Riaz Ul Haq, Mamta Rani, and Norrozila Sulaiman, Categorization of superior Koch fractal antennas, *in:* IACSIT Proc. Int. conf. on Intelligence and Information Technology, Lahore, Pakistan (ICIIT 10), Oct 28-30, 2010, vol. 1, 551-554.

[10] Riaz Ul Haq, Norrozila Sulaiman, and Mamta Rani, Superior fractal antennas, *in:* Proc. Malaysian Technical Univ. Conf. on Engg. and Tech., Melaka, Malaysia, Jun 28-29, 2010, 23-26.

[11] R. Hohlfeld, and N. Cohen, Self-similarity and the geometric requirements for frequency independence in antenna, Fractals, 7(1), 1999, 79-84.

[12] Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe**,** Chaos and Fractals: New frontiers of science (2nd ed.), Springer-Verlag, New York, 2004. MR2031217 Zbl 1036.37001

[13] Gongwen Peng, and Tian Decheng, The fractal nature of a fracture surface, J. Phys. A: Math. Gen., 23(14), 1990, 233-257.

[14] S. Prasad, Superior Koch curves, Int. J. Artif. Life Res., 2(4), 2011, 24-31.

[15] M. Rani, Fractals in Vedic heritage and fractal carpets, *in:* Proc. National seminar on History, Heritage and Development of Mathematical Sciences, 2005, 110-121.

[16] Mamta Rani, Iterative Procedures in Fractals and Chaos, Ph. D. Thesis, Department of Computer Science, Gurukula Kangri Vishwavidyalaya, Hardwar, India, 2002.

[17] M. Rani, and S. Goel, Categorization of new fractal carpets, Chaos, Solitons, Fractals, 41(2), 2009, 1020-1026.

[18] Mamta Rani, Riaz Ul Haq, Norrozila Sulaiman, Koch Curves: Rewriting System, Geometry and Application, Journal of Computer science, 7(9), 2011, 1358-1362.

[19] Mamta Rani, Riaz Ul Haq, Norrozila Sulaiman, New Koch Curves and Their Classification, Chaos & Complexity Letters, 6(3), 2011.

[20] M. Rani, and V. Kumar, New fractal carpets. Arab. J. Sci. Eng., Sect. C Theme Issues, 29(2), 2004, 125-134. MR2126593

[21] M. Rani, and S. Prasad, Superior Cantor sets and superior Devil's staircases. Int. J. Artif. Life Res., 1(1), 2010, 78-84.

[22] K. M. Roskin, and J. B. Casper, From Chaos to Cryptography, 1998. http://xcrypt.theory.org/

[23] Nicoletta Sala, Architecture and Time, Chaos Complex. Lett., 5(3), 2011, 33-43.

[24] M. Schroeder, Fractals, Chaos, Power Laws: Minutes from an infinite paradise, W. H. Freeman and Company, New York, 1991.

[25] S. L. Singh, S. N. Mishra and W. Sinkala, A new iterative approach to fractal models, Commun. Nonlinear. Sc. Numer. Simul. 17(2), 2012, 521-529.

[26] K. J. Vinoy, K. A. Jose and V. K. Varadan, Multi-band characteristics and fractal dimension of dipole antennas with Koch curve geometry, *in:* Proc. Antennas and Propagation Society International Symposium, 4, 2002, 106-109.

[27] K. J. Vinoy, K. A. Jose and V. K. Varadan, Impact of fractal dimension in the design of multi-resonant fractal antennas, Fractals, 12, 2004, 55-66.

[28] http://en.wikipedia.org/wiki/Koch_snowflake

6. AUTHORS PROFILE

Mamta Rani was born in 1976 in India. She received her master and PhD degree in computer science from Gurukula Kangri Vishwavidyalaya, Hardwar, India. This university is 110 years old. She is presently professor in Department of Computer Applications, Krishna Engineering College Ghaziabad. Before joining this college, she has served University Malaysia Pahang, Malaysia and many colleges affiliated to Uttar Pradesh Technical University, CCS Meerut University and MJP Rohilkhand University in India. She has published 32 research papers and guided 2 Ph. D. and 3 Masters students. She got first prize for the best paper presentation in a conference in India, got silver medal in an exhibition in Malaysia and has been recognized by Marquis Who's Who for scientific contribution in 2011-2012. Presently, she is guiding 3 PhD scholars and 4 master level thesis. Fractal and chaos is her research area.

Riaz Ul Haq was born in 1982 in Pakistan. He received his master degree in computer science in 2012 from University Malaysia Pahang, Malaysia and now a Ph. D. student in the same university. His research interest is Koch curves and their antenna properties. He has published 4 research papers and received got silver medal in an exhibition in Malaysia.

Deepak Kumar Verma was born in 1980 in India. He received MCA degree from U. P. Technical University, Lucknow, M. Phil degree in computer science from Maharishi Markandeshwar University Mullana, Ambala, Haryana, India and presently pursuing M. Tech. in Computer Science & Engineering from AMITY University, Noida. He is presently Assistant Professor in Department of Computer Application, Krishna Engineering College, Ghaziabad. Earlier, he has been Lecturer in GNIT, Greater Noida.