

# Least Squares Algorithms for Time of Arrival Based Mobile Source Localization and Time Synchronization in Wireless Sensor Networks

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## ABSTRACT

Accurate source localization and synchronization is of considerable interest in wireless communications. Localization and synchronization are two important issues which are traditionally treated separately in communication systems and wireless sensor networks. In this paper, we present a unified framework to solve these two problems at the same time jointly. Two algorithms are developed for accurate mobile source localization and time synchronization using the time-of-arrival measurements of the signal. The first algorithm, Least Square (LS) estimator, is derived for joint location and timing estimation which is more computationally efficient. The second algorithm is Weighted Least Square for improving estimation accuracy is proposed. For the joint source localization and time synchronization the Cramer-Rao lower bound (CRLB) is also derived.

## General Terms

Wireless Sensor Networks; Localization; Time of Arrival (TOA) Technique; Localization;

## Keywords

Synchronization; Cramer-Rao Lower Bound (CRLB); Least Square (LS) estimator; Weighted Least Square (WLS);

## 1. INTRODUCTION

Wireless Sensor Networks Consists of number of sensor nodes deployed in the sensor fields. These sensor nodes have the capabilities of sensing, computing and wireless communicating through multihop infrastructure.

Recent years have witnessed tremendous growth in both interests and applications of wireless sensor networks[1]. Various advancement in wireless communication and electronics technologies have enabled the development of low cost, low-power, multifunctional sensor nodes that are small in size and communicate in short distances.

As shown wireless sensor network architecture (see Figure 1) which consists of sensor field with sensor nodes, sink nodes, etc. Each of these sensor nodes has the capabilities to collect the data and route data back to the sink node through multihop infrastructure less architecture through the nodes [2].

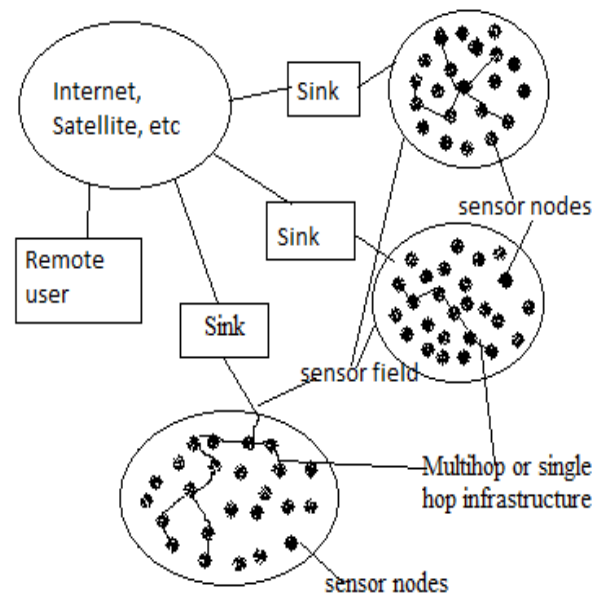


Figure 1: Wireless Sensor Network Architecture

Sink node in turn can communicate with remote user through internet, satellites, etc. Hence sink node act as a gateway between remote user and sensor nodes. Basic building block of a sensor node are a sensing unit, a processing unit, a transceiver unit, a power unit, location finding system, power generator, and mobilize.

There are wide ranges of applications of wireless sensor network due to its powerful function and low energy cost. Some of them are habitat monitoring, wildlife, pollution, natural catastrophes, machine health, human health, traffic monitoring, surveillance, tracking, detection, space exploration, complex mechanical control etc [1, 2]. With its wide range of application wireless sensor networks certainly come into each domain of people's life. Since nodes in a wireless sensor network are randomly deployed so we need to know the actual position or location of these nodes. Hence the process of finding the location of node is known as localization. Over the last few years the problem of source/sensor localization in wireless sensor networks is crucial and has assumed increasing significance. A source node generates the information or message, if something happens in their locality. Without position information, wireless sensor network cannot work properly [2,9]. Hence to make the collected information valuable we need to know the location of source nodes. Accordingly, confirming the position[20] where event happens and the node's coordinate

which capture information is WSN's the most basic function, and also has the pivotal action for WSN's validity.[4]

Traditionally, Global Positioning Systems (GPS) [9] has been widely for locating nodes in sensor field, but due to their few drawbacks like heavy-weight and expensiveness they are not supported by all the cheap sensor nodes in wireless sensor networks. Other limitations are that they cannot be implemented for non line-of -sight communication and battery-life of sensor nodes will reduce due to high power consuming Global Positioning Systems (GPS). This in turn will reduce[17,18] the effective life time of the entire network. Hence, we require an an alternate solution of GPS is required which is cost effective, rapidly deployable and can operate in diverse environments. Hence, localization has become the hot research area of wireless sensor network.

Various localization approaches are based on time-of-arrival (TOA), received signal strength (RSS), time-difference-of-arrival (TDOA) or angle-of-arrival (AOA). RSSI is the most common ranging technique, can be used to estimate the distance between two nodes based on the strength of the signal received by another node[1,2,9,13].

In TOA, the distance is calculated based on the propagation time and the speed of the signal as shown in Fig 2(a). In TDOA difference between the receive time of two separate signal can be used to estimate the distance between two nodes.[1,2,9] as shown in Fig 2(b).

Let distance between two nodes is D as shown in Fig 2(b)

$$D = (S_{rf} - S_{usf}) * (T_{usf} - T_{rf} - T_{delay}) \quad (1)$$

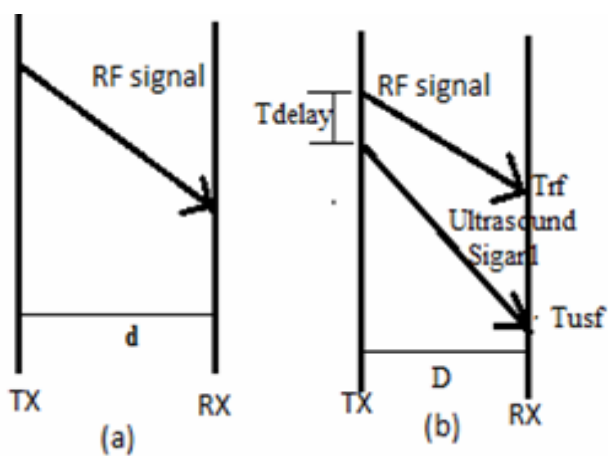


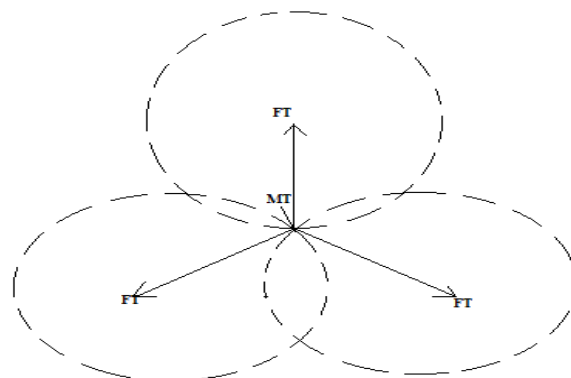
Fig 2:(a)time of arrival(TOA) b) Time difference of arrival

were,  $S_{rf}$  is speed of Rf signal and  $S_{usf}$  is speed of Ultrasound signal NLOS communication may lead to high localization errors because of different path either type of signal may follow which leads to different propagation delay. Compare to RSSI, TDOA require additional transmitter-receiver pairs in each node for the second type of signal. Direction of the received signal can be exploited for localization. Rely on directional antenna or special multiple antenna configurations to estimate AOA[2,6,9] of the received signal for beacon nodes.

Disadvantage of this method is that higher complex antenna arrays are required for direction measurement which increases the cost. Figure 3 and 4 shows the various localization approach[3,8]. In this paper, we used the time-of-arrival approach for localization.

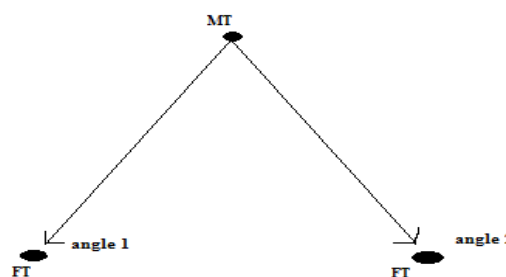
Now, if Localization helps source node to estimate its current location than time synchronization allows a source node to its estimate current time. Therefore, with localization we need to synchronize the nodes as well.

Traditionally, synchronized [4] was studied by computer science community from protocol design of view. While, localization is studied from the signal processing point of view. As a result, these two problems which have close relationship in common have been investigated separately for a long time. They also shares many aspects in common. Hence we formulate them into a unified framework [5,6,18] and solve the two problems at the same time



TOA, RSS and TDOA Measurements

Figure 3: TOA, TDOA, RSS measurements



Angle Of Arrival (AOA)Measurements

Figure 4: AOA measurements

In wireless sensor networks, hierarchical method is usually used to synchronize and localize a large sensor field[2, 5]. Some sensor nodes are synchronized and localized to the anchor first and then synchronized and localize other nodes.

The rest of the paper is organized as follows. In Section 2, the system model is described which is based on the Time-Of-arrival (TOA) measurements approach. we first derive a simple Least Square (LS) TOA-localization algorithm. After that in next section we described an improved algorithm which weighs the Least Square (LS) function and then the relation between the range and the position coordinates is then derived. Finally Cramer-Rao Lower Bound (CRLB)[6] is reviewed for performance measures of location accuracy. Finally, Future Research work and conclusions are drawn in Section 4 and Section 5 respectively.

## 2. SYSTEM MODEL

### 2.1 TOA Measurement model

In wireless sensor network, we describe a model for the localization and synchronization of the mobile node which is called as source node based on the time-of-arrival approach. In this approach the distance between the nodes is calculated based on the propagation time and the speed of the signal. All the measurements are taken for line of sight(LOS) propagation. First we estimate the position of one node only. One possible way to model physical space is to do this as a two or three dimensional real-valued vector space. Similarly, time can be modelled as a one dimensional valued vector space. These two vector spaces are often combined to form a three or four dimensional vector space known as space time[10, 11]. In this paper space is taken as 2 dimensional coordinate system.

Mobile source localization is determining based on multiple observations of the emitted signal source from N number of fixed sensor nodes represented as fixed terminals (FT) in Fig 3[10]. The accuracy is the primary goal of source Localization.

Let the mobile source location S be [Xs, Ys] and the coordinate of ith fixed sensor node be [Xi, Yi] also one sensor node is taken as reference node at coordinate [0, 0].

Total sensor node deployed is N. Distance[8] between ith node and source node is denoted by di, given as

$$d_i = \sqrt{(X_i - X_s)^2 + (Y_i - Y_s)^2} \quad i=1, 2, \dots, N \quad (2)$$

now time taken by the signal to travel from node i to source node can be calculated as

$$t_i = d_i / c \quad i=1, 2, \dots, N \quad (3)$$

where  $c=3 \times 10^8$  m/sec, speed of light.

In the presence of noise  $n_i$ , which is zero mean white process with variance  $\sigma^2$ . Now the range between two nodes is given as

$$r_i = d_i + n_i \quad (4)$$

Let,  $t_0$  is the unknown time instant at which the source transmits the signal to be measured. Then the time of arrival  $t_i$  at sensor node ith can be estimated as

$$T_i = t_i + t_0 + n_i \quad (5)$$

Now for jointly synchronized and localized the source node the expression used can be solved with Least Square estimator[8]. Let  $s'$  is the estimated location and  $t_0'$  be the estimated time then we have

$$(s', t_0') = \arg \min_{s, t_0} \sum_{i=1}^N (T_i - t_i - t_0')^2 \quad i=1, 2, \dots, N \quad (6)$$

Now if we compare the model of Time-of-arrival (TOA) to that of Time-Difference-Of-Arrival (TDOA) the subtraction of time between two nodes will solve the problem of synchronization as the factor to will get cancel but this subtraction will also increases the noise. This subtraction strengthens the noise in TDOA by exactly 3 dB and which leads to noise correlation [6,17]. For this reason, TDOA

decreases the performance as for this reason TDOA should be avoided[12].

## 3. ALGORITHMS

### 3.1 Least squares

Least square is a standard approach to the approximate solution of over determined systems i.e., sets of equation in which there are more equations than unknowns [7,14].

In this overall solution minimizes the sum of the errors made in solving every single equation.

The estimated transmission time dependent on S can be expressed using Least Square estimator with zero mean noise  $n_i$ . that is

$$t_0' = \frac{1}{N} \sum_{i=1}^N (T_i - t_i) \quad (7)$$

substitute the above equation in equation (5) for  $t_0'$ , we have now

$$(s', t_0') = \arg \min_{s, t_0} \sum_{i=1}^N (T_i - t_i - \frac{1}{N} \sum_{j=1}^N (T_j - t_j))^2 \quad (8)$$

Now for optimization we applied the relaxation technique as in [6], by squaring both sides and rearranging them we get the required estimated value for position and time both as given here

$$A\theta = b + e \quad (9)$$

$$\text{Where } \theta' = \text{argmin}(A\theta - b)^T (A\theta - b) \quad (10)$$

Where A, b and e have usual meaning which comes out after solving the above equation 7 as in [6].

### 3.2 Weighted Least squares

Now weighting matrix W is added to have better performance which leads to optimization of constrained.

$$\theta' = \text{argmin}(A\theta - b)^T W (A\theta - b) \quad (11)$$

Selection of W depends on the study of the disturbance in b.

The drawback of weighted least square (WLS) is that it is complex [7] in calculation compare to other algorithms.

### 3.3 CRAMER-RAO LOWER BOUND (CRLB)

The Cramer-Rao Lower Bound (CRLB) gives a lower bound on variance achievable by any unbiased estimators and is useful as a guideline: knowing the best an estimator can possibly do helps us judge the estimators that we implement.

Hence this analysis is done to see the effect of position error of node on the performance limit of source localization for the proposed estimator [18]. CRLB for the joint estimation problem is derived by deriving the Fisher information matrix to integrate the sensor position errors [7,8].

The CRLB for the kth parameter estimate of  $\Phi$ , is given by the kth element of  $\sigma^2(BTB)^{-1}$ , where  $\Phi^T = [X_s', Y_s', t_0']$ , can be computed from [6,8]

$$\text{CRLB}(\Phi') = \sigma^2 (BTB)^{-1} \quad (11)$$

Where, A can be derived as explained in [6].

#### 4. FUTURE RESEARCH

As a future work we intend to simulate our proposal against existing algorithms. This research paper dealt with a single mobile source scenario. In future work, we extending the work to multiple sources and also since our algorithms are line-of-sight (LOS) based schemes therefore investigating the effect of non-line of sight on the joint estimation in future. Since Least Square (LS) estimator is only applicable to single source and also this method gives noisy location estimation therefore we intend to work on different estimation methods such as Linear Interpolation (LI)[14] and cluster-based schemes [15].

#### 5. CONCLUSION

In this paper, we estimated the location and then synchronized mobile source node in unified frame in wireless sensor networks. Two TOA-based location algorithms are developed using TDOA measurements.

The first Least Square (LS) algorithm with TOA measurements. The second method is a weighted Least Square (WLS) which is an improved version of the first algorithm. But the second algorithm increases the computational complexity compare to the previous algorithm. Both this algorithms Cramer-Rao Lower bound (CRLB).

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