Fractional Order PID Controller Optimized using Bacterial Foraging Technique for Three Interacting Tank Process considered as a Third Order Process

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ABSTRACT
In this paper, an attempt has been made to tune the fractional order PID controller parameters for three interacting tank process using Bacterial Foraging Algorithm (BFA). PID controllers are tuned to satisfy three control specifications as the tunable parameters are the proportional gain, integral gain and derivative gain. The search space can be improved by the investigation of fractional order PID which involves two more parameters, the integral order and the derivative order thereby handling two additional specifications. Grunwald-Letnikov definition is used for the defining the derivative controller and Oustaloup's filter technique is used for the approximation of the function. Tuning is complicated by the mathematical approach. The tuning is effected using BFA technique. The performance index selected is Integral Square Error (ISE). The proposed BFA tuned FOPID controller will serve as a viable controller for automating three interacting tank process.

Keywords
Bacterial foraging, Fractional Order Controller, Interacting Tank Process, Integral square error, Optimization

1. INTRODUCTION
Fractional calculus is thriving for the past two decades with the research progress in the field of chaos. Fractional order modeling and control design for dynamical system are still in introductory stage. The basic concepts of fractional order calculus is dealt with by Gement, Méhauté, Oustaloup and Podlubny [1-4]. Fractional calculus finds its role in varied fields like Chaos, fractals, biology, electronics, communication, digital signal processing, dynamical system modelling and control etc. [5-8]. Design of fractional order dynamical models and controller have gained popularity in recent past with the development of fractional calculus [9-13]. Fractional order PID controller was conceptualized by Podlubny [14] and effectiveness of the proposed work was demonstrated. Realization of the FOPID controller are shown in [15-17]. Various studies have been carried out in the frequency domain approach, pole distribution of characteristic equation, pole placement technique evolved. Integrating a fractional component in a integer order controller will result in FOPID. Till date PID controllers are termed as workhorse in automation of many industrial applications. The only change to be made is the introduction of fractional order to the integral and derivative component. Tuning the fractional order PID controller parameter is complicated because of the five dimensional search requirements. Evolutionary optimization techniques like Genetic algorithm, Particle Swarm optimization, Bacterial foraging, differential evolution, Bee colony optimization, Ant colony Optimization etc. are gaining popularity in the tuning of the controller parameters. A new biological inspired computation technique namely bacteria foraging (BF) optimization technique have been proposed [21,22]. Here, the foraging behaviour of the E.coli bacteria including locating, swarming, tumbling, ingesting food is mimicked. They are classified into four stages such as chemotaxis. Swarming, reproduction and elimination and dispersal which are used to optimize the parameters. Bacteria Foraging algorithm, an attractive method giving the FOPID controller parameters is proposed in this work.

In the proposed work, the fractional order PID controller parameters is brought out by using the optimal values obtained from BFA. Section II discusses the three interacting tank process description, followed by Fractional Calculus basics. Section IV elaborates the Bacterial Foraging optimized Fractional Order Controller tuning for the three interacting tank process. Section V explains the comparison of BFA-Integral Order (IO) controller with BFA-FO controller. Conclusion section brings out the advantages of the proposed technique.

2. THREE INTERACTING TANK PROCESS DESCRIPTION
The level control of three interacting tanks involves complex design and implementation procedure, as the response of each tank depends on the response of other tanks. Moreover the process is non-linear. Using classical control technique, the level control becomes complicated. The plant has three interacting tanks interconnected by manual control valves. The flow f_{in1} to the first tank and the flow f_{in2} to the third tank are the plant inputs. The levels of the three interacting tanks are the outputs of the plant. Thus the plant is a two inputs and three output systems.

The process liquid is pumped to the first interacting tank from the sump by pump 1 through the control valve 1 and the input flow to the first interacting tank is f_{in1}. The process liquid is pumped to the third interacting tank from the sump by pump 2 through the control valve 2 and this input flow to the third interacting tank is f_{in2}. The three levels in the three interacting tank are measured using differential pressure transmitters. The three interacting tanks are interconnected through manual control valves. The inputs are voltages u_1 converted to 4-20mA current that causes the actuation of the control valve and adjusts the inflow f_{in1} into the first interacting tank and voltage u2 converted to 4-20mA current that causes the actuation of the control valve and adjusts the inflow f_{in2} to the third interacting tank. In the present work, u_2 is kept constant and u_1 is controlled. The setup diagram of the three interacting tanks is shown (see Figure 1).

Fig 1: Hardware Set-up of Three Interacting Tank Level Process

The setup consists of three cylindrical process tanks, overhead sump, submersible pump, three Differential Pressure Transmitter (DPT), rotameter, control valve and interfacing card. The process tank is cylindrical and made of a transparent glass. Provisions for water inflow and outflow are provided at the top and bottom of the tank respectively. A pump is used for discharging the liquid from the storage tank to Tank 1. The inflow to the Tank 1 is maintained by a control valve. Differential Pressure Transmitter is used for liquid level measurement. In this open vessel process, tank pressure is given to the high-pressure side of the transmitter and the low-pressure side is vented to the atmosphere. Rotameter is used for the monitoring of the level. Gate valves one each at the outflow of the tank 1, 2 and 3 are also provided to maintain the level of water in the tanks. Clockwise rotation ensures the closure of the valve, thus stopping the flow of liquid and vice versa. A 25-pin male connector is used here to interface the hardware setup with the PC. The electrical output generated from the potentiometer is first converted into a digital value before applying it to the computer. The process parameters of the three interacting tank set in tabulated in Table 1.

Table 1. Process Parameters of the three interacting tank

Sl.	Process parameters	Values
1.	Area of the Tank, A_i	615.7522cm^2
2.	Area of the connecting pipes, a_{ij}	5.0671cm^2
3.	Valve ratio	
	β_{12}	0.9
	β_{23}	0.8
	β_3	0.3
4.	Pump Gain, Ki	75cm^3/V.s

The three interacting process has two manipulated variable the inflow to the tank 1 (u_1) and tank 3 (u_2) regulated by the control valve. The output of the process are the levels (h_1,h_2, h_3) of the tanks 1, 2 and 3 respectively. The inflow u_2 is maintained constant at 0.5V and the open loop response of the three interacting tanks is obtained by varying the voltage u_1 from 0.5V to 7.5V. The steady state I/O data are tabulated in Table 2. The I/O characteristics of the three interacting tanks for varying values of u_1 are shown in figure. 2. In the present work, the inflow u_1 is considered as the manipulated variable and the height of the third tank h_3 is considered as the output of the process.

Table 2. I/O Data obtained from the Lab Scale setup

F_{in1}	h_1	h_2	h_3
0.500	1.310	1.280	1.240
1.000	3.105	2.968	2.792
1.250	4.288	4.077	3.800
1.500	5.660	5.350	4.960
2.000	9.001	8.465	7.753
2.500	13.122	12.260	11.169
3.000	18.010	16.770	15.203
3.500	23.676	22.000	19.850
4.000	30.138	27.914	25.137
4.500	37.350	34.560	31.020
5.000	45.380	41.860	37.560
5.500	54.110	49.995	44.665
6.000	63.669	58.745	52.425
6.500	73.900	68.100	60.700
6.750	79.250	73.180	65.135
7.000	85.100	78.400	69.700
7.500	96.995	89.260	79.420

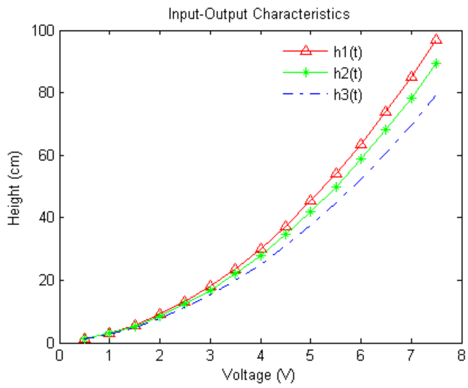

Fig 2: I/O characteristics of the three interacting tanks for varying values of u_1.

3. FRACTIONAL CALCULUS

The differintegral operator, ${}_aD_t^q$, is a combined differentiation-integration operator commonly used in fractional calculus. This operator is a notation for taking both the fractional derivative and the fractional integral in a single expression and is defined by

$$ {}_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & q>0 \\ 1 & q=0 \\ \int_a^t (d\tau)^{-q} & q<0 \end{cases} \qquad (1)$$

Where q is the fractional order which can be a complex number and a and t are the limits of the operation. There are some definitions for fractional derivatives. The commonly used definitions are Grunwald-Letnikov, Riemann-Liouville and Caputo definitions (Podlubny1999b). The Grunwald-Letnikov definition is given by

$$ {}_aD_t^q = \frac{d^q f(t)}{d(t-a)^q} = \lim_{N\to \mathrm{inf}} \left[\frac{t-a}{N}\right]^{-q} \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f\left(t-j\left[\frac{t-a}{N}\right]\right) \qquad (2)$$

The Riemann-Liouville definition is the simplest and easiest definition to use. This definition is given by

$$ {}_aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-q-1} f(\tau) d\tau \qquad (3)$$

Where n is the first integer which is not less than q i.e. $n-1 \leq q < n$ and Γ is the Gamma function.

$$ \Gamma(z) = \int_0^{\mathrm{inf}} t^{z-1} e^{-t} dt \qquad (4)$$

For functions f (t) having n continuous derivatives for $t \geq 0$ where $n-1 \leq q < n$, the Grunwald-Letnikov and the Riemann-Liouville definitions are equivalent. The Laplace transforms of the Riemann-Liouville fractional integral and derivative are given as follows:

$$ L\{{}_aD_t^q f(t)\} = s^q F(s) - \sum_{k=0}^{n-1} s^k \, {}_0D_t^{q-k-1} f(0); \qquad n-1<q<n \qquad (5)$$

The Riemann-Liouville fractional derivative appears unsuitable to be treated by the Laplace transform technique because it requires the knowledge of the non-integer order derivatives of the function at t=0. This problem does not exist in the Caputo definition that is sometimes referred as smooth fractional derivative in literature. This definition of derivative is defined by

$$ {}_aD_t^q f(t) = \begin{cases} \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q+1-m}} d\tau ; & m-1<q<m \\ \frac{d^m}{dt^m} f(t) ; & q=m \end{cases} \qquad (6)$$

Where m is the first integer larger than q. It is found that the equations with Riemann-Liouville operators are equivalent to those with Caputo operators by homogeneous initial conditions assumption. The Laplace transform of the Caputo fractional derivative is

$$ L\left\{{}_nD_t^q f(t)\right\} = s^q F(s) - \sum_0^{n-1} s^{q-k-1} f^{(k)}(0); n-1<q<n \qquad (7)$$

Contrary to the Laplace transform of the Riemann-Liouville fractional derivative, only integer order derivatives of function f are appeared in the Laplace transform of the Caputo fractional derivative. For zero initial conditions, previous equation reduces to

$$ L\{{}_0D_t^q f(t)\} = s^q F(s) \qquad (8)$$

The numerical simulation of a fractional differential equation is not simple as that of an ordinary differential equation. Since fractional order differential equations do not have exact analytic solutions, approximations and numerical techniques are used. The approximation method, Oustaloup filter is given by

$$ s^q = k \prod_{n=1}^{N} \frac{1+\frac{s}{\omega_{zn}}}{1+\frac{s}{\omega_{pn}}} \quad ; \quad q>0 \qquad (9)$$

The approximation is valid in the frequency range $[\omega_l, \omega_h]$; gain k is adjusted so that the approximation shall have unit gain at 1 rad/sec; the number of poles and zeros N is chosen beforehand (low values resulting in simpler approximations but also causing the appearance of a ripple in both gain and phase behaviours); frequencies of poles and zeros are given by

$$ \alpha = \left(\frac{\omega_h}{\omega_l}\right)^{\frac{q}{N}} \qquad (10)$$

$$ \eta = \left(\frac{\omega_h}{\omega_l}\right)^{\frac{1-q}{N}} \qquad (11)$$

$$ \omega_{zn} = \omega_{p,n-1}\eta, \quad n=2,....,N$$

$$ \omega_{pn} = \omega_{z,n-1}\alpha, \quad n=1,....,N.$$

4. BACTERIAL FORAGING TECHNIQUE BASED FOPID TUNING

The feedback controller design is formulated as an optimization problem and the solution is sort through steps of BFA. The technique uses BFA to tune the FOPID parameters online for a minimum ISE for each region separately. Due to the variety of FOPID control law permutations, it is necessary to specify a minimum set of attributes that is FOPID controller is assumed to be of non-interacting form as defined below:

$$ G_c(s) = K_p + K_i \frac{1}{s^\lambda} + K_d s^\mu \qquad (12)$$

By suitable transformation of the parameters this form is converted to interacting form. Minimizing the following error criteria generates the controller parameter

$$ ISE = \int_0^T (r(t) - y(t))^2 \; dt \qquad (13)$$

Where: r(t) = reference input,

y(t) = measured variable

At first, the ant, i.e. the FOPID parameters are randomly initialized. The fitness function is defined as ISE. Smaller the fitness function, the better performance of the system response with the specified FOPID parameters.

The block diagram for online tuning of FOPID parameters using BF minimizing ISE is shown in the figure (See Figure3).

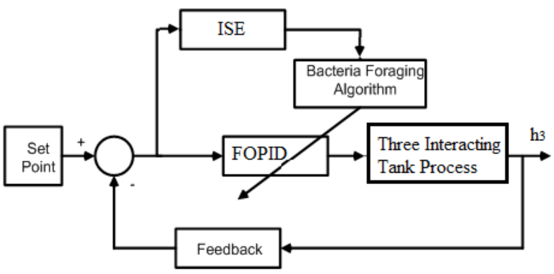

Fig 3: Block diagram of online PID tuned using BFA for CSTR

Algorithm used for the tuning of the PID controller parameters using BFA is given below:

1. To tune the PID parameters, the closed loop control scheme is constructed with the Kp, Ki, Kd, λ and μ considered as the variable.
2. The bounds of the controller parameters are fixed.
3. For evaluation of the BF tuned FOPID parameters, performance criterion ISE is selected.
4. The parameters needed by the BF algorithm S,D,Ns, N_c, N_{re}, N_{ed}, P_{ed}, , θ, C(i), $d_{attract}$, $w_{attract}$, $h_{repellent}$ and $w_{repellent}$. Where S: Number of bacteria to be used for searching the total region,

   D: Number of parameters to be optimized,

   N_s: Swimming length after which tumbling of bacteria will be undertaken in a chemotactic loop,

   N_c: is the number of iterations to be undertaken in a chemotactic loop,

   N_{re}: is the maximum number of reproductions to be undertaken,

   N_{ed}: is the maximum number of elimination and dispersal events to be imposed over the bacteria,

   P_{ed}: is the probability with which the elimination and dispersal will continue.

   θ: The location of each bacterium, which is specified by random numbers on [0,1],

   C(i): In our case, this is assumed to be constant for all the bacteria to simplify the design strategy.
5. Chemotaxis loop : For each bacterium, compute the cost function including the cell-to-cell attractant effect to the nutrient concentration. Compute the better cost via a run.
6. Perform the tumble operation by generating a random vector for each bacterium within [-1,1].
7. Move the bacterium i from its position θ with a step of size C(i) in the direction of the tumble.
8. Swim for a length N_s and determine the cost function.
9. Repeat the procedure for the next bacterium.
10. Repeat steps 5 to 9, till the chemotaxis loop N_c have reached.
11. Compute the J_{health}. Sort the bacteria in ascending order of cost J_{health} (higher cost means lower health).
12. Ignore the bacteria with te highest J_{health} and split the bacteria with the best value to maintain the number of bacteria constant.
13. Repeat step 5 to 12, till number of specified reproduction steps reach N_{re}.
14. For the elimination dispersal probability P_{ed}, eliminate the worst bacteria and disperse the split best bacteria to maintain the population constant to a random location on the optimization domain.

The following BF parameters are selected for the tuning of the FOPID

S = 20, D = 5, N_c = 30, N_s =4, N_{re} = 4, N_{ed}=2, P_{ed} = 0.25, $d_{attract}$t = 0.05, $w_{attract}$ = 0.2, $d_{repellent}$ = 0.05, $w_{repellent}$ = 8, Sr = S/2.

4.1 Comparison of BFA-IOPID Controller With BFA-FOPID Controller

The BFA algorithm was implemented by developing a dedicated software using MATLAB. The parameters of BFA are taken by trial and error to yield best solution.The heights h_1, h_2 and h_3 of the three interacting tanks for the set values fixed are 15cm, 35cm, 70cm, 61cm and 8cm at an interval of 5000s (see Figure 4a,4b). The response for the regulatory problem with set point fixed at 50cm and disturbance applied at an interval of 2500s using an integer and fractional order controller(see Figure 5a,5b).The response for the servo regulatory problem for the three interacting tank processs (see Figure 6a, 6b).

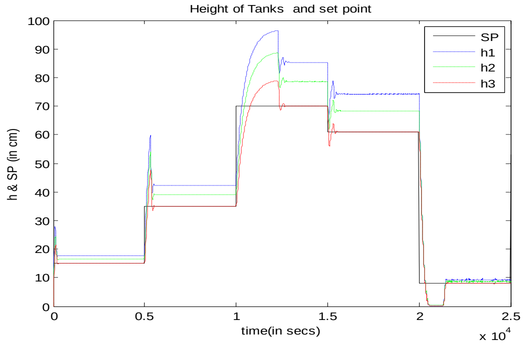

Fig 4.a: Integer order PID controller

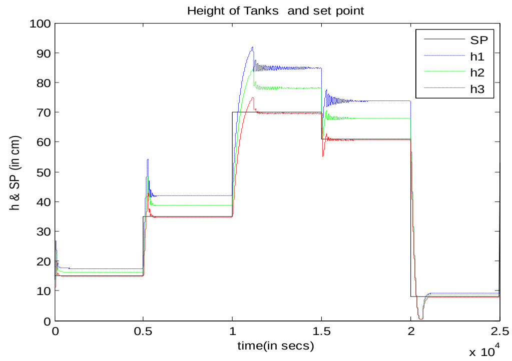

Fig 4.b: Fractional order PID controller

The height changes in all the three tanks for servo change with set point change of 15cm, 35cm,70cm, 61cm and 8 cm at an interval of 5000 secs (see Figure 4.a , 4.b).

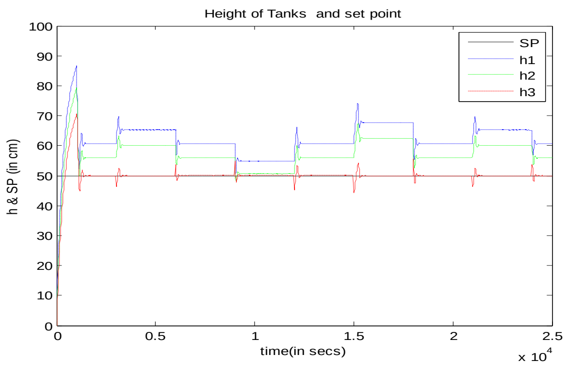

Fig 5.a: Integer order PID controller

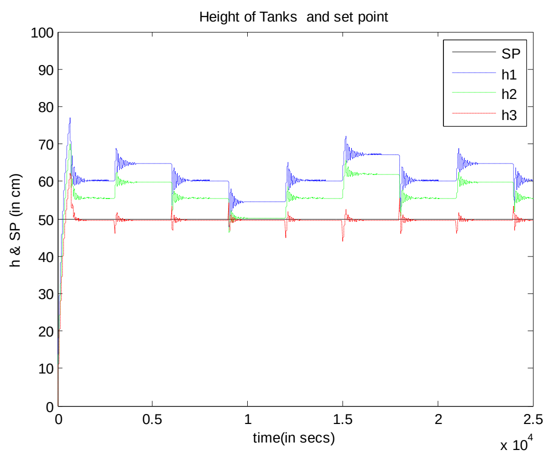

Fig 5.b: Fractional order PID controller

The height changes in all the three tanks for regulatory change with set point fixed at 50cm and disturbance applied at an interval of 2500 secs (see Figure 5a, 5b).

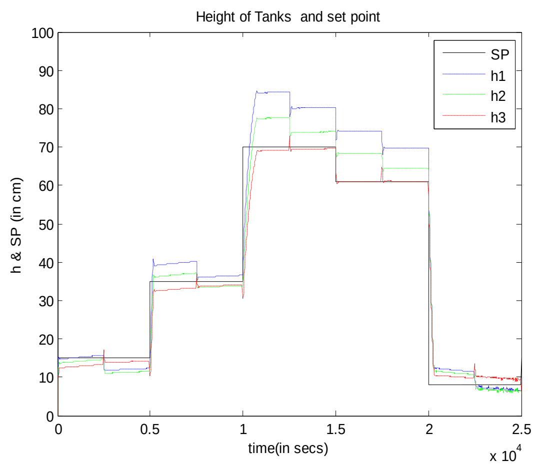

Fig 6.a: Integer order PID controller

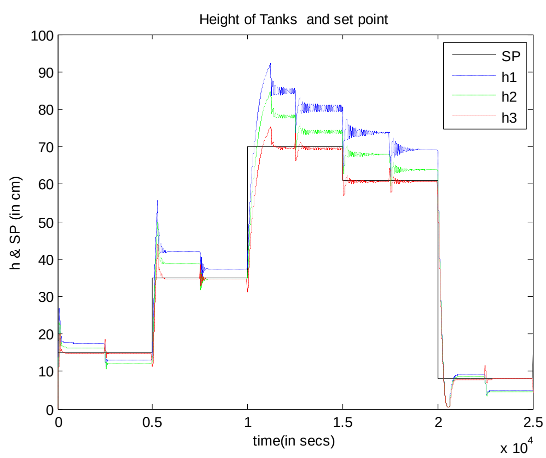

Fig 6.b: Fractional order PID controller

The height changes in all the three tanks for servo-regulatory change for set point change of 15cm, 35cm,70cm, 61cm and 8 cm at an interval of 5000 secs and disturbances applied at 2500s, 7500s,12500s, 17,500s and 22500s respectively (see Figure 6a, 6b).

On investigation of Table 3, it is evident that FOPID controller shows an optimal result compared to the IOPID controlled response.

Table 3. Performance indices for the IO-PID controller and FO-PID controller

Control	Performance Indices		
	IAE	ISE	ITAE
Integer order PID controller $2.7079+\frac{0.0938}{s}+0.075s$			
Servo	50380	71930	$6.943*10^8$
Regulatory	32460	43850	$1.742*10^8$
Servo-Regulatory	53890	74280	$7.085*10^8$
Fractional Order PID controller $9.1521+\frac{3.5294}{s^{0.493}}+4.2813s^{0.6864}$			
Servo	32970	55320	$4.433*10^8$
Regulatory	22210	28790	$1.497*10^8$
Servo-Regulatory	34410	57130	$4.447*10^8$

5. CONCLUSION

Three interacting tank process control is effected using Bacterial Foraging technique based Integer order and Fractional Order PID controller. The performance selected is Integral Square Error. The integer order PID controller involves optimization of three parameters K_p, K_i and K_d whereas the fractional order controller involves two more additional parameters λ and μ increasing the number of controller parameters as five. Applying Ant colony optimization, the integer order parameters are found to be Kp = 2.7079, Ki = 0.0938 and Kd = 0.075. The fractional order parameters are found to be Kp = 9.1521, Ki = 3.5294, Kd = 4.2813, λ= 0.493 and μ=0.6864. On investigation of the performance indices, it can be observed that the IAE, ISE and ITAE for servo and servo-regulatory problem for a fractional order controller are less compared to the integer order controller. On analysis, it can be inferred that the fractional order controller can be implemented for the control of the three interacting tank process.

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