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Closed Loop Control of Quadruple Tank Process using Fuzzy Logic PI Controller

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ABSTRACT
Quadruple tank process is a Multi-Input-Multi-Output process exhibiting both minimum phase and non-minimum phase behaviour. In this research, an attempt has been made to mathematically model and design a fuzzy controllerr for the non-minimum phase of Quadruple Tank Process. Both servo and regulatory responses are obtained for the proposed controller.

Keywords
Quadruple Tank, Jacobian Matrix, Mathematical Modeling, Fuzzy Logic Control.

1. INTRODUCTION
The PID controllers are the work horse of control field over the last decades. They are widely used in industrial applications due to its simplicity, clear functionality, applicability and ease of use. The PID controllers was introduced in 1920 and their use and popularity had grown particularly after the Ziegler- Nichols empirical tuning rules in 1942 (Ziegler-Nichols, 1942) . This control approach is an online and proven method however it requires experiences and very aggressive tuning for the process. Several approaches have been reported in the literature (Astrom and Hagglund, 2001) to tune PID parameters for SISO systems [1]. In this framework, Ziegler Nichols (Astrom and Hagglund, 2004) and Cohen-Coon (Cohen and Coon, 1953) are considered as the most commonly used methods [2]. However, most industrial processes are of multivariable nature. MIMO PID controller is much less understood and developed than single variable case. Recently, several books and surveys reported research works about tuning MIMO PID controllers, see e.g. (Luan et al., 2010), (Vilanova and Visioli, 2012), (Rames and Panda, 2012)[3][4][5]. MIMO PID controllers tuning approaches can be classified into empirical (Zhuang and Atherton, 1994), artificial intelligence (Willjuice and Baskar, 2009) and analytical approaches (Isakson and Graebe, 1999)[6][7]. Analytical approaches have particularly emerged to tune the PID parameters. The most popular techniques in this category are optimal methods (Yanchevsky, 1987), robust methods (Ho, 2003), placement pole methods (Zhang et al., 2002) and iterative methods (Lequin et al., 2003). On the other hand, Linear Matrix Inequalities (LMIs) are the most efficient tools in controller design in this framework.

A great deal of LMI based design methods have been proposed by (Geromel et al., 1994; Cao et al, 1999; Wang et al., 2007) where the Iterative Linear Matrix Inequality (ILMI) methods was proposed by Cao et al, 1998 and later used to solve several MIMO PID controller design problems (Soliman et al., 2010; Bevrani et al., 2011; Zheng et al., 2002; Lin et al, 2004; He and Wand, 2006)[8][9][10].

The fuzzy logic controller is highly flexible and intelligent controller and it has self tuning techniques feature and decision making logic to decide the control policy based on the error and change in error. Fuzzy logic controller developed and implemented by Mamdani and Assilian(1974) Takagi and sugeno(1985) introduced numerical optimization approach for selection of fuzzy parameters[11][12]. Passio Kevin M (1998) implemented fuzzy logic controller in all type of process[13]. But some cases the tuning of fuzzy parameter is difficult because control design does not require mathematical model of the process. Reznik leonid (2000) used empirical rules for the selection of fuzzy parameter [14]. The fuzzy logic controller performance is based on the input output scaling factor and membership function. The selection of input output scaling factor directly affect the stability of closed loop system. The advantage of conventional PID and fuzzy logic controller is combined to form hybrid controller to produce better control performance. The fuzzy logic controller enhances the performance of PID controller.

Section 2 describes about quadruple tank process. Section 3 elaborates fuzzy logic PI controller in detail. Conclusion is given in section 4.

2. QUADRUPLE TANK PROCESS DESCRIPTION
The schematic diagram for quadruple tank process is shown (see figure 1). The quadruple-tank process has four interconnected water tanks and two pumps. It is build by using two double-tank processes. The main aim is to control the level in the lower two tanks with two pumps. The process inputs are v_1 and v_2 i.e, input voltages to the driving circuit of the pumps and the outputs are h_1 and h_2 i.e, voltages from level measurement devices. Two way valve is used to manipulate the flow of liquid to tank 1 by proportional γ_1 and tank 3 by a factor (1- γ_1) through pump 1. Another two way valve is used to manipulate the flow of liquid to tank 2 by a ratio γ_2 and tank 4 by a factor (1- γ_2) through pump 2. In addition tank 1 gets input from tank 3 and tank2 from tank 4. Separate DPTs are provided in each tank to measure the height of the tanks[15].

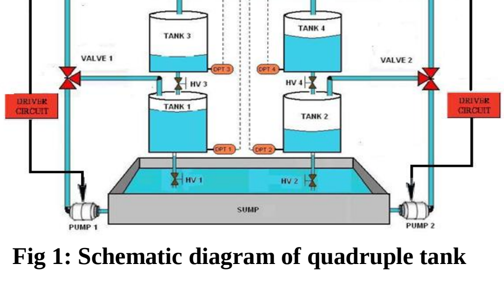

Fig 1: Schematic diagram of quadruple tank

Mathematical Modeling of the Quadruple Tank is done by formulating the mass and energy balance equations

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1}{A_1}k_1 v_1 \quad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2}{A_2}k_2 v_2 \quad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_1)}{A_3}k_2 v_2 \quad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_2)}{A_4}k_2 v_2 \quad (4)$$

where
A_i = cross sectional area of tank i;
a_i = cross-sectional area of the outlet hole i;
h_i = water height of tank i;
γ_i = flow splits;
g = gravitational constant;

The parameter values used for the process
A_1, A_3 = 28 cm^3
A_2, A_4 = 32 cm^3
a_1, a_3 = 0.071 cm^2
a_2, a_4 = 0.057 cm^2
β = 0.5 V/cm
g = 981cm/s^2

Table 1. Operating conditions for minimum phase and Non-minimum phase regions

Operating point 1-Minimum Phase	Operating point 2 - Non-minimum Phase
h_{1s}, h_{2s} = 12.4, 12.7cm	h_{1s}, h_{2s} = 12.6, 13.0 cm
h_{3s}, h_{4s} = 1.8, 1.4 cm	h_{3s}, h_{4s} = 4.8, 4.9 cm
u_{1s}, u_{2s} = 3.00, 3.00 cm	u_{1s}, u_{2s} = 3.15, 3.15 cm
k_1 = 3.33 cm^3/Vs,	k_1 = 3.14 cm^3/Vs,
k_2 = 3.35 cm^3/Vs	k_2 = 3.29 cm^3/Vs
γ_1, γ_2 = 0.7, 0.6	γ_1, γ_2 = 0.43, 0.34.

Using Jacobian matrix for linearisation

$$\dot{x} = \begin{bmatrix} \frac{-1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & \frac{-1}{T_2} & 0 & \frac{A_4}{A_3 T_4} \\ 0 & 0 & \frac{-1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 K_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 K_2}{A_2} \\ 0 & \frac{(1-\gamma_2)K_2}{A_3} \\ \frac{(1-\gamma_1)K_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} K_c & 0 & 0 & 0 \\ 0 & K_c & 0 & 0 \end{bmatrix} x$$

Table 2. State space model under minimum phase and non-minimum phase conditions

Minimum Phase
$A = \begin{bmatrix} -0.016 & 0 & 0.041 & 0 \\ 0 & -0.011 & 0 & 0.033 \\ 0 & 0 & -0.041 & 0 \\ 0 & 0 & 0 & -0.033 \end{bmatrix}$
$B = \begin{bmatrix} 0.083 & 0 \\ 0 & 0.062 \\ 0 & 0.047 \\ 0.031 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}$; $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Table 3. Transfer function model for minimum phase operating conditions

$\frac{h1}{u1}$	$\frac{0.0415s^3 + 0.00352s^2 + 8.99e^{-5}s + 6.176e^{-7}}{s^4 + 0.101s^3 + 0.00352s^2 + 4.956e^{-5}s + 2.38e^{-7}}$
$\frac{h_2}{u_1}$	$\frac{0.00051s^2 + 2.916e^{-5}s + 3.355e^{-7}}{s^4 + 0.101s^3 + 0.00352s^2 + 4.956e^{-5}s + 2.38e^{-7}}$
$\frac{h_1}{u_2}$	$\frac{0.02446s^2 + 0.00107s + 8.88e^{-6}}{s^4 + 0.101s^3 + 0.00352s^2 + 4.956e^{-5}s + 2.38e^{-7}}$
$\frac{h_2}{u_2}$	$\frac{0.031s^3 + 0.0028s^2 + 7.856e^{-5}s + 6.71e^{-7}}{s^4 + 0.101s^3 + 0.00352s^2 + 4.956e^{-5}s + 2.38e^{-7}}$

3. FUZZY LOGIC PI CONTROLLER

A common feature of the conventional control is that the control algorithm is analytically described by difference and differential algebraic equations. In general, the synthesis of such control algorithm requires a formalized analytical description of the controlled system by a mathematical model. The analytical concept is one of the main paradigms of conventional control theory. The main paradigm of fuzzy logic control is that the control algorithm is a knowledge based algorithm, described by the methods of fuzzy logic. The fuzzy logic control system is a kind of expert knowledge based system that contains the encoded rule base derived from human experience and intuition and from theoretical and practical understanding of the dynamics of controlled object.

By fuzzy controller, the interpretation is that it is described by knowledge based system consisting of IF..Then rules with vague predictions. In general, FLC is represented similar to the conventional control law as given by:

u(k) = f(e(k), e(k-1), .., e(k-v), ce(k), ce(k-1), ce(k-v), ie(k), ie(k-1),ie(k-2)...,ie(k-v))

where the function f, the control law is described by a rule base of the type mentioned. Each of the rules of the FLC is characterized by an IF part called the antecedent and with a Then part called the consequent. If the conditions of the antecedent are satisfied, then the conclusions of the consequent apply. FLC can be looked as a system that has as its input the variables that are included in the antecedents of the rules and as the output, the variable that is included in the consequents.

3.1 Steps Involved in Fuzzy Logic Control

- Determine the input and output variables of the FLC. By appropriate selection of the input and output variables, mamdani type FLC-PI is selected
- Specify the range of controlled variable and manipulated variables.

- Divide these ranges into fuzzy membership functions and attach linguistic labels which can be used to describe them using the knowledge of the expert
- Determine the rules (rule base) which relate the manipulated variable and controlled variable to specify control action
- Perform defuzzification using centroid method

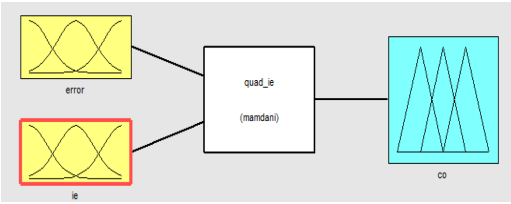

Fig 2: Fuzzy Logic PI controller designed for quadruple tank process

3.2 Fuzzification Stage

This stage converts a crisp number into a fuzzy set within an universe of discourse. Triangular membership functions with five linguistic variables for error and integral error are used as shown in figure. The linguistic variables used for error is NL (Negative Large), NS(Negative small), Zero, PS(Positive small) and PL (Positive large). The linguistic variables used for integral error is VS (Very small), S (Small), M (Medium), L(Large) and VL (Very large). The linguistic variables used for controller output is VS (Very small), S (Small), M (Medium), L(Large) and VL (Very large). The surface view and the rule viewer of the rule base is shown in figure

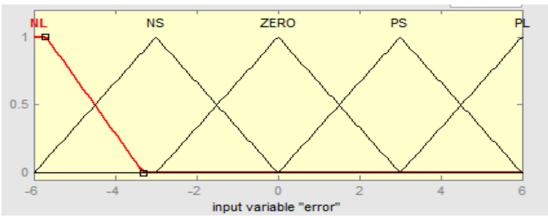

Fig.3 (a) : Input Error membership function

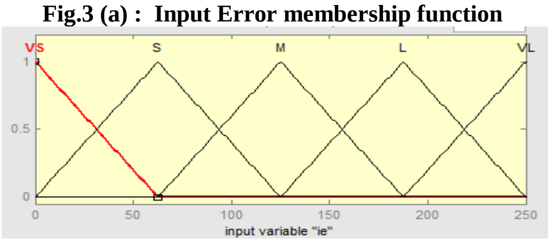

Fig.3 (b): Input Integral Error membership function

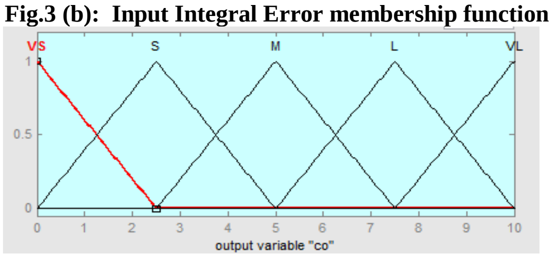

Fig.3(c) Output : Controller output membership function

Fig 3: Membership function of the input error, integral error and controller output

3.3 Fuzzy Inference Engine

This stage is the core of the fuzzy control and is constructed from the expert knowledge and experience. Based on the knowledge gained by analysing the feedback control system, decision making logic is constructed. Twenty five rules are obtained given in table 4. E refers to Error, IE refers to Integral Error and CO denotes Controller Output. AND operator is used in the rule base.

Table 4. Rule based for the Fuzzy PI controller

IE \ E (CO)	NL	NS	ZERO	PS	PL
VS	VS	VS	VS	S	M
S	VS	VS	S	M	L
M	VS	S	M	L	VL
L	S	M	L	VL	VL
VL	M	L	VL	VL	VL

.

3.4 Defuzzification Stage

In this stage the fuzzy set value is converted into a crisp value. Commonly used defuzzification technique is the centre of area method (centroid method). The calculation of the centroid defuzzified value is simplified by considering a finite universe of discourse y and a discrete membership function f(y).

$$y^* = \frac{\sum f(y_i) y_i}{\sum f(y_i)}$$

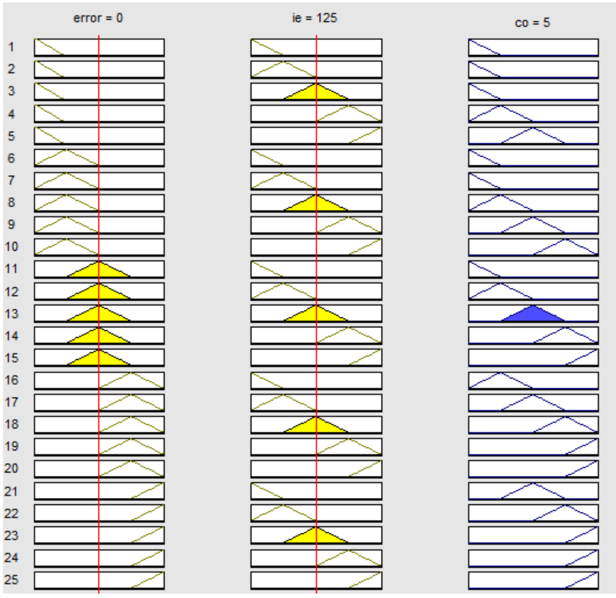

Fig. 4 (a) : Rule viewer

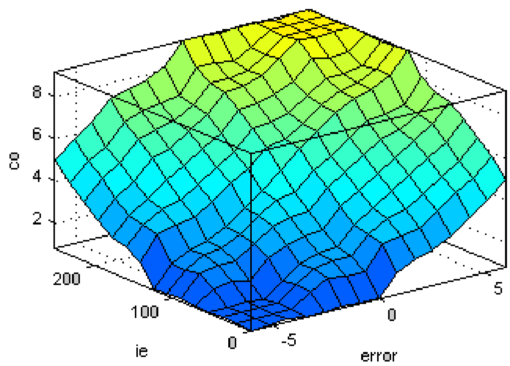

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Fig. 4 (b) : Surface viewer for the rule base

Fig 4: Surface viewer and rule viewer for fuzzy PI controller framed for quadruple tank process

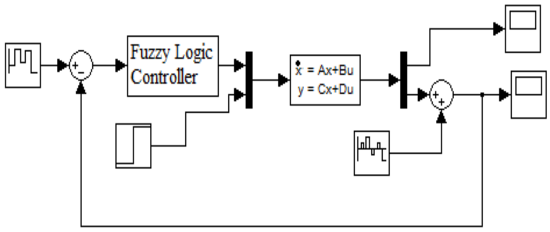

Fig 5: Block diagram of the Closed Loop Fuzzy Logic controller for Minimum phase quadruple tank process

The servo response of the proposed fuzzy controller for the Quadruple tank process is shown (see Figure 6) where a step change of 10cm, 15cm, 11cm, 14cm and 9cm are applied at an interval of 1000 secs each. The regulatory response for a disturbance applied in the range of -5% to +5% of the sensor output at an interval of 500secs each is shown (see Figure7). The servo regulatory response of the proposed controller scheme for various set points and disturbances applied is shown (see Figure 8).

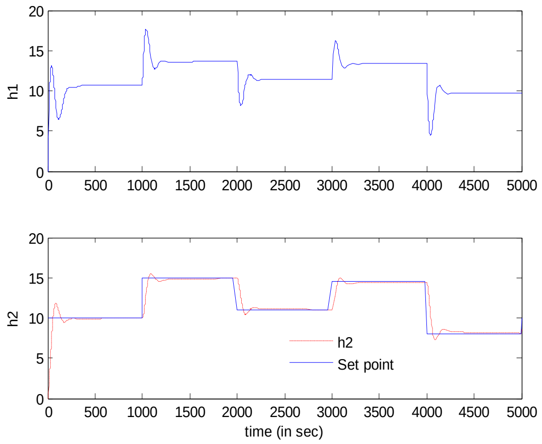

Fig 6: Servo response for fuzzy PI controlled Quadruple tank process

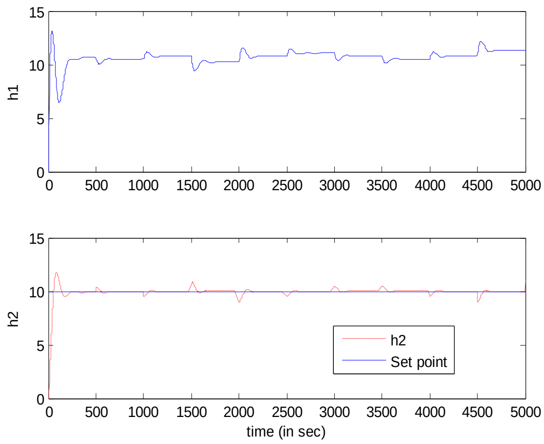

Fig 7: Regulatory response for fuzzy PI controlled Quadruple tank process

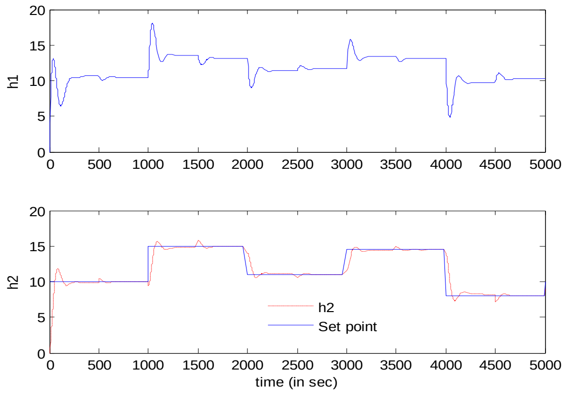

Fig 8: Servo-regulatory response for fuzzy PI controlled Quadruple tank process

4. CONCLUSION

The work proposed involves design of fuzzy logic controller design and implementation for minimum phase quadruple tank process. It provides a high level of human understanding. Perfect control of this process is highly difficult in order to achieve the outcome. The conventional PID controller shows sluggish response and takes larger time to control the multivariable nonlinear process. Due to these inherent drawbacks in the PID controller, the FLC is implemented. The proposed controller scheme is tested under servo, regulatory and servo-regulatory conditions.

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