

An Alternative Arithmetic Approach to the Water Jugs Problem

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ABSTRACT

The Water Jug problem is a famous problem in Artificial Intelligence, Problem solving, Recreational, Computer Programming and Psychology. The solution of the problem mainly based on heuristic approach or some search methods such as Breadth First Search (BFS) or Depth First Search (DFS) or Diophantine approach. In BFS, we will be certainly reaching the goal but time taken to reach the goal will be too much. In the same manner in DFS also time taken to reach the goal is not predictable. In contrast to this, in the Diophantine approach ($m x + n y = d$) normally we will be assuming the value of x and y , so it takes time to find the solution. In this paper, a new arithmetic approach, Extended Euclidean approach which is used to solve the problem, it is simple and suitable for manual calculation or programming language implementation. Analysis of the solution involves various steps and some illustrative examples are provided.

General Terms

General terms are Arithmetic approach, BFS Algorithms, DFS Algorithms, Diophantine approach.

Keywords

Water jugs problem, Artificial Intelligence, Problem solving, Diophantine approach, Extended Euclidean approach.

1. INTRODUCTION

The Artificial Intelligence is the study of how to make computers to do things better than the human being. As the term says, here the water jug problem is well-known problem in Artificial Intelligence [1], Computer Programming [2], Problem solving [3], Geometry [4], Recreational and discrete Mathematics [5,6] and Psychology [7,8,9].

You are at the side of a river. You have a 3 liter jug and a 5 liter jug. The jugs do not have marking to allow measuring smaller quantities. How can you use the jugs to measure 4 liters of water?

There are various methods to solve this problem, including Breadth First search [10], Depth First Search [11] and the Diophantine approach [12].

However each and every method has its own disadvantages that is in BFS the role is not get trapped by exploring a blind alley. In DFS, by chance we can assure that we will be getting the goal earlier and the memory space is less. In Diophantine approach the goal totally depends on the assumption that are made over the value of X and Y .

In this paper a simple Arithmetic approach to solve the problem that was introduced. A novel feature of this approach

is that one can deduce the total amount of water in jugs at each step by getting the value of X and Y by using the backward approach. When these values are just substituted in the Extended Euclidean equation our goal will be reached. Due to its simplicity it is very suitable for manual calculation of the proving steps.

The whole paper is organized as follows. In section 2 the Arithmetic approach for solving the general two water jugs problem and it describe the mathematical background behind. In the third section, illustration of how to use the new approach with some examples is explained. Then in the final section some concluding remarks are explained.

2. AN ARITHMETIC APPROACH TO THE WATER JUG PROBLEM

A simple arithmetic approach to the general two water jugs problem was introduced which is applied to solve the problem below:

Let m, n be positive integers. You are at the side of a river. You have a m -liter jug and a n -liter jug, where $0 < m < n$. The jugs do not have markings to allow measuring smaller quantities. How can you use the jugs to measure $d (< n)$ liters of water?

The Extended Euclidean algorithm is usually used simply to find the greatest common divisor of two integers. The standard Euclidean algorithm gives the greatest common divisor and nothing else.

However, if we keep track of a bit more information as we go through the algorithm, we can discover how to write the greatest common divisor as an *integer linear combination* of the two original numbers. In other words, we can find integers s and t such that

$$\gcd(a, b) = sa + tb.$$

[Note that, since $\gcd(a, b)$ is usually less than both a and b , one of s or t will usually be negative.]

From this equation we will be obtained only the GCD (m, n) but after getting it we have to follow a new procedure Back Substitution Method. The conclusion of this will be $m(x') + n(y')$ where x' = number of moves to be carried out by x and y' = number of moves to be carried out by y .

As a reminder, here are the steps of the standard Euclidean algorithm to find the greatest common divisor of two positive integers a and b :

Algorithm 2.1

Input: The integers m, n and d , where $0 < m < n$ and $d < n$.

Output: An integer sequence corresponding to a feasible solution obtained by filling the m -litre jug and n -litre jug.

Procedure:

1. Set the value of the variable c to the larger of the two values a and b , and set d to the smaller of a and b .
2. Find the quotient and the remainder when c is divided by d . Call the quotient q and the remainder r . Use the division algorithm and expressions for previous remainders to write an expression for r in terms of a and b .
3. If $r = 0$, then $\text{gcd}(a, b) = d$. The expression for the previous value of r gives an expression for $\text{gcd}(a, b)$ in terms of a and b . Stop.
4. Otherwise, use the current values of d and r as the new values of c and d , respectively, and go back to step 2.

In this algorithm, we will be getting the GCD (m, n). Using the algorithm 2.1 we obtain the integer sequence.

Finding out the GCD is the first process and after that we have to use the Back substitution method.

Algorithm 2.2:

1. In each of the equations, subtract the product term from the number on the left-hand-side and set it equal remainder on the right-hand-side.
2. Backwards: Write down the last flipped equation, the one with the gcd in it, and then alternate it by subbing the smallest underlined term and collecting terms the remaining terms.

From this algorithm, the number of subtractions and additions involved will provide a solution to the problem. By using this Extended Euclidean Approach and the Backward Substitution method actual pouring sequence can be determined easily by looking at the numbers appeared in the integer sequence obtained.

3. EXAMPLES

We now illustrate how to use the arithmetic approach to solve the two water jugs problem below.

Example 3.1. There are a 5-litre jug and a 3-litre jug. We want to use them to measure 1 liters of water, as described in the introduction before. Using the above notations, we have $m=5, n=3, d=1$ and the associated Extended Euclidean approach is $5x+3y=1$. Applying Algorithm 2.1, we can obtain an integer sequence.

5	=	1	*	3	+	2
3	=	1	*	2	+	1
2	=	1	*	1	+	1
1	=	1	*	1		

Fig 1: finding the GCD of m and n

Here as the first step we are finding out the GCD of the $m=5$ and $n=3$ as 1 [see Fig 1]. Next by using the Algorithm 2.2 we are able to perform the Extended Euclidean approach.

1	=	2	-	(1	*	(3	-	2))
	=	2	-	(3	-	2)				
	=	2	*	2	-	3						
	=	2	*	(5	-	3)	-	3		
	=	2	*	5	-	3	*	3				

Fig 2: finding out the values of x and y

So by using this approach we can easily find out the value for both x and y [see Fig 2]. The number of additions and

subtractions involved are 2 and 3 respectively, so $x = 2, y = -3$ is a solution to the equation $5x+3y=1$.

0	→	5	→	2	→	0	→	5	→	4	→	1
				+5		-3		+5		-3		-3

Fig 3: the follow of values

Since the integers inside the boxes in the sequence refer to the total amount of water in the jugs at different stages, we can work out the corresponding pouring steps very easily [see Fig

3]. If we use a coordinate (x, y) to represent the amounts of water inside the 5-litre jug and the 3-litre jug in each pouring step [see Fig 4], then the successive pouring steps are as

$$(0,0) \rightarrow (5,0) \rightarrow (2,3) \rightarrow (2,0) \rightarrow (0,2) \rightarrow (5,2) \rightarrow (4,3) \rightarrow (4,0) \rightarrow (1,3) \rightarrow (0,1)$$

Fig 4: successive pouring steps

Thus with the help of the Extended Euclidean approach without any confusion we are able to find out more optimal solution to this particular water jug problem.

$m=3, n=7, d=5$ and the associated Extended Euclidean approach is $3x+7y=5$.

Applying Algorithm 2.1, we can obtain an integer sequence,

Example 3.2. There are a 3-litre jug and a 7-litre jug. We want to use them to measure 5 liters of water, as described in the introduction before. Using the above notations, we have

$$\begin{array}{rclclcl} 7 & = & 2 & * & 3 & + & 1 \\ 6 & = & 1 & * & 5 & + & 1 \\ 5 & = & 1 & * & 5 & & \end{array}$$

Fig 5: finding the GCD of m and n

Here as the first step we are finding out the GCD of the $m=3$ and $n=7$ as 1[see Fig 5].

Next by using the Algorithm 2.2 we are able to perform the Extended Euclidean approach.

$$\begin{array}{rclclcl} 5 & = & 6 & - & 1 & (& 7 & - & 6 &) \\ 3 & = & 6 & - & 7 & + & 6 & & & \\ 2 & = & 12 & - & 7 & & & & & \\ 5 & = & 3 & * & 4 & - & 1 & * & 7 & \end{array}$$

Fig 6: finding out the values of x and y

So by using this approach we can easily find out the value for both x and y . The number of additions and subtractions

involved are 4 and 1 respectively [see Fig 6], so $x = 4, y = -1$ is a solution to the equation $3x+7y=5$.

$$\begin{array}{cccccc} 0 & \rightarrow & 3 & \rightarrow & 6 & \rightarrow & 9 & \rightarrow & 2 & \rightarrow & 5 \\ & & +3 & & +3 & & +3 & & -7 & & +3 \end{array}$$

Fig 7: the follow of values

Since the integers inside the boxes in the sequence refer to the total amount of water in the jugs at different stages, we can work out the corresponding pouring steps very easily. If we

use a coordinate (x, y) [see Fig 7] to represent the amounts of water inside the 3-litre jug and the 7-litre jug in each pouring step [see Fig 8], then the successive pouring steps are as

$(0,0) \rightarrow (3,0) \rightarrow (0,3) \rightarrow (3,3) \rightarrow (0,6) \rightarrow (3,6) \rightarrow (2,7) \rightarrow (0,2) \rightarrow (3,2) \rightarrow (0,5)$

Fig 8: successive pouring steps

Thus with the help of the Extended Euclidean approach without any confusion we are able to find out more optimal solution to this particular water jug problem.

4. CONCLUSION

In Breadth First Search all nodes has to be evaluated and instead of finding a single node we will be forced to find many answers and it is waste of time. It is its main disadvantages.

In Depth First Search, this method single and unfruitful path is being carried out for a very long time and longer path never explored until the entire shorter one has been examined.

So we choose Diophantine equation here the values for both x and y is just assumption so that we have to work from the first and by chance if sometimes the equation satisfies means then we will consider it. But only by chance we will work on it.

The new approach the Extended Euclidean Approach that is being founded in this paper has many and more number of advantages. The first and the foremost one is we don't want to do any guess, by having the input itself we are able to get the output. By using the Extended Euclidean Approach we are able to get the appropriate result with less work, complexity and memory. We can assure that the answer what is found out is efficient and effective.

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