Maximum Sustainable Yield Policy in Prey-Predator System-A Study

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Abstract:

In this paper, maximum sustainable yield policy in prey-predator system is discussed where the prey population follows logistic law of growth. Here a model is proposed involving linear prey–predator interaction and intra-specific competition among predator populations. The growth rate of the predator depends upon predation on the modelled and alternate prey. In traditional prey–predator system, fishing under combined harvesting effort at MSY level may be a sustainable policy, but if MSY does not exist then it is due to the extermination of the predator species only. In MSY level harvesting of one species is sustainable policy. Therefore it can be summed up that MSY (or MSTY) policies in prey predator models are unlikely to fit requirements of biological diversity conservation in all cases.

Keywords:

Maximum sustainable yield, Prey, Predator, Harvesting effort, Intraspecific compition.

1. Introduction:

Maximum sustainable yield (MSY) policy is a simple way to manage resources considering that over-exploited resources lead to reduction in productivity. But, it has been observed that the traditional MSY approach to fishery management depends on single species model of population dynamics and overlooks species interactions with other member of the communities [1]. At the beginning Schaefer (1954) introduced MSY policy for a single species fishery having logistic law of growth and subject to proportional harvesting [11]. Clark (1990) also discussed the importance of the concept of MSY policy for fishery management [2]. Recently, Kar and Matsuda (2007) investigated the MSY policy for single species fishery with strong Allee effect [12, 13]. Legovic (2008) mentioned the effects of harvesting of MSY policy in a single species fishery. In this paper, the impacts of MSY policy have been studied on two different prey–predator systems under different harvesting scenarios. **Section1** Provide understanding about MSY policy and revision of the outcomes due to Legovic et al. (2010a) [7]. In Section2,an intra-specific competition term have been introduced to the predator equation and the impacts of MSY policy have been studied when either prey or predator species is harvested or MSTY policy when the combined harvesting effort of both prey and predator species are taken into account [9]. Therefore, it is not at all obvious whether higher or lower yields should be expected from an entire ecosystem that would be predicted from the application of single species yield policy. The increasing study of realistic and practically useful mathematical model in population biology, whether dealing with a fishing population with or without its age distribution, population of an endangered species, bacterial or viral growth and so on, is a reflection of their use in helping to understand the dynamic process involved and in making practical prediction.

2. Outcome of MSTY policy's implimentation on normal prey and predator model:

Here, Study of the effect of MSY policy in multispecies system is done, following prey–predator system considered by Legovic et al. (2010a) [7,8]:

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - pxy$$
$$\frac{dy}{dt} = pxy - dy \qquad (1)$$

where x and y are respectively the prey and predator biomass at any time t. r is a constant intrinsic growth rate (biotic potential) and k is the environmental carrying capacity of prey population, p is the predation rate, d is the natural death rate of the predator.

Legovic. et al. (2010a) proposed the following results for proportional harvesting to either prey or predator or both the populations with the equal fishing effort as [5,6]:

In any prey–predator model, fishing the prey population at the MSTY level will require the predator population to be exterminated.

In any prey–predator model, fishing the predator population at MSTY level is not likely to cause extermination of neither prey nor predator species.

In any prey–predator model, if similar fishing attempt of prey and predator population is made, the final MSTY will be, MSTY of a single species population consisting of prey only resulting in complete extermination of predator population.

3. Outcome of MSTY policy implimentation on intra-specific competition:

Legovic et al. (2010a) provide inputs on effects of MSTY policy in prey–predator model; they have considered predation as the only interaction in their prey–predator model. Intra-specific competition is the most important aspect in the predator growth dynamics [4, 10]. It is assumed to induce additional instantaneous deaths to the predator population and the increased death rate is proportional to the square of the predator density. It is biologically unrealistic to fully eliminate density-dependent mortality [1]. Intra-specific competition enhances the death rate of the species, it is expected that the MSTY policy will be considerably differ from the MSTY policy in a simple prey–predator model. Hence, MSTY policies in a prey–predator model have been studied having intra-specific competition in the predator growth dynamics when either prey or predator or both the prey and predator populations are subject to harvest [2].

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - pxy$$
(2)
$$\frac{dy}{dt} = pxy - dy - \mu y^2$$

Where μ is the coefficient of intras-pecific competition.

The Coexistence equilibrium of the equation (2) is $P(x^*, y^*)$, where

$$x^* = k\left(\frac{pd + \mu r}{p^2k + \mu r}\right)$$

And

$$y^* = \frac{r(pk - d)}{p^2k + \mu r}$$

It is now examined whether MSY or MSTY policy can be implemented in the proposed prey–predator system for a possible sustainable fishing activity [9]. Various scenarios associated with MSY and MSTY are observed successively.

3.1. Prey harvesting:

When the target species is the prey only then after introducing the proportional harvesting of prey species [12] then the system (2) becomes

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - pxy - ex$$
(3)
$$\frac{dy}{dt} = pxy - dy - \mu y^2$$

Here e is the harvesting effort.

The Coexistence equilibrium of the equation (3) is $P({x^*}_1, {y^*}_1)$, where

$${x^*}_1 = k\left(\frac{pd + \mu r - e\mu}{p^2k + \mu r}\right)$$

And

$${y^*}_1 = \frac{r(pk - d) - pke}{p^2k + \mu r}$$

The yield in the equilibrium is,

$$Y(e) = e{x^*}_1$$

$$= ek\left(\frac{pd + \mu r - e\mu}{p^2k + \mu r}\right)$$

Here the MSTY is at $e = \frac{pd+\mu r}{2\mu}$

3.2. Predator harvesting:

Now, it is considered that the target species is the predator species only [9]. Then introducing a proportional harvesting, system (2) becomes

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - pxy$$
(4)
$$\frac{dy}{dt} = pxy - dy - \mu y^2 - ey$$

The Coexistence equilibrium of the equation (4) is $P({x^*}_2, {y^*}_2)$, where

$${x^*}_2 = k\left(\frac{pd + \mu r + e\mu}{p^2k + \mu r}\right)$$

And

$${y^*}_2 = \frac{r(pk - d - e)}{p^2k + \mu r}$$

The yield in the equilibrium is,

$$Y(e) = e\,{y^*}_2 = e\frac{r(pk - d - e)}{p^2k + \mu r}$$

Here the MSTY is at $e = \frac{pk-d}{2}$

3.3. Combined harvesting of both Prey & predator:

Now, if both the species is considered then the species are subjected to proportional harvesting with equal effort [2]. So the system (2) becomes

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - pxy - ex$$
(5)
$$\frac{dy}{dt} = pxy - dy - \mu y^2 - ey$$

The Coexistence equilibrium of the equation (5) is $P({x^*}_3, {y^*}_3)$, where

$${x^*}_3 = k\left(\frac{(pd + \mu r) - (\mu - p)e}{p^2k + \mu r}\right)$$

And

$${y^*}_3 = \frac{r(pk - d) - (pk + r)e}{p^2k + \mu r}$$

The yield in the equilibrium is,

$$Y(e) = e{x^*}_1, + e{y^*}_1$$

$$= e\{k\left(\frac{(pd+\mu r)-(\mu-p)e}{p^2k+\mu r}\right) + \frac{r(pk-d)-(pk+r)e}{p^2k+\mu r}\}$$

Here the MSTY is at $e = \left(\frac{k(pd+\mu r)+r(pk-d)}{2(\mu k+r)}\right)$

4. Results and discussions:

In equation (3), If the ecological parameters are fixed as r = 2.3, k = 50, p= 0.55, d= 0.75, μ= 0.8 and e is varied from 0 to 2, in suitable units. For these the Fig. 1 is plot. Now from Fig. 1, it is observed that prey biomass and predator biomass decreases when effort increases from zero and ultimately predator goes to extermination at e = 2 and at that moment the density of prey is 1.8. The yield curve is increasing for any effort lying in (0, 2). The maximum sustainable yield is 4.5 approximately at e=1.4.

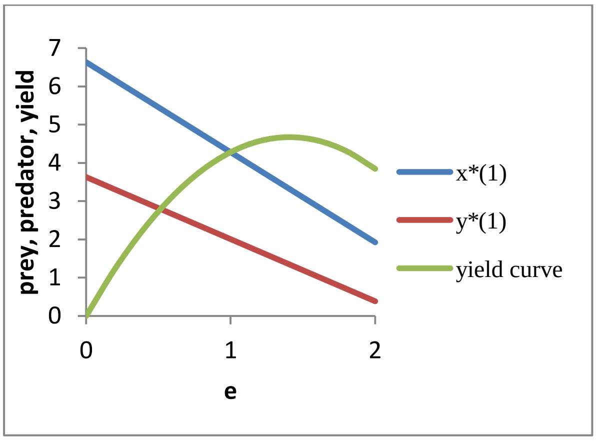

Fig1: Equilibrium values of prey and predator biomass and yield as a function of common harvesting effort e. MSY can be obtained.

In equation (4), the ecological parameters is now assumed to be fixed as r = 5, k = 10.5, p= 0.25, d= 0.8,μ= 0.13 and e is varied from 0 to 2, in suitable units. For these values the Fig. 2 is plotted. Now from Fig. 2, it is observed that prey biomass increases and predator biomass decreases when effort increases from zero and ultimately predator goes to extermination at e =1.8 and at that moment the density of prey is maximum. Here the maximum sustainable yield is 3 approximately at e=0.9.

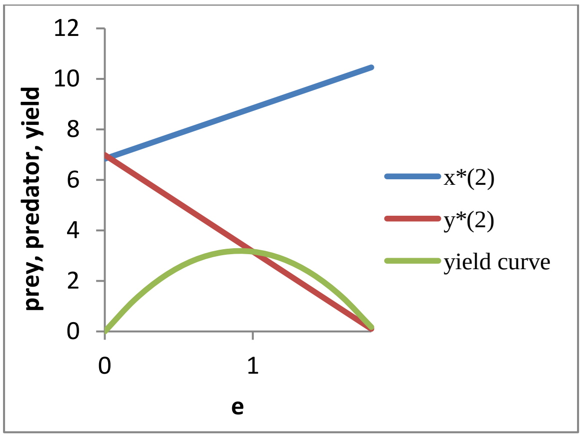

Fig 2: Equilibrium values of prey and predator biomass and yield as a function of common harvesting effort e. MSTY can be obtained.

In equation (5), the ecological parameters is now assumed to be fixed as r = 7, k = 44, p= 0.45, d= 0.76, μ= 0.53 and e is varied from 0 to 4.5, in suitable units. For these the Fig. 3 is plotted. Now from Fig. 3, it is observed that prey biomass and predator biomass decreases when effort increases from zero and ultimately predator goes to extermination at e = 4.5 and at that moment the density of prey is 15. The yield curve is increasing for any effort lying in (0, 4.5). The maximum sustainable yield is 64 at e=4.5.

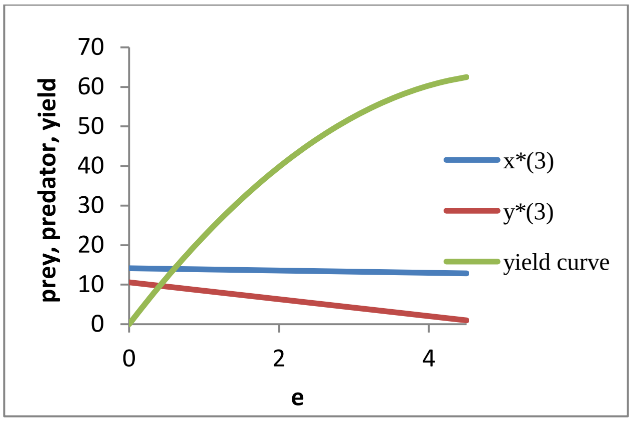

Fig 3: Equilibrium values of prey and predator biomass and yield as a function of common harvesting effort e. MSY can be obtained.

5. Conclusion :

1. In this paper it is formulated that a basic mathematical model which is based on the existence of intra-specific competition [3].
2. The objective of this study is to analyze the impacts of fishing with the aims of both yield maximization and species conservation. According to Legovic et al. (2010) [7]. It is known "In any prey–predator system, subject to equal fishing effort on both the prey and predator populations, the ultimate MSY will be the MSY of a single isolated population composed of prey only, which means that the predator population has gone to extermination".
3. Some potential consequences of MSY or MSTY policy have been drawn for the model [9]. Harvesting of both the species, with increasing effort, reduces the predator biomass and increases the prey biomass at equilibrium, which is termed as Volterra's principle, can be found in Lotka-Volterra model (Legovic, 2008).
4. Here it is also obtained maximum sustainable total yield from both the prey and predator populations under combined harvesting effort.

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