## Dynamic Hybrid Harmony Search Algorithm for Layup Sequence Design of Laminate Composites

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## ABSTRACT

Recently, a popular metaheuristic algorithm called Harmony Search, conceptualized using the musical process of searching for a perfect state of harmony is undergoing faster developments. In this paper, a new variant of harmony search called Dynamic Hybrid Harmony Search algorithm (DHHS) is proposed. The novelty introduced in this algorithm is the division of the whole population into few sets of smaller populations/ groups within which the harmony improvisation occurs, before they are regrouped for a new better search. The intensification mechanism is improved by adding a customized neighborhood search algorithm. This proposed algorithm is applied to a combinatorial problem namely lay-up sequence optimisation of laminate composite stiffened cylinder. The results of the numerical studies prove that the proposed algorithm is efficient in handling the design of layup sequence problem.

## **Keywords:**

Metaheuristic algorithms, harmony search, combinatorial optimization, neighborhood search, stiffened cylinder, laminate composites.

## **1. INTRODUCTION**

In general, most of the scientific and engineering problems have been solved by using calculas. However, for optimization problems, the differential calculus technique sometimes has a drawback when the objective function is step-wise, discontinuous, or multi-modal, or when decision variables are discrete rather than continuous. Thus, researchers have recently turned their interests into metaheuristic algorithms that have been inspired by natural phenomena such as evolution, animal behavior, or metallic annealing.

One of the most practical approaches for solving combinatorial optimization problems is the metaheuristic algorithms. Metaheuristics are therefore developed specifically to find a solution that is "good enough" in a computing time that is "small enough". Metaheuristics have been demonstrated by the scientific community to be a viable, and often superior, alternative to more traditional (exact) methods of mixed-integer optimization such as branch and bound and dynamic programming. Especially for complicated problems or large problem instances, metaheuristics are often able to offer a better trade-off between solution quality and computing time. Modern nature-inspired metaheuristic algorithms including ant and bee algorithms, bat algorithm, cuckoo search, differential evolution, firefly algorithm, genetic algorithms, harmony search, particle swarm optimization, simulated annealing and support vector machines have started to demonstrate their power in dealing with tough optimization problems and even NPhard problems. Among them, the harmony search algorithm [1], which has derived the inspiration from the process of A. Rama Mohan Rao CSIR-Structural Engineering Research Centre, Council of Scientific and Industrial Research, Taramani, Chennai, India. - 600 113.

improvisation of music composed by a set of musicians, has seen the faster development in the recent years [2].

This paper especially focuses on a music-inspired metaheuristic algorithm, Harmony Search (HS). Interestingly, there exists an analogy between music and optimization: When a group of musician gather to compose something new, they may start it with a random note, each. Then they evaluate the harmony of their played notes and if it is not well enough they may give it up. In the next step, they may play some note from their memory or something else regarding to their available choices. Aesthetic sense of musicians rules all over this period as the fitness function and so they optimize the harmony of their composition. From the optimization point of view, every musician could be considered as the decision parameter of algorithm, and whatever they play could be model of the value these parameters may retain. Finally, as mentioned before, the aesthetic sense of musicians is the model of fitness function in an optimization problem [5]. Just like musicians in Jazz improvisation play notes randomly or based on experiences in order to find fantastic harmony, variables in the harmony search algorithm have random values or previously-memorized good values in order to find optimal solution.

Harmony search algorithm is fast growing with many applications. The HS algorithm is simple in concept, few in parameters, and easy for implementation. One of the key success factors of the algorithm is the employment of a novel stochastic derivative, which can be used even for discrete variables. Instead of traditional calculus-based gradient, the algorithm utilizes musician's experience as a derivative in searching for an optimal solution. This can be a new paradigm and main reason in the successes of various applications such as robotics (robot terrain and manipulator trajectory); visual tracking; web text data mining; power flow planning; fuzzy control system; hybridization (with Taguchi method or SQP method); groundwater management; irrigation ; logistics; time tabling ;bioinformatics (RNA structure prediction) [2] and structural design problems [3].

It has been successfully applied to various benchmarks and realworld optimization problems [4-12] including parameter estimation of the nonlinear muskingum model and vehicle routing, optimal cost design of water distribution networks and so on. To improve the performance of the HS algorithm and eliminate the drawbacks of invariable HMCR and PAR, Mahdavi et al. [10] proposed an improved harmony search (IHS) algorithm that uses variable PAR (variable pitch adjusting rate) and bw (bw is an arbitrary distance bandwidth for the continuous design variable) in improvisation step. Also, Omran and Mahdavi [11] proposed a new variant of harmony search, called the global best harmony search (GHS), in which concepts from swarm intelligence are borrowed to enhance the performance of HS algorithm such that the new harmony can mimic the best harmony in the harmony memory (HM). HS is good at identifying the high performance regions of the solution space at a reasonable time, but gets into trouble in performing local search for numerical optimization. A major drawback of the HS algorithm is that the user needs to specify the minimum and maximum values of bw, which are difficult to guess and are problem dependent. In the GHS algorithm, HM uses the best harmony to search, sometimes easy to converge to a local optimum.

Inspired by the above said works in HS, in this paper, a dynamic improved version of HS developed is presented. In dynamic HS the whole population is divided into sub populations and the harmony improvisation is performed in each of the population independently[13,14]. In order to preserve the diversity aspect, the subpopulations are shuffled after prescribed number of iterations of harmony improvisation and regrouped again to improve further through harmony search. The process is repeated till the convergence criteria is satisfied. The developed metaheuristic algorithm is employed to solve the layup sequence design problem of laminate composite stiffened cylinder, which is a difficult combinatorial optimization problem.

Currently, the focus is on solving the problem at hand in the best way possible, rather than promoting a certain metaheuristic. This has led to an enormously fruitful cross-fertilization of different areas of optimization. This cross-fertilization is documented by a multitude of powerful hybrid algorithms that were obtained by combining components from several optimization techniques. Hereby, hybridization is not restricted to the combination of different metaheuristics, but includes, for example, the combination of exact algorithms and metaheuristics. In view of this, in order to solve the complex layup sequence design problem in the best possible way, the proposed algorithm is hybridized with a customised local (or neighborhood) search algorithm. Although generally the metahueristic algorithms provide promising results, it remains clear that, in many cases, these algorithms cannot compete with customised local (or neighborhood) search algorithms [15,16]. However, local search methods often suffer from the initialization problem. That is, the performance of a local optimizer is often a function of the initial solution to which it is applied. Therefore, hybridization of meta-heuristic algorithm with an effective (application specific) local search algorithm is likely to provide much superior solutions as metaheuristic algorithms generate good initial solutions for the local search algorithm to explore further to provide a good local optimised solution. Keeping this in view, we propose to hybridise the proposed algorithm with a customized neighborhood search algorithm called variable depth neighborhood search algorithm and is described in the subsequent sections as Dynamic Hybrid Harmony Search (DHHS). To show the efficiency of this method, DHHS algorithm is applied to a combinatorial optimization problem of layup sequence design of laminate composite stiffened cylindrical shell. Numerical results reveal that the proposed algorithm is a powerful search algorithm for the combinatorial optimization.

## 2. DYNAMIC HYBRID HARMONY SEARCH ALGORITHM (DHHS)

This section describes the proposed Dynamic Hybrid Harmony Search (DHHS) algorithm. The proposed algorithm has the basic procedures of harmony improvisation as that of Improved harmony search algorithm [10] except for the employment of the customized variable depth neighborhood search and the divison of whole population into a defined number of smaller groups, within which the improvisation takes place. The step by step procedure of the proposed algorithm is shown in Figure 1 and discussed in this section below.

#### Step 1. Initialize the problem and Parameters

Initialize the optimization problem and algorithm parameters. First, the optimization problem is specified as follows:

Minimize or Maximize f(x) (1)

subject to  $x_i \in X_i$ , i=1, 2...N, where f(x) is the objective function; x is the set of each design variable  $(x_i)$ ;  $X_i$  is the set of the possible range of values for each design variable (continuous design variables), that is,  $Lx_i \leq X_i \geq ux_i$  and N is the number of design variables.



Fig 1: Flow chart of DHSS algorithm

#### Step 2. Form the groups from population

The population is sorted and divided into  $K_{max}$  groups, each holding M solution vectors such that population ,  $P = k_{max} \times M$ . The division is done in round robin fashion i.e., the first vector going to the first group. Second one going to the second group, the  $k^{th}$  vector to the  $k^{th}$  group and  $(k+1)^{th}$  vector back to the first group. This way of distributing vectors to groups preserves diversity among vectors within each group. In this algorithm, each group is allowed to evolve independently to search locally at different regions of the solution space. In addition, shuffling all the groups and re-dividing them again into a new set of groups results in a global search through changing the information between groups. As such, the proposed algorithm attempts to balance between a wide search of the solution space and a deep search of promising locations.

#### Step 3. Initialize HS Parameters for each group

The HS algorithm parameters that are required to solve the optimization problem (i.e., Eq. (1)) are also specified in this step: the harmony memory size (number of solution vectors in harmony memory, HMS) equal to the size of a group (M), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), and termination criterion (maximum number of searches). Here, HMCR and PAR are the parameters that are used to improve the solution vector. Both are defined in Step 5.

#### Step 4. Perform variable depth neighborhood search

In the variable depth neighborhood search algorithm, a random cut-off point within the user defined range is used and all the design variables to the left of the cut-off point in the candidate solutions are considered for improvement. Series of solutions are then generated from each candidate solution, by using all possible combinations (i.e. 1, 2 or 3) for the chosen variables. Among the several solutions derived from each existing candidate solution, the best one based on objective value is chosen to compete as the replacement solution. Since the influence of extreme fibers in the composite laminate is significant in altering the stiffness of the laminate, the variables to the left of cut-off point in the existing candidate solution are considered for swapping to obtain a local optimal value. In the numerical studies carried out in this paper, the cut-off point is chosen randomly between 1 and 4.

#### Step 5. Improvise a new harmony adaptively.

To improvise a new harmony from the HM, a new harmony vector,  $x' = (x'_1, x'_2, \ldots, x'_N)$  is generated from the HM based on memory considerations, pitch adjustments, and randomization. For instance, the value of the first design variable (x'\_1) for the new vector can be chosen from any value in the specified HM range (x'\_1 - x\_{1HMS}). Values of the other design variables (x'\_1) can be chosen in the same manner. Here, it is possible to choose the new value by using the HMCR parameter, which varies between 0 and 1 as follows:

$$\begin{array}{c} x_{i} \leftarrow \begin{cases} x_{i} \in \{x_{i}^{1}, x_{i}^{2}, ..., x_{i}^{HMS}\} & with \quad probabilityHMCR \\ \\ x_{i} \in X_{i} & with \quad probability \quad (1-HMCR) \end{cases}$$

$$\begin{array}{c} (2) \end{array}$$

The HMCR is the probability of choosing one value from the historic values stored in the HM, and (1- HMCR) is the probability of randomly choosing one feasible value not limited to those stored in the HM. For example, an HMCR of 0.95 indicates that the HS algorithm will choose the design variable value from historically stored values in the HM with a 95% probability and from the entire feasible range with a 5% probability. An HMCR value of 1.0 is not recommended, because of the possibility that the solution may be improved by values not stored in the HM. This is similar to the reason why the genetic algorithm uses a mutation rate in the selection process. Every component of the new harmony vector,  $x'=(x'_1, x'_2, ..., x'_N)$ , is examined to determine whether it should be pitch-adjusted. Pitch adjusting decision for

$$\mathbf{X}_{i}^{\prime} \quad \clubsuit \begin{cases} yes with probability PAR, \\ No with probability (1-PAR). \end{cases}$$

The pitch adjusting process is performed only after a value is chosen from the HM. The value (1-PAR) sets the rate of doing nothing. To improve the performance of the HS algorithm and eliminate the drawbacks lies with fixed values of PAR and  $b_{w}$ .

DHHS algorithm uses variables PAR and bw changing dynamically with generation number as illustrated in IHS [10] as follows.

$$PAR(gn) = PAR_{\min} + \frac{(PAR_{\max} - PAR_{\min})}{NI} \times gn$$

(4)

where PAR is the pitch adjusting rate for each generation, PARmin is the minimum pitch adjusting rate, PARmax is the maximum pitch adjusting rate ,NI is number of solution vector generations and gn is generation number.

$$bw(gn) = bw_{max} \exp(c.gn)$$
(5)
$$c = \frac{L_n\left(\frac{bw_{max}}{bw_{min}}\right)}{NI}$$

(6)

where bw(gn) is the bandwidth for each generation, bwmin is the minimum bandwidth and bwmax is the maximum bandwidth.

If the pitch adjustment decision for x'i is Yes, and x'i is assumed to be xi(k), i.e., the kth element in Xi, the pitch-adjusted value of xi(k) is:

$$x_i \leftarrow x_i \pm \alpha$$
(7)

where  $\alpha$  is the value of bw × U(-1, 1), bw is an arbitrary distance bandwidth for the continuous design variable, and U(-1, 1) is a uniform distribution between -1 and 1.

A detailed flowchart for the new dynamic hybrid harmony search meta-heuristic algorithm is given in Figure 1. The HMCR and PAR parameters in addition with the variable depth neighborhood search, introduced in the harmony search, help the algorithm to find globally and locally improved solutions, respectively.

#### Step 6. Update the harmony memory.

In Step 5, if the new harmony vector is better than the worst harmony in the HM in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM. The HM is then sorted by the objective function value.

#### Step 7. Check the stopping criterion.

Repeat Steps 4 to 6 until the termination criterion is satisfied.

#### Step 8: Shuffle and regroup

Once the harmony improvisation is completed for defined number of iterations in all the groups, they are replaced in the same positions in the original population and the population is sorted. A new set of groups are formed and are subjected to new harmony improvements by performing the steps 2 to step 7.

#### Step 9. Check for Stopping/Maximum Iterations

Repeat steps 2 to 8 till the maximium iterations specified is achieved.

## **3. LAMINATE COMPOSITE MODELS**

Fiber reinforced composites are extensively used in many modern engineering applications especially after the emergence of high performance composites fibers like high modulus/high strength carbon fibers, Kevlar etc. The composite materials have found its applications in aerospace, automobile, shipbuilding and other industries. Although the high strength-to-weight properties of composite materials are attractive, their greatest advantage is that they provide the designer with the ability to tailor the directional strength and stiffness of a material to a given loading environment of the structure. Unlike conventional materials, directional strength can be imparted to a composite component by appropriate selection/design of stacking sequence. This allows the designer to achieve reduced cost and weight, while ensuring fail-safe design. Therefore, a very important issue in the design of laminated FRP composites is to minimize the cost and weight, while maximizing the strength of the components. This can be achieved by optimal selection of ply thickness and hence that of laminate thickness, ply angles and material of each layer. However, due to manufacturing constraints, ply angles and ply thickness are to be selected from a set of discrete values and the design process becomes a combinatorial optimization problem, which is difficult to solve.

The classical laminate theory [17] is used in the present work to describe the behaviour of laminate composite stiffened shells with a particular laminate configuration. Since the composite laminate shell is considered as symmetric with respect to the mid-surface and also is balanced, the bending-extensional coupling stiffness matrix [B] can be neglected. The extensional stiffness matrix [A] and the bending stiffness matrix [D] are given as

$$\begin{aligned} A_{ij} &= \sum_{l=1}^{k} \overline{Q}_{ij} (h_{l} - h_{l-1}) \\ D_{ij} &= \frac{1}{3} \sum_{l=1}^{k} \overline{Q}_{ij} (h_{l}^{3} - h_{l-1}^{3}) \\ (8) \end{aligned}$$

where  $h_i$  is the thickness of the ith lamina and  $\overline{Q}$  denotes the reduced stiffness of the layers of the laminate. The extensional stiffness matrix and bending stiffness matrix, which are functions of design variables, are the major factors that influence the stability and strength design of the laminated composite structures. These matrices will be used in frequency and buckling calculations of laminate composites.

## 3.1 Frequency of a fiber-reinforced stiffened composite cylinder

The laminate composite stiffened cylinder is shown in Figure 2. The frequency is computed for stiffened laminate cylinders by using smeared stiffness theory. According to the smeared stiffness theory [18], the stiffeners are assumed to be closely distributed over the shell surface such that local buckling is not taken into consideration. Due to smeared stiffener assumption, the effect of stiffening can be represented by the equivalent laminate stiffness matrices  $[\overline{A}], [\overline{B}], [\overline{D}]$  in which the components  $[\overline{A}_{11}], [\overline{A}_{22}], [\overline{B}_{11}], [\overline{B}_{22}], [\overline{D}_{11}], [\overline{D}_{22}]$  are obtained by modifying the

corresponding shell laminate stiffness coefficients in the following way:

$$\overline{A}_{11} = A_{11} + \frac{A_s}{d_s t}; \overline{B}_{11} = B_{11} + e_s \frac{A_s}{d_s t^2}; \overline{D}_{11} = D_{11} \frac{I_s}{d_s t^3} + e_s^2 \frac{A_s}{d_s t^3}$$
  
$$\overline{A}_{22} = A_{22} + \frac{A_r}{d_r t}; \overline{B}_{22} = B_{22} + e_r \frac{A_r}{d_r t^2}; \overline{D}_{22} = D_{22} \frac{I_r}{d_r t^3} + e_r^2 \frac{A_s}{d_s t^3}$$
  
(9)

where  $A_s$  and  $A_r$  are cross-sectional areas of stringer and ring,  $d_s$ and  $d_r$  are distances between stringers and rings.  $I_s$  and  $I_r$  are moments of inertia of stringer and ring about their centroidal axes and  $e_s$  and  $e_r$  are eccentricities of stringer and ring respectively. It is assumed that the normal strains  $\varepsilon_x(z)$  and  $\varepsilon_y(z)$  vary linearly in the stringer as well as in the shell and the normal strains are continuous at the interface of the stiffener and shell. The lateral bending stiffness of the stiffener is also neglected and shear membrane force Nxy is assumed to be carried entirely by the sheet. The stiffeners are made up of unidirectional fibre-reinforced epoxy. The fibre direction coincides with the stiffener axis and E11 is the longitudinal modulus. The shell laminate is composed of the same fibre volume fraction as stiffener.

Frequency for stiffened cylindrical shell is given by equation 10.

$$\omega^{2} = \omega_{mn}^{2} = \frac{1}{\rho h} = \begin{cases} \left[\overline{D}_{11} \left(\frac{m\pi}{l}\right)^{4} + 2\left(\overline{D}_{12} + \overline{D}_{33}\right) \left(\frac{n}{a}\right)^{2} \left(\frac{m\pi}{l}\right)^{2} + \overline{D}_{22} \left(\frac{n}{a}\right)^{4}\right] \\ + \left(\overline{A}_{11} \overline{A}_{22} - \overline{A}_{12}^{2} \left(\frac{m\pi}{l}\right)^{4}\right) / a^{2} \begin{bmatrix} \overline{A}_{11} \left(\frac{m\pi}{l}\right)^{4} + \overline{A}_{22} \left(\frac{n}{a}\right)^{4} \\ + \left(\overline{A}_{11} \overline{A}_{22} - \overline{A}_{12}^{2} - 2\overline{A}_{12} \overline{A}_{33}\right) \left(\frac{n}{a}\right)^{2} \left(\frac{m\pi}{l}\right)^{2} / \overline{A}_{33} \end{bmatrix} \end{cases}$$

$$(10)$$



Fig 2: Stiffened Laminate composite cylinder

where L is shell length, R is the radius of the middle surface of the shell and h is the total thickness of the shell. m is the number of half waves in the length of the shell, n is the number of half waves in circumference.

## 4. NUMERICAL STUDIES

Numerical simulation studies have been carried out by solving layup sequence design problem of a laminate composite stiffened cylinder.

#### 4.1 Stiffened laminate composite cylinder

This numerical study is concerned with the design of a stiffened laminate composite cylinder for optimum weight, cost, ply angle sequence and stiffener configuration. The stiffened shell is made up of a graphite/epoxy material system with lamina properties given in Table 1. In the stiffener, all fibres are laid parallel along the stiffener axis. The shell laminate is composed of N orthotropic layers of equal thickness. The vibration behaviour of the shell may be influenced by a number of factors, such as the shell geometry, the shell laminate stacking sequence, the stiffener eccentricity, the stiffener cross-section parameters, stiffener bending stiffness parameters etc. In the present work, it is proposed to investigate the effect of stacking sequence and also the stiffener configuration on the frequency of the stiffener cross section properties are kept constant. The fiber-reinforced stiffened composite cylinder considered for evaluation has an outer radius R = 0.60 m and a length L = 1.25 m. The cross sectional areas of both the stiffeners i.e., stringer and rings

are taken as same, with width, bs as 0.03 m and depth, ds as 0.0182 m. The external stiffener configuration is used for this study. The ply orientations considered for the optimisation of cylindrical skirt are  $0^0$ ,  $\pm 45^0$  and  $90^0$ . The laminates are considered to be symmetric and balanced. Only half of the plies of a laminate are considered as design variables because of symmetry. In addition to ply balancing and ply contiguity constraints, an additional combinatorial constraint is considered for this problem, which does not permit the ply angle difference between two contiguous plies greater than 45°. This additional constraint contributes in preventing delamination. In order to handle the additional combinatorial constraint related to ply angle difference, each ply need to be represented with an integer. Each element in the string is an integer between 1 and 4. The integers 1, 2, 3 and 4 represent  $0^{\circ}$ ,  $+45^{\circ}$ ,  $90^{\circ}$  and  $-45^{\circ}$  plies of graphite-epoxy. The population size, P is 48, number of groups, M is 6 and number of candidate solution in each group, K<sub>max</sub> is 8. The design variables for this problem are the ply orientations, number of stringers, number of ring stiffeners, and material type of the stiffeners. While the design objective considered is the maximization of frequency, the combinatorial constraints considered for this problem are ply contiguity constraint, ply balancing and ply angle difference. The combinatorial constraints are handled by using the correction operator [ 19-22].

The optimization problem considered here is to optimize the stacking sequence of the stiffened cylindrical shell with varied number of stringers and rings to maximize the frequency. Table 2 shows the stacking sequences of stiffened laminate composite cylindrical shell with varied predefined number of stringers (NST) and rings (NRG). The number of function evaluations is also recorded in Table 2. To compare the efficiency of the proposed algorithm, this numerical example is also solved by Genetic Algorithm(GA), with Tournament selection scheme and two point cross over. Mutation in GA is replaced with the variable depth local search algorithm for fair comparison. Figure 3 shows the convergence performance of DHHS and GA for one of the cases of above said problem. From the results presented in Table 2 and Figure 3, the performance of the proposed DHHS is clearly superior.

#### Table 1. Material properties for stiffened composite cylinder

S. No.	Composites	E <sub>L</sub> KN/mm <sup>2</sup>	E <sub>T</sub> KN/mm <sup>2</sup>	$\mathbf{v}_{\mathrm{LT}}$	G <sub>LT</sub> KN/mm2	Mass Density Kg/mm <sup>3</sup>
1	Graphite/Epoxy (AS/3501)	140.68	9.13	0.30	7.24	0.01605e <sup>-04</sup>
2	Glass/Epoxy (Generic S-glass- epoxy)	43.00	9.07	0.27	4.54	0.01992e <sup>-04</sup>

Table 2. Optimal stacking sequences of stiffened laminate composite cylinder with varied number of stiffener configurations by using proposed DHHS algorithm

S.No	Stacking Sequence	Number of plies	Frequency (HZ)	NST	NRG	Function Evolution (DHHS)	Function Evolution (GA)
1	$[45, 45, 0, -45, 0, -45, -45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	12.75	7	3	1570	2050
2	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	12.83	8	3	3942	4678
3	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, -45, 0, 45, 0, -45]_s$	32	12.90	9	3	3193	4326
4	$[45, 45, 0, 45, 45, 0, 45, 0, -45, -45, 0, -45, 0, -45, 0, -45]_s$	32	12.98	10	3	2858	3761
5	$[-45, -45, 0, 45, 0, 45, 45, 0, 45, 45, 0, -45, 0, -45, 0, -45]_s$	32	13.62	7	4	1547	2865
6	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	13.69	8	4	3946	4635
7	$[45, 45, 0, -45, 0, 45, 0, -45, -45, 0, 45, 45, 0, -45, 0, -45]_s$	32	13.76	9	4	1798	2653
8	$[-45, -45, 0, 45, 45, 0, 45, 0, 45, 45, 0, -45, 0, -45, 0, -45]_s$	32	13.83	10	4	1557	1984
9	$[-45, -45, 0, 45, 0, 45, 45, 0, 45, 45, 0, -45, 0, -45, 0, -45]_s$	32	14.44	7	5	1547	1961
10	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	14.51	8	5	3946	4535
11	$[-45, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45, 0, -45]_s$	32	14.57	9	5	2269	3265
12	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	14.64	10	5	1517	2198
13	$[45, 45, 0, 45, 45, 0, -45, 0, -45, -45, 0, 45, 0, -45, 0, -45]_s$	32	15.21	7	6	2077	2567
14	$[-45, -45, 0, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, -45, 0, -45]_s$	32	15.28	8	6	3946	4120
15	$[-45, -45, 0, 45, 45, 0, 45, 45, 0, 45, 0, 45, 0, -45, 0, -45]_s$	32	15.34	9	6	2269	3521
16	$[-45, -45, 0, 45, 45, 0, 45, 0, 45, 0, 45, 45, 0, -45, 0, -45]_s$	32	15.40	10	6	1536	2673

S. No	Stacking sequence	Number of plies	Cost	Weight	Frequency	NRG	NST	Stiffener Material type
1	$\begin{matrix} [45_{(gl)}, 0_{(gl)}, 45_{(gl)}, 0_{(gl)}, -45_{(gl)}, 0_{(gl)}, -45_{(gl)}, 90_{(gl)}, -45_{(gl)}, 0_{(gl)}, \\ 45_{(gl)}, 0_{(gl)} \end{matrix} \bigr]_{s}$	24	518.87	518.87	5.95	5	7	2
2	$\begin{matrix} [-45_{(g)}, -45_{(gl)}, 90_{(gl)}, 45_{(gl)}, 45_{(gl)}, 0_{(gl)}, 45_{(gl)}, 45_{(gl)}, 90_{(gl)}, -45_{(gl)}, \\ -45_{(gl)}, 90_{(gl)} \end{matrix} \bigr]_s$	24	658.53	504.56	6.17	4	7	2
3	$[-45_{(g)},90_{(g)},45_{(gl)},45_{(gl)},0_{(gl)},0_{(gl)},45_{(gl)},0_{(gl)},-45_{(gl)},90_{(gl)},-45_{(gl)},90_{(gl)},-45_{(gl)},90_{(gl)},-45_{(gl)},0_{(gl)},0_{(gl)},-45_{(gl)},0_{(gl)},$	24	783.17	500.04	6.31	4	7	2
4	$\begin{matrix} [45_{(g)}, 45_{(g)}, 0_{(g)}, -45_{(gl)}, 90_{(gl)}, -45_{(gl)}, 90_{(gl)}, -45_{(gl)}, 90_{(gl)}, 45_{(gl)}, \\ 90_{(gl)} \end{bmatrix}_s$	22	882.57	472.25	6.55	4	7	2
5	$[45_{(g)}, 45_{(g)}, 0_{(g)}, -45_{(g)}, -45_{(gl)}, 0_{(gl)}, -45_{(gl)}, 90_{(gl)}, 45_{(gl)}, 90_{(gl)}]_s$	20	982.00	444.44	6.87	4	7	2
6	$[-45_{(g)}, -45_{(g)}, 90_{(g)}, -45_{(g)}, 0_{(g)}, 45_{(g)}, 0_{(gl)}, 45_{(gl)}, 90_{(gl)}]_s$	18	1206.20	412.11	7.08	4	7	2
7	$[90_{(g)}, 90_{(g)}, 45_{(g)}, 0_{(g)}, 45_{(gl)}, 90_{(gl)}, -45_{(gl)}, 0_{(gl)}, -45_{(gl)}, 0_{(gl)}]_s$	20	2314.30	409.94	8.34	5	7	2
8	$[90_{(g)}, 45_{(g)}, 0_{(g)}, -45_{(g)}, 0_{(g)}, 0_{(gl)}, 45_{(gl)}, 90_{(gl)}, -45_{(gl)}]_s$	18	2327.80	371.43	8.70	4	7	2

 Table 3. Trade-off solutions of hybrid laminate composite stiffened cylinder for multi-objective optimization of both weight and cost using proposed DHHS algorithm with constraint on Frequency (5.5HZ)

NRG: Number of Rings; NST: Number of Stringers



Fig 3: Convergence characteristics of DHHS and GA

# 4.2 Hybrid laminate composite stiffened cylinder

Hybrid laminates incorporating two or more fiber composite materials in their construction can offer improved designs and better tailoring capabilities, as it is possible to combine the desirable properties of the two materials. In the present example, two different materials are considered. One is graphite-epoxy, which is expensive, but has high stiffness properties and the other is the glass-epoxy, which is not as stiff, but is relatively cheaper. The material properties used in this study for graphite-epoxy and glass-epoxy are given in Table 1. The stiffness-to-weight ratio of graphite-epoxy is about four times higher than E

that of glass-epoxy, with 
$$\frac{E_1}{\rho} = 345$$
 against  $\frac{E_1}{\rho} = 87.5$ .

However, it is also more expensive, with a cost per unit weight is 8 times higher than that of glass-epoxy.

The optimization problem considered here is a multi-objective optimization of hybrid laminate composite stiffened cylinder for simultaneous optimization of cost and weight with a design constraint on its frequency. The stiffened cylinder considered here is same as the one considered in the earlier numerical study. The frequency is considered as a design constraint and the constraint value is taken as 5.5 Hz for the problem solved in this

Material Type =1: Graphite-epoxy; Material Type =2: Glass-epoxy

paper. The multi-objective optimization problem of simultaneous optimization of cost and weight is accomplished by using weighted aggregating approach, where two objective functions are combined into one overall objective function and is given as:

Objective function= $\beta$  x weight+(1- $\beta$ )cost

(11)

where  $\beta$  is the weight factor. The optimisation problem is to minimise the weighted sum of the total weight and cost of the hybrid laminate stiffened composite cylinder.

The ply angles, number of horizontal stiffeners, number of vertical stiffeners, stiffener material type are the design variables in the optimisation problem. The ply orientations are considered as  $0^{\circ}$ ,  $\pm 45^{\circ}$ , and  $90^{\circ}$ . As mentioned earlier, the solution is represented in the form of an encoded string representing ply angles. Each element in the string is an integer between 0 and

8, where 0 represents a pair of empty plies, 1, 2, 3 and 4 represents  $0^{\circ}$ , +45°,  $90_{2}^{\circ}$  and -45° plies of Graphite-epoxy.

Similarly 5, 6, 7 and 8 represents  $0^{\circ}$ ,  $+45^{\circ}$ ,  $90_2^{\circ}$  and  $-45^{\circ}$  plies of Glass-epoxy. It is necessary to introduce 0 to represent empty plies in the string as the number of individuals in a string

is constant, whereas the number of plies in a laminate is not. The balancing and ply contiguity and ply angle difference are considered as combinatorial constraints. While the combinatorial constraints are handled using the correction operator, the design constraint is handled using penalty approach. The trade-off solutions obtained using the proposed DHHS is shown in Table 3. The results furnished in the table include the stacking sequences, number of plies and their corresponding weight, cost and buckling load factor. A close look at the results indicates that, the minimum weight of the hybrid laminated stiffened cylinder is 371.43N. It is obtained when majority of plies (10 out of 18 plies) are made of graphite-epoxy. The corresponding cost is 2327.80. The minimum cost is found to be 518.87 which is about 22.29% of the cost of stacking sequence corresponding to the optimal weight. The optimum ply sequence corresponding to the minimum cost consists of only glass-epoxy plies. Its weight is found to be 518.87, which is approximately 39.69% heavier than the optimum weight laminate sequence. This study clearly confirms the effectiveness of the proposed DHHS in obtaining optimal solutions for simultaneous cost/weight minimisation. The trade-off solutions given in Table 3, can be used by the designer to determine the optimal configurations for his problem. The final choice of the best design will depend on additional information that will enable him to evaluate all the

points on the Pareto curve and prioritise these values depending on the application on hand.

## 5. CONCLUSION

A meta-heuristic algorithm called Dynamic Hybrid Harmony search (DHHS) has been proposed in this paper for solving the layup sequence optimization problem that derives from multimaterial composite design. A stiffened cylindrical shell is considered for the illustration of the proposed algorithm. Here divided into smaller groups and the population is the improvisation is performed in each group independently. They are shuffled dynamically to exchange the improved vectors after a predefined number of iterations and regrouped to perform the next level of improvisation. This way, a right balance of the diversification and intensification is emphasized in this algorithm. The studies indicate that improved multi-group based harmony search, hybridized with the customized variable depth neighborhood search is quite effective in obtaining practical solutions and in lesser functional evolutions, when compared with GA.

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