PID Controller Design using BB-BCOA Optimized Reduced Order Model

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ABSTRACT
A Big Bang-Big Crunch Optimization Algorithm (BBBCOA) is availed in the design of PID controller. A sixth order system is reckoned and is scaled down to second order with the help of BB-BCOA, Particle Swarm Optimization(PSO), Genetic Algorithm (GA) Hankel Norm Approximation (HNA). Later, a controller is designed by approximate model matching technique in the Pade sense. The procedure followed is justified by the step responses of the closed loop transfer functions obtained. In the indirect case, initially controller is designed for the original system under test and the overall closed loop model is reduced to third order. The concept is exemplified and the responses are seen to be comparable.

General Terms
Model Order Reduction, PID Controller, Optimization Technique.

Keywords
Big Bang - Big Crunch Optimization Algorithm, PID controller, Particle Swarm Optimization, Genetic Algorithm, Hankel Norm Approximation

1. INTRODUCTION
The interminable tendency for systems with higher complexity driven with the demand for miniaturization has created furor in simulation process during the initial design validation stage [1-2]. Abundant techniques of order reduction are available in literature today [3-8] but, choosing the best technique is still at large because of various reasons. In the past decade the increase of computing power manifold has aided the analyzation of alternative designs efficiently. Here, a promising and a novel order reduction technique is being hunted to design a reduced order controller R_C(s) to preserve the crucial dynamics of the system under consideration. Further, simplify the best model in light of the purpose for which the model is to be used, such that the designed systems abide by stipulations laid down. This is due to the fact that the designed system can only be accepted if it satisfies so called design constraints. Consequently this results in simple, low-order approximations both for plant as well as controller models, without sacrificing accuracy.

The heart of this paper lies in the design of a controller G_C(s) for an uncontrolled plant G_P(s). The G_C(s) designed should be in a position to drive the system in stable mode, when the response of the closed loop system is considered with feedback being unity. In spite of a desired quicker response, the designed controller G_C(s) must also be able to closely match the time responses of the controlled system with those of the reference model. In order to carry out the above mentioned task, a recently erupted evolutionary technique [9] called as Big Bang - Big Crunch Optimization Algorithm (BB-BCOA) was sought for the purpose. In other words, BB-BCOA being another type of evolutionary computation is being roped in to assist in the design of PID controller. Such numerical technique not only aids in rationally searching but also in selecting an appropriate combination of the best parameters among the available collection, so as to satisfy the design requirements.

The two different types of approaches for controller design dealt with are Process and Controller reduction. The same is being reflected in figure1. In the former approach the controller is obtained on the basis of reduced order model and in the latter, it is obtained for the original system under consideration [10]. Further, the closed loop system function of controller (higher order) and original system with unity feedback is obtained and is reduced to a lower order [11]. In the process reduction approach, the propagation of error during the design steps hinders, as the reduction is carried out in early stages of design. Meanwhile, in the controller reduction approach, the issue of error propagation dosen't persists as the reduction process is carried out in the final stage of the design. Although the method of Hankel Norm Approximation (HNA) has proved to be suitable for the controller design [12], another alternate approach on evolutionary procedure called BB-BCOA is proposed here. This algorithm relies on one of the theories of the evolution of the universe; namely, the Big Bang and Big Crunch Theory and then realized to be useful for optimization [9]. This approach comes out to be better than the other conventional techniques including HNA.

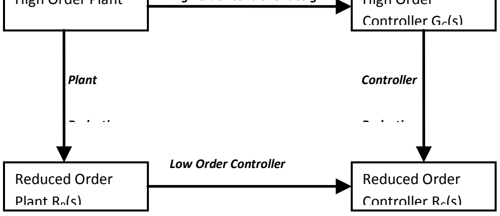

Fig 1. Approaches for controller

2. BIG BANG - BIG CRUNCH (BB-BC)
BB-BC, a relatively recent method of evolutionary computation was introduced by Erol and Eksin [9] and has been quite popular especially in the area of structural engineering for the design of RC frames, space trusses, , optimum design of complex composite laminates, target motion analysis [13-17]. Further the same technique is employed for course timetabling problem and reduced order modelling [18-19]. Hence, it is seen to be used in solving

mixed integer optimization problems that are typical of complex engineering systems. Similar to Particle Swarm Optimization(PSO), Genetic Algorithm (GA) it works well with any dimension problem and in finding the optimum for single objective and multi-objective functions (nonlinear and linear). Conceptually BB-BCOA is stochastic population based nature, but is easier to implement. In addition, this stochastic population based optimization technique comes with a simple memory component. To conclude, BB-BCOA has similar or better results than the existing methods [19].

The BB-BCOA is basically inspired by the theories in physics and astronomy, portraying how the universe was created, evolved and would end. It comprises of creating initial population randomly as in GA called as Big Bang phase. According to the theory of evolution, the energy is dissipated randomly and haphazardly during the initial phase. Similarly in GA, the individual solutions are scattered all over the entire space uniformly [9]. The individuals are henceforth flown through the multi-dimensional search space with each individual representing a possible solution to the multi-dimensional optimization problem .This phase is accompanied by the Big Bang phase or a convergence phase that has single output with many inputs. During this stage, randomly distributed individual sin the entire space are strained in an order or to a single representative point via a minimal approach. This point can be named as the center of the 'mass' and is further referred as the inverse of the fitness function value. Each solution's fitness is based on a performance function related to the optimization problem being solved. The point representing the center of mass that is denoted by 'X_c' of the population can be calculated as follows[20]

$$X_c = \frac{\sum_{k=0}^{N-1}\left(\frac{1}{J}\right)X_K}{\sum_{k=0}^{N-1}\frac{1}{J}} \tag{1}$$

where 'X_k ' is a dot within entire search space generated and is related to the numerator polynomial coefficients, J is a objective function of the candidate k, N is the population size in the initial phase. The convergence operator in the crunching phase is different from wild selection since the output term may contain additional information (new candidate or member having different parameters than others) than the participating ones. In the next cycle of the big bang phase new solutions are created by using the fitness values function 'J' [9] as

$$J = \sum_{i=0}^{M-1}[y(i\Delta t)^2 + y_r(i\Delta t)^2 - 2y(i\Delta t)y_r(i\Delta t)] \tag{2}$$

where

$$M = \frac{T}{\Delta t}$$

$y(i\Delta t)$ and $y_r(i\Delta t)$ are the unit step responses of the higher order and the reduced order models at time $t=\Delta t$. Here, the time T and Δt is assumed to be 10 and 0.1 second respectively.

The basic flowchart of BB-BCOA is shown in the figure 2.

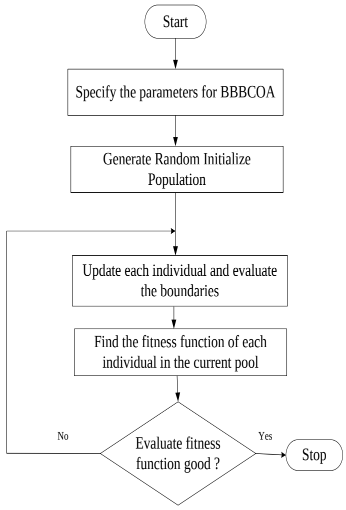

Fig. 2 Optimization process of BB-BCOA

3. HANKEL NORM APPROXIMATION

Hankel norm reductions are among the fanciest sort of model reduction procedures that exist today, from the mathematical and system theoretical point of view. It is one of the very few model approximation procedures that produce optimal approximate models. Glover[21] introduced state space ideas and characterized all stable approximations of a linear time variant degree r (r<n) linear time invariant system which minimize the stable system *G*(s) of McMillan degree *n* by *G*(s) of McMillan "Hankel norm" error

$$\left\| G(s) - \hat{G}(s) \right\|_H$$

Consider a linear time invariant system

$$\frac{dx}{dt} = Ax + Bu \;;\; y = Cx + Du \tag{3}$$

Where, $x \in R^n, u \in R^m, y \in R^p$ are vectors of the states, inputs and outputs.

The matrices $A \in R^{nxn},\ B \in R^{nxm},\ C \in R^{pxn}\ D \in R^{pxm}$ are assumed to be constant matrices. The n^{th} order transfer function G(s) is given by

$$G(s) = C(sI - A)^{-1}B + D \tag{4}$$

The problem is to find a reduced order model in the form of equation (5) such that the reduced model retains the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs. The r^{th} (r<n) order model in time domain form is given by

$$\frac{dx_r}{dt} = \hat{A}x + \hat{B}u \qquad y_r = \hat{C}x + \hat{D}u \tag{5}$$

The corresponding reduced model in transfer function form is given by

$$\hat{G}(s) = \hat{C}(sI - \hat{A})^{-1}\hat{B} + \hat{D} \tag{6}$$

If the eigen values of A are assumed to be strictly in the left half plane then controllability grammian *(P)* and observability grammian *(Q)* are defined as

$$P = \int_0^\infty e^{At}BB^Te^{At}dt \;;\; Q = \int_0^\infty e^{A^Tt}C^TCe^{At}dt \tag{7}$$

Where, P and Q satisfy the following linear matrix equations (Lyapunov equations)

$$AP + PA^T = -BB^T \quad ; \quad A^TQ + QA = -C^TC \tag{8}$$

The Hankel norm of stable rational transfer function *G(s)* is defined by [21] as

$$\|G_r(s)\|_H = \lambda_{\max}^{1/2}(PQ) \tag{9}$$

where, λ_{max}(M) stands for the largest eigen value of matrix M. It provides a measure of most controllable/observable state. This is fundamental for model reduction and its main objective is to discard the less relevant states from input-output perspective that is the less controllable/observable states. This is also an important measure of the minimality of a realization from numerical point of view.

The hankel singular values of a stable rational transfer function *G(s)* are the square roots of the eigen values of the matrix product PQ. These indicate the respective state energy of the system. For convenience these singular values are usually ordered for the truncation of the states that corresponds to smaller hankel singular values as

$$\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq \sigma_{r+1} \geq ... \geq \sigma_n > 0 \tag{10}$$

These singular values represents the fundamental measures of gain and complexity of a linear time invariant system.

For all stable G(s) of McMillan degree ≤ r, the hankel norm approximation error is

$$\left\| G(s) - \hat{G}(s) \right\|_H \geq \sigma_{r+1} \tag{11}$$

By implementing the additive decomposition the reduced model is computed as:

$$\hat{G}(s) = \hat{D}_1 + \hat{C}_1(sI - \hat{A}_{11})^{-1}\hat{B}_1 \tag{12}$$

4. DESIGN PROCEDURE

The direct and indirect approaches of controller design [22] are shown in the fig.1. Initially a controller is designed for high order system and is reduced using BB-BCOA to obtain a low order controller. Then the closed loop response of higher order controller with original plant and low order controller with original plant are compared with the reference model. The controller parameters are obtained using approximate model matching in the Pade sense. The performance of full order controller is then compared with that of the reduced order controller as shown in fig 3.

The structure and complexity of the controller depends on the choice of the reference model which is considered as a desired closed loop system with certain specifications associated with it. The reference model should ensure the stability and acceptable performance of closed loop system. The reference model may be chosen to meet the following design specifications [23]

1. The time domain specifications e.g., rise time, overshoot, settling time and steady state error.
2. The frequency domain specifications e.g., bandwidth, cut off rate, gain margin and phase margin.
3. The complex domain specifications e.g., damping ratio, damping factor, undamped natural frequency and location of Closed loop poles.

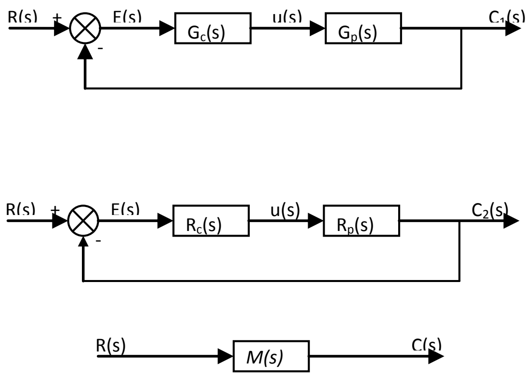

Fig 3.Closed Loop configurations with reference

4.1 Direct Approach: Plant Reduction and Controller Design

The design procedure is based on approximate model matching in Pade sense and consists of the following steps.

Step1: For the plant having a transfer function G_p(s), construct a reference model M(s) on the basis of specifications as discussed above. The closed loop response of the controlled system with unity feedback approximates the reference model response.

Let the transfer function of the plant G_p(s) and the reference model M(s) are given by

$$G_P(s) = \frac{a_0 + a_1 s + ... + a_m s^m}{b_0 + b_1 s + ... + b_n s^n} \tag{13}$$

$$M(s) = \frac{g_0 + g_1 s + ... + g_u s^u}{h_0 + h_1 s + ... + h_v s^v} \tag{14}$$

Step 2: Determine an equivalent open loop specification model. If M(s) is the desired closed loop system (reference model) then the equivalent open loop specification model transfer function is obtained by

$$\tilde{M}(s) = \frac{M(s)}{1 - M(s)} \tag{15}$$

Step 3: Specification of the structure of the controller.

Let the controller structure G_c(s) is given by

$$G_C(s) = \frac{p_0 + p_1 s + ... + p_k s^k}{q_0 + q_1 s + ... + q_l s^l} \tag{16}$$

Step 4: For determining the unknown controller parameters, the response of the closed loop system is matched with that of the reference model as

$$G_C(s)G_P(s) = \tilde{M}(s) \; ; \; G_C(s) = \frac{\tilde{M}(s)}{G_P(s)} = \sum_{i=0}^{\infty} e_i s^i \tag{17}$$

Where e_i's are the power series expansion coefficients about s=0. Now the unknown control parameters p_i and q_i are obtained by equating the (16-17) in Pade sense.

$$p_0 = q_0 e_0$$
$$p_1 = q_0 e_1 + q_1 e_0$$
$$p_2 = q_0 e_2 + q_1 e_1 + q_2 e_0$$
$$.$$
$$p_i = q_0 e_i + q_1 e_{i-1} + ... + q_i e_0$$
$$0 = q_0 e_{i+1} + q_1 e_i + ... + q_{i+1} e_0$$
$$.$$
$$0 = q_0 e_{i+j} + q_1 e_{i+j-1} + ... + q_j e_i \quad (18)$$

The controller having the desired structure is obtained by solving above linear equations.

Step 5: After obtaining the controller parameters, the closed loop transfer function can be obtained as

$$G_{CL}(s) = \frac{G_C(s) G_P(s)}{1 + G_C(s) G_P(s)} \quad (19)$$

Step 6 : Reduce the plant $G_P(s)$ to $R_P(s)$ using the method of BBBCOA. Repeat steps 4 and 5. The closed loop transfer function for the reduced order model is

$$R_{CL}(s) = \frac{R_c(s) R_p(s)}{1 + R_c(s) R_p(s)} \quad (20)$$

4.2 Indirect Approach: Controller Design and Reduction

In this approach a high order controller is designed for the original high order plant and closed loop transfer function with unity feedback is obtained. Then, the closed loop transfer function $G_{CL}(s)$is reduced to a lower order to obtain reduced closed loop transfer function $R_{CL}(s)$.

5. NUMERICAL EXAMPLES

5.1 Direct Method

Ex 1: Consider the regulator problem, whose transfer function and the reference model are given as [5]

$$G_P(s) = \frac{s^5 + 8s^4 + 20s^3 + 16s^2 + 3s + 2}{2s^6 + 36.6s^5 + 204.8s^4 + 419s^3 + 311.8s^2 + 67.2s + 4}$$

$$M(s) = \frac{0.023s + 0.0121}{s^2 + 0.21s + 0.0121}$$

The equivalent open loop transfer function is

$$\tilde{M}(s) = \frac{0.023s + 0.0121}{s^2 + 0.187s}$$

The desired controller is given by

$$G_C(s) = \frac{\tilde{M}(s)}{G_P(s)}$$
$$= \frac{1}{s}(0.064707 + 0.76697s + 0.82153s^2 - 4.9633s^3 + ...)$$

Taking the PID controller structure as

$$G_C(s) = K_1 + \frac{K_2}{s} + K_3 s \;\; = \frac{K_1 s + K_2 + K_3 s^2}{s}$$

Comparing the controller $G_c(s)$with the power series expansion the parameters K_1, K_2 and K_3 of the controller are obtained, which gives the PID controller as

$$G_C(s) = \frac{0.064707 + 0.76697\, s + 0.82153 s^2}{s}$$

The corresponding closed loop transfer function is

$$G_{CL}(s) = \frac{0.8215s^7 + 7.339s^6 + 22.63s^5 + 29\, s^4 + 16.03s^3 + 4.979s^2 + 1.728s + 0.1294}{1.822s^7 + 25.64s^6 + 125\, s^5 + 238.5s^4 + 171.9s^3 + 38.58\, s^2 + 3.728s + 0.1294}$$

The original system is reduced to second order model using proposed BB-BCOA, PSO, GA, HNA and is given by

$$R_{PBBBCOA}(s) = \frac{0.0233s + 0.01176}{s^2 + 0.2035s + 0.01176}$$
$$R_{PPSO}(s) = \frac{0.02555s + 0.01036}{s^2 + 0.4756s + 0.01036}$$
$$R_{PGA}(s) = \frac{0.01414s + 0.009369}{s^2 + 0.1436s + 0.009369}$$
$$R_{PHNA}(s) = \frac{0.0113s + 0.0736}{s^2 + 0.1436s + 0.009369}$$

Now, the controller structure is obtained as

$$R_{CBBBCOA}(s) = \frac{\tilde{M}(s)}{R_{PBBBCOA}(s)}$$
$$= \frac{1}{s}(0.06191 + 0.76252s + 0.5764s^2 - ...)$$

Taking the PID controller structure as

$$R_C(s) = K_1 + \frac{K_2}{s} + K_3 s \quad or \quad R_C(s) = \frac{K_1 s + K_2 + K_3 s^2}{s}$$

Comparing the coefficients with the power series expansion the parameters, K_1, K_2 and K_3 of the controller are obtained, which gives the PID controller as

$$R_{CBBBCOA}(s) = \frac{0.06191 + 0.7625s + 0.5764\, s^2}{s}$$

The closed loop transfer function of the reduced second order model and the controller using BBBCOA, PSO, GA and HNA is obtained as

$$R_{CLBBBCOA}(s) = \frac{0.01343s^3 + 0.02455\, s^2 + 0.01041s + 0.000728}{1.013s^3 + 0.2281\, s^2 + 0.02217\, s + 0.000728}$$
$$R_{CLPSO}(s) = \frac{0.04853s^3 + 0.03861\, s^2 + 0.00933s + 0.0006703}{1.049s^3 + 0.2142\, s^2 + 0.01969\, s + 0.0006703}$$
$$R_{CLGA}(s) = \frac{0.01165s^3 + 0.07664\, s^2 + 0.005604s + 0.004762}{1.012s^3 + 0.417\, s^2 + 0.0792\, s + 0.004762}$$
$$R_{CLHNA}(s) = \frac{0.05177s^3 + 0.0445\, s^2 + 0.007102s + 0.0006152}{1.052s^3 + 0.1885\, s^2 + 0.0167\, s + 0.0006152}$$

Fig 4 shows the comparison of step response of closed loop transfer function of the original plant, the reduced model using BBBCOA, PSO, GA and HNA with that of the reference model. It is seen that all the three responses are matching in both steady state and transient regions.

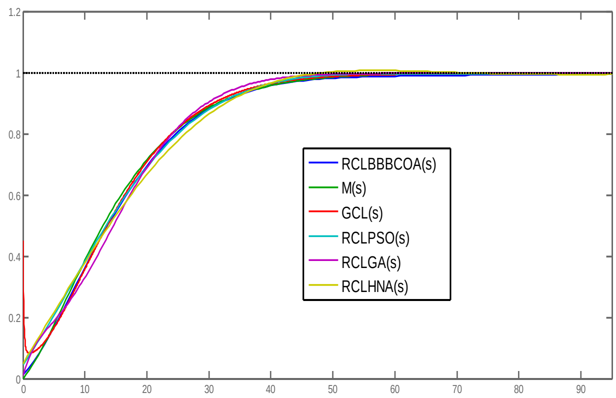

Fig 4: Comparison of step responses

5.2 Indirect Method

Ex 3: Consider a 6th order rational minimum phase stable practical system taken from Prasad [24] having transfer function and the reference model as

$$G_P(s)=\frac{248.05s^4+1483.3s^3+91931s^2+468730s+634950}{s^6+26.24s^5+1363.1s^4+26803s^3+326900s^2+859170s+528050}$$

$$M(s)=\frac{4}{s^2+4s+4}$$

The equivalent open loop transfer function is

$$\tilde{M}(s)=\frac{4}{s^2+4s}$$

The transfer function of the controller is given as

$$\tilde{M}(s)=G_C(s)G_P(s) \text{ or } G_C(s)=\frac{\tilde{M}(s)}{G_P(s)}$$

$$=\frac{1}{s}(0.8316+0.5313s-0.2841s^2+0.1159s^3+..)$$

Matching controller structure with power series expansion coefficients gives

$K=0.8316,\ K_1=1.1735,\ K_2=0.5347$

Hence the controller $G_C(s)$ for the original plant is given as

$$G_C(s)=\frac{0.976s+0.8316}{0.5347s^2+s}$$

Then the corresponding closed loop transfer function $G_{CL}(s)$ is

$$G_{CL}(s)=\frac{242.5s^5+1656s^4+9.111e04s^3+5.347e05s^2+1.011e06s+5.28e05}{0.5363s^8+15.07s^7+757.3s^6+1.598e04s^5+2.038e05s^4+8.788e05s^3+1.677e06s^2+1.539e06s+5.28e05}$$

This high order closed loop transfer function $G_{CL}(s)$ is reduced to third order using BB-BCOA, PSO, GA, HNA technique and is given by

$$R_{CL3BBBCOA}(s)=\frac{-0.04996\ s^2+7.323s+25.83}{s^3+14.13\ s^2+33.04\ s+25.83}$$

$$R_{CL3PSO}(s)=\frac{0.8622\ s^2+2.05s+0.9609}{s^3+3.258\ s^2+3.172\ s+0.9609}$$

$$R_{CL3GA}(s)=\frac{0.4844\ s^2+2.393s+1.674}{s^3+3.233\ s^2+4.045\ s+1.674}$$

$$R_{CL3HNA}(s)=\frac{0.9633\ s^2+3.88s+1014}{1.176\ s^3+1.404\ s^2+1190\ s+1013}$$

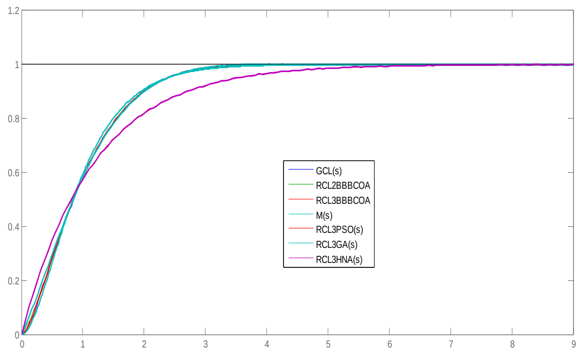

Fig 5: Comparison of step responses

Further, $G_{CL}(s)$ is also reduced to second order effectively using BB-BCOA and is given by

$$R_{CL2BBBCOA}(s)=\frac{0.2917s+2.797}{s^2+3.083s+2.797}$$

The fig 5 shows the comparison of step response of closed loop transfer function of the original plant, reduced model (third order) by BB-BCOA, HNA method, PSO, GA technique and the reference model. The response for the second order reduced model RCL2BBBCOA is also plotted and is seen to be comparable with other techniques.

5. REFERENCES

[1] Dia Abu-Al-Nadi, Othman MK Alsmadi, Zaer S Abo-Hammour," Reduced order modeling of linear mimo systems using particle swarm optimization", The Seventh International Conference on Autonomic and Autonomous Systems ICAS 2011,. ISBN: 978-1-61208-134-2, IARIA, pp. 62-66.2011.

[2] S. M. Giriraj Kumar, R. Sivasankar, T. K. Radhakrishnan, V. Dharmalingam, and N. Anantharaman ,“ Particle Swarm Optimization Technique Based Design of Pi Controller for a Real-Time Non-Linear Process”, Instrumentation Science and Technology, 36, pp.525-542, 2008.

[3] R. Genesio and M. Milanese, “A note on the derivation and use of reduced order models”, IEEE Trans. Automat. Control, Vol. AC-21, No. 1, pp. 118-122, February 1976.

[4] M. S. Mahmoud and M. G. Singh, Large Scale Systems Modelling, Pergamon Press, International Series on Systems and Control 1st ed., Vol. 3, 1981.

[5] M. Jamshidi, Large Scale Systems Modelling and Control Series, New York, Amsterdam, Oxford, North Holland, Vol. 9, 1983.

[6] S.Panda , S.K. Tomar, R Prasad, C. Ardil ," Model reduction of Linear systems by conventional and Evolutionary Techniques" International Journal of Computational and Mathematical Sciences, Vol.3,pp 28-34, 2009.

[7] C. B. Vishwakarma and R. Prasad,"MIMO System Reduction Using Modified Pole Clustering and Genetic Algorithm", Modelling and Simulation in Engineering, Vol 2009,Article ID 540895, Hindalvi publishing corporation ,doi:10.1155/2009/540895, 2009.

[8] Boby Philip, Jayanta Pal,“An Evolutionary Computation Based Approach for Reduced Order Modelling of Linear Systems", IEEE International Conference on Computational Intelligence and Computing Research (ICCIC), 2010, Coimbatore 28-29 Dec. 2010.

[9] O.K. Erol and I. Eksin,"New optimization method: Big Bang–Big Crunch", Advances in Engineering Software, 37, 106–11, 2006.

[10] Shiv Kumar Tomar and Rajendra Prasad **“**Indirect approach of model order reduction of linear Time-invariant systems using truncation method", XXXII National Systems Conference, NSC 2008, Dec 17-19, 2008

[11] Yousuff A. and Skelton R. E. “A note on balanced controller reduction", IEEE Trans. Automatic Control, vol-AC-29, pp 254-257, Oct 1984.

[12] Singh Nidhi,“Reduced order modelling and controller design", Ph D. Thesis, Indian Institute of Technology Roorkee, Roorkee, India, 2007

[13] Ali Kaveh and Omid Sabzi," A comparative study of two meta-heuristic algorithms for optimum design of reinforced concrete frames", International Journal of Civil Engineering, Vol. 9, No. 3, Sept. 2011.

[14] Ali Kaveh and S. Talatahari ," Size optimization of space trusses using Big Bang–Big Crunch algorithm", Computers and Structures, pp. 1129–1140, 2009.

[15] Charles V. Camp,"Design of Space Trusses Using Big Bang–Big Crunch Optimization", Journal of Structural Engineering,ASCE, pp.999-1008, July 2007.

[16] Pavel Y. Tabakov, "Big Bang–Big Crunch Optimization Method in Optimum Design of Complex Composite Laminates", World Academy of Science, Engineering and Technology, 2011.

[17] Hakkl M. Gene;, Ibrahim Eksin and Osman K. Erol," Big Bang - Big Crunch Optimization Algorithm Hybridized With Local Directional Moves and Application to Target Motion Analysis Problem", IEEE International Conference on Systems Man and Cybernetics (SMC), pp. 881- 887, 2010.

[18] Ghaith M. Jaradat and Masri Ayob,"Big Bang-Big Crunch Optimization Algorithm to Solve the Course Timetabling Problem", 10th International Conference on Intelligent Systems Design and Applications,IEEE,2010,pp.1448-1452,2010.

[19] Boby Philip, Jayanta Pal, "An Evolutionary Computation Based Approach for Reduced Order Modelling of Linear Systems", IEEE International Conference on Computational Intelligence and Computing Research (ICCIC), 2010, Coimbatore 28-29 Dec. 2010.

[20] Rashmi Singh and H.K. Verma, "Big Bang–Big Crunch Optimization Algorithm for Linear Phase Fir Digital Filter Design", International Journal of Electronics Communication and Computer Engineering, Vol. 3.2011.

[21] K.Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L"-error bounds", Int. J. Contr., Vol 39, no. 6,1984, pp. 1115-1193.

[22] Anderson B. D. O. and Liu Y., "Controller Reduction : Concept and Approaches", IEEE Trans. On Automatic Control, vol-34, No-8, pp 802- 812, 1989.

[23] S. R. Desai, and Rajendra Prasad," Design of PID Controller using Particle Swarm Optimized Reduced Order Model", Eighth Control Instrumentation System Conference (An International Conference) CISCON-2011, Nov 3 - 6, 2011.

[24] Prasad Rajendra, "Analysis and design of control Systems using reduced order models", Ph.D Thesis, University of Roorkee, Roorkee, India,1989