

# Bayesian MAP Model for Edge Preserving Image Restoration: A Survey

Greeshma T. R.  
Mtech Scholar, Dept of CSE  
MES College of Engineering,  
Kuttipuram, Kerala

Ameeramol P. M.  
Assistant Professor, Dept of CSE  
MES College of Engineering,  
Kuttipuram, Kerala

## ABSTRACT

Image restoration is a dynamic field of research. The need for efficient image restoration methods has grown with the massive production of digital images and movies of all kinds. It often happens that in an image acquisition system, an acquired image has less desirable quality than the original image due to various imperfections and/or physical limitations in the image formation and transmission processes. Thus the main objective of image restoration is to improve the general quality of an image or removing defects from it. The two main considerations in recovery procedures are categorized as blur and noise. In the case of images with presence of both blur and noises, it is impossible to recover a valuable approximation of the image of interest without using some *a priori* information about its properties. The instability of image restoration is overcome by using a priori information which leads to the concept of image regularization. A lot of regularization methods are developed to cope up with the criteria of estimating high quality image representations. The Maximum A posteriori Probability (MAP) based Bayesian approach provides a systematic and flexible framework for this. This paper presents a survey on image restoration based on various prior models such as Tikhonov, TV, wavelet etc in the Bayesian MAP framework.

## General Terms

Image processing: Automated restoration system.

## Keywords

Image restoration, Image Regularization, prior, Bayesian model, MAP estimation, total variation (TV), wavelet.

## 1. INTRODUCTION

With the advancement of digital technology, digital images became an important part of our day-to-day life. These images span many industries like medical, military, scientific etc. They include images taken from a low cost CCD camera to expensive and complex magnetic resonance imaging. Noise and blur are inherent to most of the imaging domains. Millions of images and movies are either taken in poor conditions or transferred over different communication channels which are highly prone to noise. Restoration has been a well-studied problem in the image processing community and continues to attract researchers with an aim to achieve a better estimate in the presence of noise.

## 2. Digital Image

A digital image is usually represented as a matrix of grey or color values. Each element in the matrix is called pixel, i.e. picture element. Each pixel may consist of one or more bits of information, representing the brightness of the image at that point. For color images there are 3 matrices each for red, green and blue (RGB). An  $N \times N$  image can be represented as a vector in  $R^{N^2}$  space.

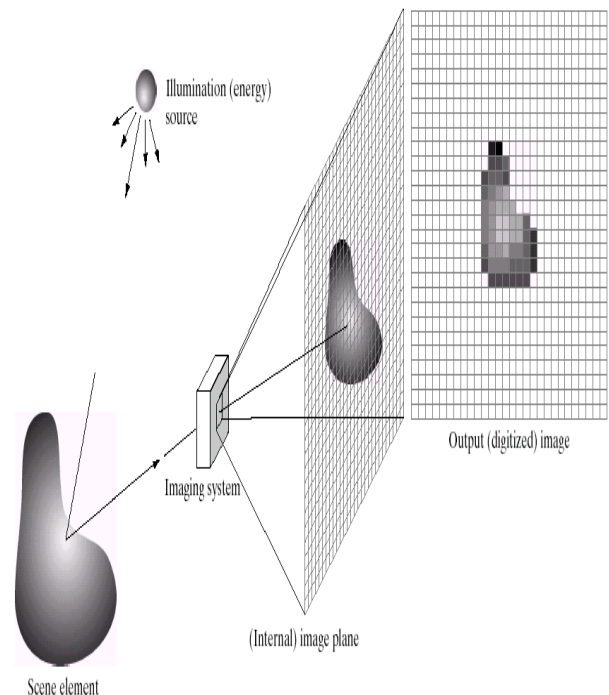


Fig 1: Digital image representation

Image Processing is a technique to enhance the raw images received from cameras/sensors to make it suitable for various day-to-day life applications. Image restoration is a classical problem in image processing. This deals with the reconstruction of a clean image from a blurred and noisy image contaminated with additive white Gaussian noise (AWGN). This gets a lot of attention from the image processing research community due to the fact that millions of images and videos are taken in noisy atmosphere or communicated through noisy channels and also they have blurred differently. It requires a systematic approach that takes into account the entire process of image formation and provides a foundation for the subsequent steps of image processing.

Normally images are degraded by two major effects such as blur and noise. Blurring is fundamental to imaging process and it occurs due to various phenomenon like atmospheric turbulence, camera out of focus, motion etc. Noises are also inherent to images while transmission. This necessitates image restoration since the degraded images are visually annoying and found to be wrong target for compression and analysis. Image restoration refers to removal or minimization of degradations in an image. This includes deblurring of images, noise filtering, and correction of geometric distortion or non-linearity due to sensors. The ultimate goal of image restoration is to get a high quality image estimate by compensating or undoing defects which degrade an image.

The rest of this paper is organized as follows. In section 2, we describe about Image Restoration Problem. Section 3 presents different Image regularization concept, section 4 focuses on Bayesian MAP concept, 5 and 6 depicts related works and Conclusion.

### 3. IMAGE RESTORATION PROBLEM

An ideal image  $x$  is measured in the presence of additive white Gaussian noise. The measured image  $y$  is thus defined as  $y = Hx + \eta$  Where  $H$  represents the convolution matrix that models blurring effect and  $\eta$  represents the random error which is taken as noise. Image restoration problem deals with designing an algorithm that can remove the unwanted information from  $y$ , getting as close to the original image  $x$ .

Figure below is a pictorial representation of an image degradation restoration model. First half shows the degradation operation in which image is being blurred with some degradation function  $H$  and a noise vector  $\eta$  is added to that. The second half is the area under consideration of this work.

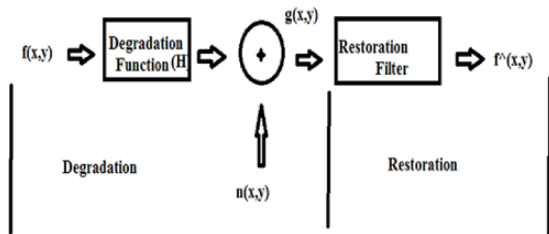


Fig 2: Basic image degradation restoration model

In the case of images with blur, it is possible to come up with a very good estimate of the actual blurring function and undo the blur to restore the original image by the effective inverse filtering operation. Inverse filtering is a deconvolution method in which the inverse operation of degradation function is performed. It is evident that inverse operation does not go well in the cases of images which are corrupted by both blur and noise. This is due to the fact that the inverse operation results in amplification of noise. By employing any kind of denoising methods in such situation initiates smoothening of fine details of the image. The best method that can be employed in such situation is to use some prior information to calculate the image estimates by mathematical or statistical approaches. The Bayesian formulation offers a systematic and flexible way or image regularization and it provides a rigorous framework for estimation of the model parameters.

### 4. IMAGE REGULARIZATION

Image regularization is an effective field of research under image restoration field. This method focuses on incorporating some prior information to solve the ill-posedness of the restoration problem. These priors are normally taken as penalty for complexity such as smoothing limitation.

The general formulation for regularization techniques is

$$\| Hx - y \|_2^2 + \lambda \| Lx \|_2^2$$

Where  $\| Hx - y \|_2^2$  is the Error term,  $\lambda$  is the regularization parameter and  $\| Lx \|_2^2$  is the penalty term.

### 5. BAYESIAN MAP FRAMEWORK

The Bayesian formulation of the image restoration problem provides a systematic and flexible way for regularization using MAP estimation. In the Bayesian framework, the inverse problem can be expressed as a problem to find the posterior density of unknown data from the realization observed. The best method that can be employed for this purpose is the MAP which is a mode of the posterior distribution. By Maximum A posteriori probability (MAP), the estimate of unobserved quantity from empirical data is calculated. It is regularization to maximum likelihood estimation which focuses on prior distribution. This method is best used for a range of image restoration problems, as it provides a computationally efficient means to deal with the shortcomings of filtering operation without sacrificing the computational simplicity of the filtering approach.

The Bayesian estimation is based on the equation

$$P(x|y) \propto P(y|x) P(x)$$

$\swarrow$                        $\swarrow$                        $\swarrow$   
 Posterior  $\propto$  Likelihood  $\times$  Prior

In this framework inverse problem is expressed as problem to find posterior density  $P_{\text{posterior}}(x)$  from the realization observed  $y_{\text{observed}}$  as

$$P_{\text{posterior}}(x) = \frac{P_{\text{prior}}(x) \cdot P(y_{\text{observed}}|x)}{P(y_{\text{observed}})}$$

There will be infinite number of solutions for this equation. From that the more optimized result is to be extracted out. Here by using the concept of MAP method, it is easy to calculate the posterior of the data as

$$\text{argmax}_x P(x|y) = \text{argmax}_x P(y|x) P(x).$$

By the MAP method the underlying scene  $x$  which maximizes the Bayesian expression is calculated which will be the accurate result.

### 6. RELATED WORK

There are extensive works on the development and evaluation of various image restoration techniques. Edge preserving image reconstruction has been proposed to recover the original image from its degraded version while this method also preserves the fine details in the image such as edges. The problem is that enhancement of fine detail (or edges) is equivalent to enhancement of noise. So we have to reconstruct the image in such a way that it is recovered from blur and noise and preserve its edge details. All the restoration methods are focusing on prior based Bayesian MAP estimation.

In the Bayesian estimation there is a wide freedom in the choice of prior where the quality of restoration varies according to different prior. This work provides an analytical survey on various prior based image reconstruction methods in the Bayesian framework.

S. Derin Babacan et al. [3] proposed a novel variation methods based on the hierarchical Bayesian formulation, and provide approximations to the posterior distributions of the image, blur, and model parameters rather than point estimates. TV-based deconvolution into a Bayesian estimation problem provides advantages in blind deconvolution, such as means to estimate the uncertainties of the estimates. The blind deconvolution problem is formulated using a hierarchical Bayesian model, and variational inference is utilized to approximate the posterior distributions of the unknown parameters rather than point estimates. Approximating the posterior distribution makes evaluating the uncertainty of the estimates possible. The unknown parameters of the Bayesian formulation can be calculated automatically using only the observation or using also prior knowledge with different confidence values to improve the performance of the algorithms.

The experimental results shown that the quality of the restorations can be improved by utilizing prior knowledge about the parameters. By comparing the restoration result of Lena image with the same confidence parameters for TV approach (ISNR=3.19dB) and SAR1 approach (ISNR=1.26dB) it is noted that TV-based approaches are more successful at removing the blur while providing smooth restorations with less ringing. However this method still have presence of edge smoothening. Moreover the rate of convergence of the TV algorithm is defined by the image size.

M. Dirk Robinson et al. [4] discussed about a wavelet based deconvolution and restoration algorithm. The wavelet methods are developed based on the idea that image representations based on small set of wavelet is accurate. Here the image is represented by DWT with convolution operators are specified in Fourier domain. And the method of Fourier Wavelet super resolution combines multiple aliased low quality images to produce a high-resolution high-quality image. The efficiency of algorithm stems from separating the multiframe deconvolution or restoration step from the wavelet-based denoising step allows achieving nonlinear denoising in a non iterative fashion.

In this method, first a set of aliased low quality images are captured. These images are then broken down into small tiles and process them separately. To each tiles apply the wiener filter to fuse and deblur. Then estimate the wavelet signal power by first very coarsely denoising the sharpened image. A simple hard thresholding wavelet denoising approach is applied to obtain the coarsely denoised image using a different set of scaling and wavelet functions and wavelet. Update the wavelet and apply new wavelet to efficiently denoise the images thus results in high contrast super-resolved images. The main defect of wavelet method is the presence of ringing artifacts.

Nelly Pustelnik et al. [5] introduce an image restoration algorithm based on combined or hybrid methods of total variation and wavelet regularization. The main advantage of hybrid method is that it compensates the drawbacks of both the method while making use of advantages of both. Here the image is decomposed into a number of wavelets and to each wavelet effective total variation image estimation is applied. Then the proximity operators can be easily computed. The

main advantages of the method are: 1) To deal directly with the true noise likelihood without requiring any approximation of it 2) To permit the use of sophisticated regularization functions, 3) Can be implemented on multiple architecture. This work is being tested on multiple natural and synthetic images.

This method is quantitatively measured in terms of PSNR for the blurred Boat image with PSNR 11.2dB. The results shows that this method provides a better restoration with improved PSNR of 18.8dB where as the TV approach provides an improved PSNR of 17.8dB and wavelet provide a better PSNR of 18.0dB. But this method is relatively complex one.

David Humphrey and David Taubman. [6], reviews a restoration method based on the MAP estimation of images. In this method, a piecewise stationary Gaussian prior is used in calculating the Maximum A posteriori Probability estimate of the underlying data from the observer data by the Linear Shift Invariant Wiener Filter. This method deals with images which are degraded by both blur and noise.

In this paper, it is seen that the image is first segmented and model each region in the segmentation with independent Gaussian priors and maintaining some auxiliary data to make use of Wiener filter at all points in the image. The notion of the extension of a region is introduced which could be created by cutting out the appropriate parts from the other parts. Segmentation is the most important stage of this system since it helps to detect the region of interest by using some prior information. Then to each region a MAP estimate is performed to get the hidden data in each region. A local cost function is applied and iterates this method until we get the intended image estimate for each segment. Finally all the segments that are of interest are combined. This method is efficient in the problem of demosaicking of digital camera images and results shown that this method efficiently preserve edges.

From the literature survey, it is observed that the Bayesian framework is a best environment to work with the image restoration problems in the case of images with the presence of both blur and noises [1]. Bayesian algorithm marginalizes the hidden variables using maximum likelihood function. The MAP based estimation in the Bayesian framework support more for calculating the posterior probability of the hidden data from the observed information [2]. In this framework there is a lot of freedom in the choice of prior. This framework provides a systematic and flexible way for image restoration with a minimum ISNR of 5dB as in table below.

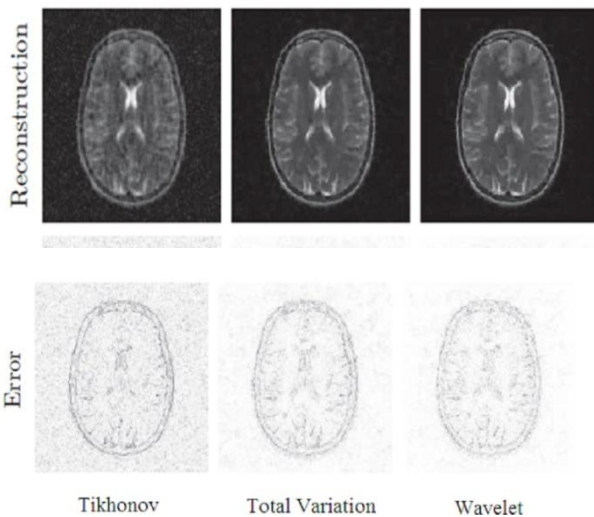
**Table 1: SNR improvement using Bayesian MAP methods**

Method Opted	Image Analyzed	Degraded Image SNR	Restored Image SNR
Bayesian Method	Cameraman Image	PSNR=35.0dB	PSNR=43.03dB
MAP estimation	Lena Image	PSNR=23.23dB	PSNR=29.99dB

Here quality of regularization varies according to the choice of prior. Lot of time and energy has been invested by image processing community for the modelling of adequate priors. A common way to construct the prior is based on intuitive expectation on data content where it is seen that a healthier prior add more to the quality of the image reproduced. In the earlier years edge preserving image regularization were based on the L2 norm based Tikhonov regularization method. Standard Tikhonov regularization, corresponding to a quadratic regularization term with a linear solution. This quadratic cost functionals are of limited use for measuring the plausibility of natural images. So this prior causes over smoothness which strongly penalizes large edges in images. This L2 norm was replaced by L1 norm in later years.

A popular one is the differential L1 norm that is used in total variation which aims at the modulus of the gradient and is an optimal regularization for piecewise constant solutions. The use of L1 norm formally introduced the idea of sparsity in image processing. This sparsity concept expands to the development of wavelet approach. The underlying idea of wavelet regularization is that natural images tend to be sparse in the wavelet domain. Hence, among all the possible candidates, a solution has only few significant (non-zero) wavelet coefficients. ie. The image can be represented by lesser number of coefficients.

The figure below shows a qualitative or visual comparison of restoration result of various regularization methods discussed above in the case of a T2 image degraded with a PSNR of 30dB [7]. Here takes three regularization concepts tikhonov regularization, total variation and wavelet approach for result analysis.



**Fig 3: Visual comparison of different regularization with respect to T2 image [7]**

From the survey it is seen that wavelet is competitive with TV regularization both in terms of SNR and computation time. It also appears that the prior corresponding to L1 regularization may be better adapted to images than the classical L2 term. Wavelet reconstruction usually outperforms TV for images that contain textured areas and/or many small details. Even though the hybrid method accommodates more complex procedures to restore the image, it results in negligible improvement in restoration both qualitatively and

quantitatively. Table below shows the comparison of different regularization methods.

From the survey it is seen that wavelet is competitive with TV regularization both in terms of SNR and computation time. It also appears that the prior corresponding to L1 regularization may be better adapted to images than the classical L2 term. Wavelet reconstruction usually outperforms TV for images that contain textured areas and/or many small details. Even though the hybrid method accommodates more complex procedures to restore the image, it results in negligible improvement in restoration both qualitatively and quantitatively. Table below shows the comparison of different regularization methods.

**Table 2: SNR improvement with respect to various priors**

METHOD	Image Analyzed	Degraded Image SNR	Restored Image SNR
Tikhonov	Boat image	PSNR=11.dB	ISNR=15.0dB
Total Variation	Boat image	SNR=11.2dB	SNR=17.8dB
Wavelet	Boat image	SNR=11.2dB	SNR=18.0dB
Hybrid	Boat image	SNR=11.2dB	SNR=18.8dB

## 7. DISCUSSION AND CONCLUSION

In the imaging applications, it often happens that images are subject to various imperfections and/or physical limitations such as noise and blurs generally so that the resulting image is not what we desire. To reduce noise amplification via common methods of inverse operations, regularization in the Bayesian framework with the concept of Maximum A posteriori Probability (MAP) is a first class approach to allow realistic restoration with preserving fine details of image preferably edges. In the Bayesian framework there is a good freedom in selection of priors.

Earlier works were focused on different regularization methods both piecewise and unbroken and they still have need for lots of improvements in the contest of preserving edges while getting rid of noise effects. The first method was focused on the Tikhonov regularization scheme in which L2 based prior are focused. Due to it's over smoothness another method called Total Variation is developed based on the variational difference between images. This TV concept aim to the development of sparsity concept which guided to the development of wavelet method in which the image domain. All the recent works were developed in the wavelet basis. But this method still has some smoothing effect. Nowadays edge preserving restorations are focusing on how to implement sparsity in the pixel domain rather than wavelet to achieve a better restoration. This method is assumed to be competent both qualitatively and quantitatively.

## **8. REFERENCES**

- [1] Javier.Mateos, W.Tom. E. Bishopi, Rafael Molina and Aggelos. K. Katsaggelos , \Local Bayesian Image Restoration Using Variational MethodsS And Gamma-Normal Distributions," IEEE Transactions On Image Processing, 2009.
- [2] Masayuki Tanaka,Takafumi Kanda and Masatoshi Okutomi, \Progressive MAP-Based Deconvolution with Pixel-Dependent Gaussian Prior," in 2010 International Conference On Pattern Recognition , 2010
- [3] S. Derin Babacan,Rafael Molina and Aggelos .K. Katsaggelos, \Variational Bayesian Blind Deconvolution Using A Total Variation Prior," in IEEE Transactions On Image Processing, 2007.
- [4] M. Dirk Robinson,Cynthia. A. Toth, Joseph. Y. Lo, and Sina Farsiu, \Efficient Fourier-Wavelet Super-Resolution," in IEEE Transactions On Image Processing, vol. 19, no 10, OCT 2010.
- [5] Nelly Pustelnik,Caroline Chaux and Jean-Christophe Pesquet, \Parallel Proximal Algorithm for Image Restoration Using Hybrid Regularization," in IEEE Transactions On Image Processing, vol. 20, no 9,SEP 2011
- [6] David Humphrey, and David Taubman, \A Filtering Approach to Edge Preserving MAP Estimation of Images," IEEE Transactions On Image Processing,vol. 20, no. 5, MAY 2011.
- [7] M. Guerquin-Kern, D. Van De Ville, C. Vonesch, J.C. Baritoux, K. P. Pruessmann and M. Unser, \Wavelet Regularized Reconstruction For Rapid MRI,"IEEE Transactions On Image Processing, MAY 2009.