Analytical Model of Intercell Interference in Relay Based Cellular OFDMA Networks

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ABSTRACT
In order to improve the coverage and capacity of next generation cellular networks, low cost relays are deployed in the area, where users do not get required Signal to Noise Ratio (SNR) from the base station (BS), especially at the cell edge. The deployment of relays not only reduces the infrastructure cost of setting up new BSs but also supports the rapidly growing number of subscribers. However introduction of Relays introduces additional interferences, which affects the system capacity. In this paper, we analyze this interference in Relay based Orthogonal Frequency Division Multiplexing Access (OFDMA) system. We present an analytical model to characterize the interference experienced by a particular user in a reference cell from all interfering cells irrespective of the position of user. We consider the effect of path loss, shadowing and fading on interference powers from various cells. Then, we determine the Cumulative Distribution Function (CDF) of interference.

General Terms
Inter Cell Interference modeling, Relay based OFDMA, Signal to Interference Ratio, CDF.

Keywords
Relays, OFDMA, ICI characterization, CDF, Outage probability

1. INTRODUCTION
In order to meet the throughput and coverage requirements of future generation cellular networks, with a rapid growth of the number of cellular subscribers, and the scarcity of frequency spectrum, new schemes are required. One solution is to decrease the cell radius to support the increasing number of subscribers per cell. But it requires large number of base stations (BS) per area thus increases the infrastructure costs. Also, smaller cell radius causes higher inter-cell interference (ICI); thereby system capacity degrades and thus requires better frequency planning techniques such as sectorization to minimize interference. An alternate solution is proposed in IEEE 802.16j standard that is the deployment of fixed or nomadic Relay Stations (RS).

RSs improve the network coverage to users, especially at the cell edge, where the signal quality is poor from BS, but being closer to the RS, they receive strong signal from the RS. Because of the improvement in signal quality experienced by users at the cell edge, these users require lesser resources from the BS. For example, in cellular OFDMA systems, the number of subcarriers required by the user is smaller when the MS is served via an RS. Thus, the same resources can be shared among a larger number of users resulting in overall capacity improvement. However, introduction of RSs causes additional interference, which affects the system capacity. In order to analyze the impact of interference on system capacity it is necessary to know the characteristic of interference with relays. In this paper, we model this interference in Relay based cellular OFDMA system.

The modeling of ICI and outage probability computation for cellular OFDMA has been recently considered in the literature. Traditionally, time consuming simulations are used to find the Signal to Interference Ratio (SIR) statistics. In some work either the large path loss [1] or multipath propagation [2] is taken which are incomplete for the study of real wireless environment. The authors in [2] calculate the capacity of Nakagami multipath fading channels assuming SNR to be Gamma distributed. In this the effect of shadowing and path loss is neglected. In [3], ICI resulting from multiple cells has been approximated by Gaussian and Binomial distribution. The results have been verified by simulations. The distribution of ICI for various positions of the user has been calculated in [4]. In this semi-analytical approach, impact of power allocation on downlink inter-cell interference has been presented. In [5], an iterative method is presented to calculate the distribution of ICI for a universal frequency reuse cellular OFDMA system with voice over IP traffic. In [6], it is shown that for random subcarrier allocation, the overall interference on all subcarriers will be the geometric mean of the interferences on individual subcarriers.

In [7], an analytic expression for CDF of SINR in multi-cellular system is derived with full transmit diversity using Alamouti scheme. In [8], the Probability Distribution Function (pdf) of interference power on a subcarrier is derived and then pdf of a sub-channel is determined by geometric mean of pdfs of interferences on every subcarrier present in a sub-channel. This pdf is used to calculate the outage probability of the user for a given position in the cell. However, this outage probability should be independent of user positions, therefore, it is necessary to consider the randomness of users. Also interference powers from various cells are not independent but they are correlated. This fact is considered in [9] for computation of CDF of ICI in single hop and two hop cellular OFDMA networks. However, no analytical model exists to characterize the interference experienced by a user when served by a RS, in cellular OFDMA system. In this paper, we propose an analytical model to determine the SIR experienced by a user and compute its Cumulative Distribution Function (CDF). We compute ICI for the users when served by the BS directly and served by BS and RS.

The remainder of the paper is organized as follows. Section II introduces the system model for the downlink of relay assisted cellular OFDMA network and gives the problem description. In section III, we present a model to characterize the ICI. The Cumulative Distribution Function (CDF) of ICI is derived in

IV In section V, we present the analytical and simulation results. Finally section VI concludes the paper.

2. SYSTEM MODEL

Consider the downlink of a relay based cellular OFDMA system with hexagonally shaped cells of radius R as shown in Fig. 1. Six RS s are added in the conventional single hop cellular network as shown in Fig.1. We define the reference cell as the combination of central sub-cell containing a BS and its surrounding six sub-cells containing RSs. The RSs are deployed in the network such that there is a good channel condition between BS and RS as Line of Sight (LOS) environment and the BS-MS link and RS-MS link are in non line of sight environment. All RSs are regenerative repeaters, so they decode the data received from BS and then forward to the target MS. When a user experiences SIR above threshold from BS, then it is served by BS, we call such users as direct users lying in base region and transmission takes place in single hop only. However, if a user experiences SIR above threshold from RS, then it is served by RS, we call such users as hopped users lying in relay region and transmission takes place in two hops via BS and RS.

We consider the interference from the first tier surrounding the reference cell. The reference cell and six neighboring cells in first tier are numbered as 0 and 1, 6 respectively. A network with universal frequency reuse is considered, i.e. all the cells use the same spectrum. This relaxes the advanced frequency planning otherwise necessary among different cells. However in network with frequency reuse of one, users will experience interference from RS and BS of neighboring cells. We analyze this interference on BS-MS,

BS-RS and RS-MS link and compute their CDFs. A simple path loss based channel model is assumed in this paper. If the power transmitted by the BS is P, then a user at a distance d receives power P d−β, where β is path loss exponent. The signals coming from BSs experiences fading and shadowing (which follows a lognormal law with σ (dB) = 6 to 12 dB. All paths are assumed to be independently faded and shadowed. We use the natural base for lognormal Random Variables.

3. INTERCELL INTERFERENCE MODELING

In this section, we derive the CDF of ICI on BS-MS link. SIR on BS-MS link can also be written as,

3.1 Group1: Users present in base region, (Direct users)

When a user enters in the base region, it is served by the BS directly and transmission takes place via single hop only. The SIR for a user on a sub carrier in a reference cell on BS-MS link is expressed as,

$$\gamma_{bs-ms} = \frac{P_{bm} r_{bm}^{-\beta} \xi \alpha^2}{\sum_{i=1}^{N} P_{bm}\, d_{ibm}^{-\beta} \xi_i \alpha_i^2}.$$

Where, P_{bm} is the power transmitted by BS to user (MS), r_{bm} is the distance between BS and MS present in the inner region, d_{ibm} is the distance between i^{th} interfering BS (which is using the same subcarrier as used by the reference BS) and MS, β is the path loss exponent and N is the no. of interferers from neighboring cells, $d_{i\,bm} = \sqrt{(x-x_i)^2 + (y-y_i)^2}$, (x, y) and (x_i, y_i) are the coordinates of MS in base region and the i^{th} interfering BS respectively. ξ_i and α_i^2 represent shadowing and fading between i^{th} interfering BS and reference user. ξ and α represents the shadowing and fading between reference BS and reference user. N is the number of interferers in the no. of tiers considered. Fig.1 illustrates the ICI on one subcarrier to MS in base region from all interfering BS.

3.2 Group2: Users present in relay region, (Hopped users)

When a user enters in one of the relay regions, it is served by the RS and BS via BS-RS link and RS-MS link and transmission takes place in two hops. The SIR for user on a subcarrier in a reference cell on BS-RS link is expressed as,

$$\gamma_{bs-rs} = \frac{P_{br} r_{br}^{-\beta} \xi \alpha^2}{\sum_{i=1}^{N} P_{br} d_{ibr}^{-\beta} \xi_i \alpha_i^2}.$$

Where, P_{br} is the power transmitted by BS to RS, r_{br} is the distance between BS and RS in which MS is present , d_{ibr} is the distance between i^{th} interfering BS (which is using the same subcarrier as used by the reference BS) and RS serving the MS, β is the path loss exponent and N is the no. of interferers from neighboring cells, $d_{i\,br} = \sqrt{(x-x_i)^2 + (y-y_i)^2}$, (x, y) and (x_i, y_i) are the coordinates of MS in relay region and the i^{th} interfering BS respectively. Fig.1 illustrates the ICI on one subcarrier to serving RS from all interfering BS. Similarly, the SIR for user on a subcarrier in a reference cell on RS-MS link is expressed as,

$$\gamma_{rs-ms} = \frac{P_{rm} r_{rm}^{-\beta} \xi \alpha^2}{\sum_{i=1}^{N} P_{rm}\, d_{irm}^{-\beta} \xi_i \alpha_i^2}.$$

Where, P_{rm} is the power transmitted by RS to MS present in relay region, r_{rm} is the distance between RS and MS, d_{irm} is the distance between i^{th} interfering RS (which is using the same subcarrier as used by the serving RS) and MS present in relay region, Fig.1 illustrates the ICI on one subcarrier to MS in relay region from all interfering RS.

4. COMPUTATION OF CDF OF INTERCELL INTERFERENCE

In this section, we derive the CDF of ICI on BS-MS link. SIR on BS-MS link can also be written as,

$$\gamma_{bs-ms} = \frac{1}{\sum_{i=1}^{N} \left(\frac{d_{ibm}}{r_{bm}}\right)^{-\beta} \frac{\xi_i}{\xi} \left(\frac{\alpha_i}{\alpha}\right)^2} = \frac{1}{I_{bs-ms}}.$$

Since, MSs are uniformly distributed in reference cell and position of interfering BSs is fixed, thus all d_{ibm}s are correlated random variables. To calculate the CDF of γ_{bs-ms} , we compute the CDF of total interference to signal ratioI_{bs-ms}. We solve it by the method of moments in two steps. Since d_{ibm} s are not independent therefore,

First, we group the interference to signal ratio I_{bs-ms} into two components, B_is, which are having different distributions and C_is, which are iid random variables

$$I_{bs-ms} = \sum_{i=1}^{N} B_i C_i$$

$$B_i = \left(\frac{d_{ibm}}{r_{bm}}\right)^{-\beta} = \left[\frac{(x-x_i)^2+(y-y_i)^2}{x^2+y^2}\right]^{\frac{-\beta}{2}} \text{ and}$$

$$C_i = \frac{\xi_i}{\xi}\left(\frac{\alpha_i}{\alpha}\right)^2$$

and we find the exact moments of the interference to signal ratio I_{bs-ms}. Since the subscript is the notation of link, the procedure to compute CDF for all links is same, so we drop the subscript without loss of generality. The first and second order moments of I_{bs-ms} are calculated as,

$$E[I] = E[C_i]E\left[\sum_{i=1}^{N} B_i\right] \text{ And}$$

$$E[I^2] = E\left[\sum_{i=1}^{N} B_i^2\right]E[C_i^2] + (E[C_i])^2\left[E\left(\sum_{i=1}^{N} B_i\right)^2 - E\left(\sum_{i=1}^{N} B_i^2\right)\right]$$

The last two terms of the above equation can be solved as follows. Let, $M = E\left[\sum_{i=1}^{N} B_i^2\right]$ Where, expectation is taken over all the position of MS, being uniformly distributed over hexagon cell H_0 centered at (0, 0) and area is $3\sqrt{3}/2$. These integrals are evaluated separately for each interfering BS and then summed for all integrals to get *M.*

$$= \sum_{i=1}^{N} \frac{2}{3\sqrt{3}} \iint_{x,y\varepsilon H_0} \left(\sqrt{\frac{(x-x_i)^2+(y-y_i)^2}{x^2+y^2}}\right)^{-2\beta} dx\, dy.$$

Let, $A_s = E\left(\sum_{i=1}^{6} B_i\right)^2$, Since the distance between a MS and the interfering fixed BSs are not independent, therefore can not be separated into a sum of terms, therefore it is integrated for the entire geometry as follows ,

$$= \frac{2}{3\sqrt{3}} \iint_{x,y\varepsilon H_0} \left(\sum_{i=1}^{6}\left(\sqrt{\frac{(x-x_i)^2+(y-y_i)^2}{x^2+y^2}}\right)^{-\frac{\beta}{2}}\right)^2 dx\, dy$$

We have obtained the first and second moments of $I = I_{bs-ms}$, and its distribution can be approximated by lognormal distribution with parameter $(\mu_{bs-ms}, \sigma_{bs-ms}^2)$, as it is often used to model the total interference in cellular systems.

In general n[th] moment can be written as,

$$E[I^n] = e^{n\mu_{bs-ms}+\frac{n^2}{2}\sigma_{bs-ms}^2}$$

On inverting we obtain,

$$\mu_{bs-ms} = 2\ln E[I] - \frac{1}{2}\ln E[I^2] \text{ and}$$

$$\sigma_{bs-ms}^2 = -2\ln E[I] + \ln E[I^2]$$

Using μ_{bs-ms}, σ_{bs-ms}^2 we find the distribution as,

$$F_{I_{bs-ms}}(x) = N\left[\frac{\ln x - \mu_{bs-ms}}{\sigma_{bs-ms}}\right], \quad x > 0$$

Here, *N(x)* is the standard normal cumulative distribution function. Similarly, CDF of $I = I_{bs-rs}$ and $I = I_{rs-ms}$ can be determined and are given as

$$F_{I_{bs-rs}}(x) = N\left[\frac{\ln x - \mu_{bs-rs}}{\sigma_{bs-rs}}\right], \quad x > 0$$

$$F_{I_{rs-ms}}(x) = N\left[\frac{\ln x - \mu_{rs-ms}}{\sigma_{rs-ms}}\right], \quad x > 0$$

We have thus determined the distribution of interference to signal ratio on a subcarrier for the MS present in base region served by the BS directly and MS present in one of the relay region served by BS via RS.

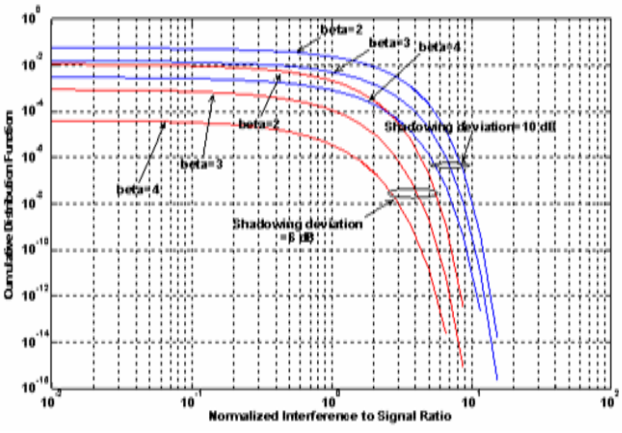
5. NUMERICAL AND SIMULATION RESULTS

We validate our analytical results by Monte-Carlo simulations. We calculate the M and As parameters and use them to determine the first and second order moments. We observe the downlink interference for MS present in base region on BS-MS link and MS present in relay region on BS-RS link and RS-MS link. We plot the normalized interference CDF in log-log scale for both direct and hopped users in relay based cellular OFDMA system. Fig. 2 and 3 show the plot of CDF of interference to signal ratio for direct users (on BS-MS link) and hopped users on (BS-RS link and RS-MS link) for the cases, β = 2, 3 and 4 and the shadowing parameter over the range of 6-10 dB (σ =1.382 to 2.763). Fig. 4 and 5 show the plot of CDF of interference to signal ratio for direct users (on BS-MS link) and hopped users on (BS-RS link and RS-MS link) with and without power control.

6. CONCLUSION

. We derived a general expression for CDF of interference to signal ratio on BS-MS, BS-RS and RS-MS links in relay based cellular OFDMA network analytically and determined CDF by including Log normal shadowing and Rayleigh fading. The Rayleigh fading model can be replaced by any other fading model. The computation is very simple as it involves only numerical integrations which depend upon the position of the MS and path loss exponent β. They are calculated once and used to calculate exact moments. This method of modeling ICI is a tool to understand interference in real wireless scenario and to find mitigation techniques. It is also used to compute outage and blocking probability accurately and hence capacity of the system.

7. ACKNOWLEDGMENTS



. Fig.2: CDF of Normalized interference to Signal ratio for direct users (served by BS)

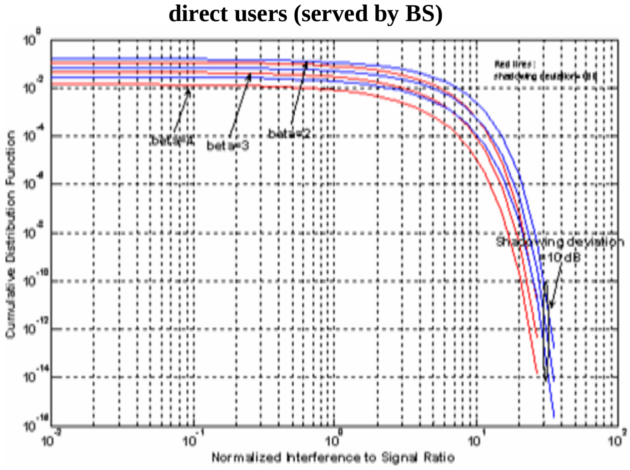


Fig.3: CDF of Normalized interference to Signal ratio for hopped users (served by Relays)

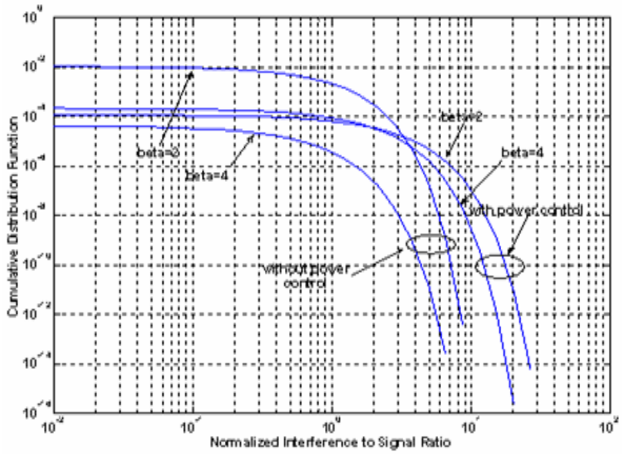


Fig.4: CDF of Normalized interference to Signal ratio for direct users with and without power control

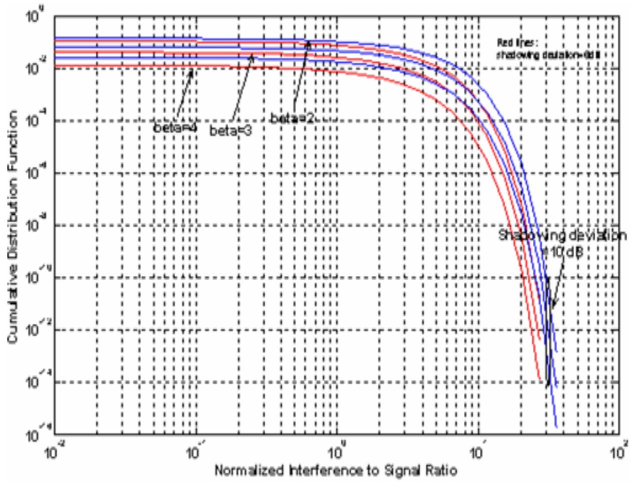


Fig.5: CDF of Normalized interference to Signal ratio for hopped users with and without power control

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