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Enumeration of Basic Hamilton Cycles in the Mangoldt Graph

Levaku Madhavi
Assistant Professor
Department of Applied Mathematics
Yogi Vemana University-Kadapa-516003
Andhra Pradesh, India

Tekuri Chalapati
Department of Mathematics
Sree Vidyanikethan Engineering
College, Tirupati – 517 502
Andhra Pradesh, India

ABSTRACT
The Mangoldt graph M_n is an arithmetic function, namely, Mangoldt function $\wedge(n)$, $n \geq 1$ an integer. In this paper the notion of a basic Hamilton cycles in M_n is introduced and their number is enumerated.

Keywords
Mangoldt Graph, Hamilton Cycle, Basic Hamilton Cycles.

AMS(MOS) Subject Classification: 68R05

Index Terms
Graph Theory, Discrete Mathematics

1. INTRODUCTION
Nathanson [11] introduced the concepts of Number Theory, particularly, the theory of congruences into Graph Theory, which paved the way for the emergence of **Arithmetic Graphs.** Maheswari and Madhavi[8,9,10] introduced the Mangoldt graph M_n which is an arithmetic graph associated with the Mangoldt function $\wedge(n)$, $n \geq 1$, an integer. It is shown that M_n is connected and neither bipartite nor a tree for $n \geq 6$. Further they have studied the vertex domination and edge domination and gave a formula for the number of triangles In this study the Hamiltonian nature of M_n is established for various forms of n by exhibiting the Hamilton cycles in the general setup and using these Hamilton cycles the notion of basic Hamilton cycle is introduced in M_n and their number is enumerated.

2. HAMILTONIAN PROPERTY OF THE MANGOLDT GRAPH MN

Definition 2.1: Let $n \geq 1$ be an integer. The **Mangoldt function** $\wedge(n)$, is defined as follows:

$$\Lambda(n) = \begin{cases} \log p, if\ n\ is\ a\ power\ of\ prime \\ 0, \qquad otherwise \end{cases}$$

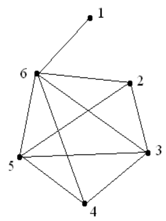
For example, $\wedge(4) = \wedge(2^2) = \log 2$. $\wedge(6) = 0$

and $\wedge(8) = \wedge(2^2) = \log 2$.

Definition 2.2[1]: Let $n \geq 1$ be an integer. The **Mangoldt graph** M_n is the graph whose vertex set is { 1, 2, …, n } and the edge set is $\{ (x , y) : \wedge (x . y) = 0 , 1 \leq x, y \leq n$ and $x \neq y$.

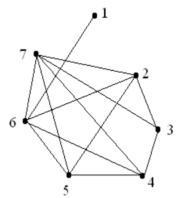
That is, the vertices x and y of are adjacent if and only if x . y is not a power of prime. Clearly M_n is a simple undirected graph without loops.

Example 2.3: The graphs M_6 and M_7 are given below:



M_6

Fig 2.2.1



M_7

Fig 2.2.2

Lemma 2.4: Let $n \geq 1$ be an integer. For all vertices u, $1 < u < n$, (u, u + 1) is an edge of M_n . .

Proof: For all vertices M_n in, one of u and u + 1 is even and the other odd. So u (u + 1) is not a power of a single prime so that there is an edge between u and u + 1. ■

It is evident that the vertex 1 is an isolated vertex in the graphs M_1 , M_2 , M_3, M_4 and M_5 and thus these graphs are not Hamiltonian. In M_6 , M_7 , M_8 and M_9, the vertex 1 is adjacent only to the vertex 6 and hence its degree is one. So the vertex 1 does not belong to any cycle in these graphs, as the degree of a vertex in a cycle must be two. Thus these graphs do not contain Hamilton cycles and hence they are not Hamiltonian. So M_n is not Hamiltonian for $n < 10$. In the following it is established that for $n \geq 10$ the graph M_n is Hamiltonian. The following number theoretic result is needed.

Lemma 2.5 : For an integer $m > 1$, (m-1) (m+1) is not a power of a single prime.

Proof: If m is odd then m – 1 and m + 1 are both even so that m – 1 = 2r and m + 1 = 2r + 2 = 2(r + 1) for some positive integer r. So $(m – 1) (m + 1) = 2^2 r (r + 1)$ and this is not a power of single prime since one of r and r + 1 is odd the other even.

If m is even m – 1 and m + 1 are both odd. Suppose $(m – 1) (m + 1) = q^l$ for some odd prime q and integer $l > 1$. Then $m – 1 = q^s$ and $m + 1 = q^t$ for some integers s, t, $s < t$, $s \geq 1$ and $s + t = l$. Also we have $q^t – q^s = 2$ with $s < t$. Since $q > 2$, $q^t – q^s = 2$ with $s < t$. Since $q > 2$, $q^t – q^s = q^s (q^{t-s} – 1) > 2$, for all values of q, t, s which is a contradiction to $q^t – q^s = 2$. So (m – 1) (m + 1) is not a power of a single prime.

Theorem 2.6: For $n \geq 10$ and $n = 2^r$, r is a positive integer, M_n is Hamiltonian.

Proof : Let $n \geq 10$ and $n = 2^r$, r a positive integer. There is no edge between 1 and 2 as well as 1 and n in M_n since $1 \times 2 = 2$ and $1 \times n = 2^r$ are powers of the prime 2.

For positive integers l and m, $l \neq m$ which are less than n and not power of a single prime, consider the following vertex sequence in M_n.

(2, 3, 4,…., $l - 1$, $l + 1$, …., m – 1, m +1 , ….., n – 3, n – 2, n, n – 1 , $\boxed{l,\ 1,\ m}$, 2)

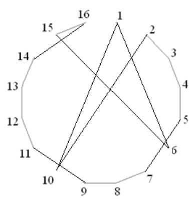
By the Lemma 2.4, there exist an edge between u and u + 1 for all vertices u, 1 < u < n, in M_n . By the Lemma 2.6,($l - 1$) ($l + 1$) and (m – 1) (m + 1) are not powers of a single prime. So there is an edge between $l - 1$ and $l + 1$ as well as m – 1 and m + 1 . Also $n (n - 2) = 2^r (2^r - 2) = 2^{r+1}(2^{r-1} - 1)$. This is a product of an even number 2^{r+1} and an odd number 2^{r+1} and an odd number $2^{r-1} - 1$ so that it is not a power of a single prime. So there is an edge between n – 2 and n.

Since each of l and m is not a power of a single prime. (n – 1) x l , l x 1 , l x m and m x 2 are also not powers of a single prime. Therefore (n – 1, l), (l, 1), (l, m) and (m, 2) are edges in M_n . Thus the vertex sequence (2, 3, 4,., $l - 1$, $l + 1$,…., m – 1, m +1 , …., n – 3, n – 2 , n, n – 1 , $\boxed{l,\ 1,\ m}$,2) is a Hamilton cycle in M_n and hence M_n is Hamiltonian.

Example 2.7: For $n = 2^4 = 16$ the vertex sequences

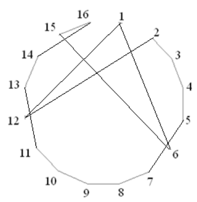
(i) (2,3,4,5,7,8,9,11,12,13,14,16,15,$\boxed{6,1,10}$, 2)
(ii) *(2,3,4,5,7,8,9,10,11,13,14,16,15,*$\boxed{6,1,12}$, 2)

are Hamilton cycles in M_{16}. The graphs of these Hamilton cycles are given below.



Hamilton cycle (i)

Fig. 2.2.3



Hamilton cycle (ii)

Fig. 2.2.4

Theorem 2.8: For $n \geq 10$ and $n = p^r$, p is a prime, $p \neq 2$ and r a positive integer, the graph M_n is Hamiltonian.

Proof: For $n \geq 10$ and $n = p^r$, p is a prime, $p \neq 2$ and r a positive integer. There is no edge between 1 and 2 since 1 x 2 is a power of 2 and there is no edge between 1 and n since $1 \times n = p^r$, p a prime.

Let l and m , $l \neq m$ be a positive integers which are not powers of a single prime. Evidently $l \neq 1$, n and $m \neq 1$, n. Consider the following vertex sequence in M_n .

(2, 3, 4,…., $l - 1$, $l + 1$, …., m – 1, m +1 , ….., n – 2, n – 1 , n, $\boxed{l,\ 1,\ m}$,2) .

By the Lemma 2.4 and 2.5, there edges in M_n between the pairs ($l - 1$, $l + 1$), (m – 1, m + 1) and (u, u + 1), $u \in M_n$, 1 < u < n. Also there is an edge between n and 2 since $2 \times n = 2p^r$, $p \neq 2$.

Further n x l , l x 1, l x m and m x 2 are not powers of a single prime so that (n , l), (l, 1), (l, m) and (m , 2) are edges in M_n . So the vertex sequence (2, 3, 4,, $l - 1$, $l + 1$, …., m – 1, m +1 , ….., n – 2, n – 1 , n, $\boxed{l,\ 1,\ m}$,2) is a Hamilton cycle in M_n and M_n is Hamiltonian.

Example 2.9: For $n = 5^2 = 25$ the vertex sequences

(i) (2,3,4,5,7,8,9,10,11,12,13,14,15,16,17,18,19,21,22, 23,24,25,$\boxed{6,1,20}$, 2)
(ii) (2,3,4,5,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21, 22,23,25,$\boxed{6,1,24}$, 2)

are Hamilton cycles in M_{25}.

Theorem 2.10 : For $n \geq 10$ and n not a power of a single prime the graph M_n is Hamiltonian.

Proof : Let $n \geq 10$ and n not a power of a single prime. Let l and m, $l \neq m$ be positive integers less than or equal to n, which are not powers of a single prime. The following three cases will arise.

Case (1) : Let $l \neq m$ and $m \neq n$. As in the Theorem 2.8 one can see that the vertex sequence

(2, 3, 4,…., $l - 1$, $l + 1$, …., m – 1, m +1 , ….., n – 2, n – 1 , n, $\boxed{l,\ 1,\ m}$,2) is a Hamilton cycle in M_n.

Case (2) : Let $l = n$ and $m \neq n$. The following two sub cases will arise.

Subcase (1) : Let n – 1 be not a power of 2. It is easy to see that the vertex sequence

(2, 3, 4,…., m – 1 , m + 1 , …., n – 2, n – 1 , $\boxed{n,\ 1,\ m}$,2) is a Hamilton cycle in M_n.

Subcase (2) : Let n – 1 be a power of 2. Consider the following vertex sequence in M_n (2, 3, 4,…., m – 1 , m + 1 ,., n – 3, n – 1, n – 2 , $\boxed{n,\ 1,\ m}$,2).

Clearly the pairs (2, 3) , (3, 4) , …….(m – 1 , m + 1) ….., (n – 1 , n – 2), (n – 2 , n) (n , 1) , (l , m) and (m , 2) are adjacent in M_n . The vertices n – 2 and n – 1 are also adjacent if (n – 3) (n – 1) is not a power of a single prime. Suppose that (n – 3) (n – 1) is not a power of a single prime. Suppose that $(n - 3) (n - 1) = p^r$ for some prime p and r a positive integer. Then $n - 3 = p^s$ for some positive integers s , t and s < t. This gives $p^t - p^s = (n - 1) - (n - 3) = 2$ which is true only for the least values of p, s and t namely, p = 2 , s = 1 and t = 2 . But for p = 2 , s = 1 and t = 2 we have $n - 3 = p^s = 2$, or, n = 5 which is a contradiction to $n \geq 10$. So (n –3)(n – 1) is not a power of a single prime and thus n – 2 and n – 1 are adjacent in M_n. These show the vertex sequence.

(2, 3, 4,., m – 1 , m + 1 , ., n – 3, n – 1, n – 2, $\boxed{n,\ 1,\ m}$,2) is a Hamilton cycle in M_n.

Case (3) : Let $l \neq n$ and m = n .

Subcase (1) : Let n – 1 be not a power of 2. As in Subcase (1) of Case (2), it is easy to see that the vertex sequence

(2, 3, 4,…., $l - 1$, $l + 1$, ., n – 3, n – 3 , n – 1 $\boxed{l,\ 1,\ n}$, 2) is a Hamilton cycle in M_n.

Subcase (2) : Let n – 1 be a power of 2. Again as in Subcase (2) of Case (2), one can see that the vertex sequence

(2, 3, 4,., $l - 1$, $l + 1$, …., n – 3, n – 1 , n – 2 , $\boxed{l,\ 1,\ n}$, 2) is a Hamilton cycle in M_n . So M_n is Hamiltonian in this case also.

Example 2.11 : For n = 15 the vertex sequences

(i) (2,3,4,5,7,8,9,11,12,13,14,15,$\boxed{6,1,10}$, 2)
(ii) (2,3,4,5,6,7,8,9,11,12,13,14,$\boxed{15,1,10}$, 2)

are Hamilton cycle in M_{15}.

3. ENUMERATION OF BASIC HAMILTON CYCLES IN Mn

In this Section the concept of basic Hamilton cycle in the Mangoldt graph M_n is introduced and the number of basic Hamilton cycles in M_n is determined for various forms of n.

Definition 3.1: Let $n \geq 10$ be an integer. A Hamilton cycle of the form

(.., $l-1$, $l+1$,….., m – 1, m + 1, …., $\boxed{l, 1, m}$,….) where l and m are positive integers less than are equal to n, which are not powers of a sing prime, is called a **basic Hamilton cycle** in the Mangoldt graph M_n .

The Hamilton cycles given in the Theorem 2.6, 2.8 and 2.10 are examples of the basic Hamilton cycles in M_n for various forms of n.

Lemma 3.3 : Let n be a positive integer. The number N of positive integers less than or equal to n which are not powers of a single prime is equal to $n - (\alpha_1 + \alpha_2 + . + \alpha_k + 1)$, where $p_1 < p_2 < \ldots < p_k \leq n$ are primes and $\alpha_1, \alpha_2, \ldots, \alpha_k$ are the largest positive integers such that $p_i^{\alpha}i \leq n$, $l \leq i \leq k$.

Proof: Let n be a positive integer and let $p_1 < p_2 < \ldots < p_k$ be primes and $\alpha_1, \alpha_2, \ldots, \alpha_k$ be the largest positive integers such that $p_i^{\alpha}i \leq n$, $l \leq i \leq k$, the powers of the prime p_i which are less than or equal to n are $p_i^1, p_i^2, \ldots, p_i^{\alpha}i$, and their number is equal to α_i . Deleting these $\alpha_1, \alpha_2, \ldots, \alpha_k$ number of positive integers, which are powers of a single prime from 1, 2, 3,…..,n we get 1 and the positive integers less than or equal to n which are not powers of a single prime. So number of positive integers less than or equal to n which are not powers of a single prime equal to $n - (\alpha_1, \alpha_2, \ldots, \alpha_k + 1) = N$.

Theorem 3.4 : For $n \geq 10$ and $n = 2^r$, r a positive integer, the number of basic Hamilton cycles in the Mangoldt graph M_n is equal to $(n-3) \times {}^{N}P_2$.

Proof: Let $n \geq 10$ and $n = 2^r$, r a positive integer. By the Theorem 2.6, the cycle

(2, 3, 4,…., $l-1$, $l+1$, …., m – 1, m + 1 , n – 3, n – 2, n, n – 1 , $\boxed{l, 1, m}$, 2) …… (3.1)

is a basic Hamilton cycle in M_n , where l and m, are positive integers less than n which are not powers of a single prime.

Clearly $l \neq n$ and $m \neq n$. By the Lemma 5.4.3 the number of positive integers $\leq n$ which are not powers of a single prime is equal to $N = n - (\alpha_1 + \alpha_2 + \ldots + \alpha_k + 1)$ where α_i , $1 \leq i \leq k$, are positive integers such that $p_i^{\alpha}i \leq n$ for primes $p_1 < p_2 < \ldots$, $p_k \leq n$.

For every choice of l and m in triad (l, 1, m) from this collection of N positive integers which are not powers of a single prime there is a basic Hamilton cycle of the form (3.1). There are ${}^{N}P_2$ choices for l and m from the above N positive integers. So the number of basic Hamilton cycles of the form (3.1) in the Mangoldt graph M_n is equal to ${}^{N}P_2$.

Taking any one these ${}^{N}P_2$ basic Hamilton cycles and replacing the triad (l, 1, m) in any one of the n – 3 places between 2,3 ; 3,4 ; …; $l-1$, $l+1$, ….; m – 1 , m + 1 ; ….; n – 2 , n ; n, n – 1 and n, 2 in this basic Hamilton cycle one gets the following n – 3 basic Hamilton cycles, since $(n-1)2 = (2^r - 1)2$ and this is a product of an odd number $2^r - 1$ and an even number 2 so that it is not a power of a single prime and there is an edge between n – 1 and 2.

(2, $\boxed{l, 1, m}$, 3, 4, …., $l-1$, $l+1$, …, m – 1, m + 1 ,…, n – 3, n – 2 , n, n – 1 , 2),

(2, 3, $\boxed{l,1, m}$, 4, …., $l-1$, $l+1$, …, m – 1, m + 1 , …, n – 3, n – 2 , n, n – 1 , 2),

……… ………….

(2, 3, 4,….,l -1, $\boxed{l,1, m}$, $l+1$, …, m – 1, m + 1 ,.. , n – 3, n – 2 , n, n – 1 , 2),

……….. …………...

(2, 3,4,…$l-1$, $l+1$, …, m – 1, $\boxed{l,1, m}$, m + 1 , …, n – 3, n – 2 , n, n – 1 , 2),

……… ………….

(2, 3, 4, …., $l-1$, $l+1$, …, m – 1, m + 1 , …, n – 3, n – 2 , n, n – 1, $\boxed{l, 1, m}$, 2).

So that the number of basic Hamilton cycles is in M_n equal to $(n-3) \times {}^{N}P_2$ ■

Example 3.5 : Consider the Mangoldt graph M_{16} . Here $n = 16 = 2^4$ and n – 1 = 15 which is not a prime power. Also $\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 1$. So the number of positive integers less than 16 which are not powers of a single prime and 1 is equal to N = 16 – (4 + 2 + 1 + 1 + 1 + 1 + 1) = 16 – 11 = 5 and the number of basic Hamilton cycles in the Mangoldt graph M_{16} is equal to

$(n-3) \times {}^{N}P_2 = (16-3) \times {}^{5}P_2 = 13 \times 5 \times 4 = 260$.

Theorem 3.6 : Let $n \geq 10$ be an integer such that $n = p^r$, p a prime $p \neq 2$ and r a pisitive integer. The number of basic Hamilton cycles in the Mangoldt graph M_n is equal to $(n-3) \times {}^{N}P_2$.

Proof: Let $n \geq 10$ be an integer such that $n = p^r$ p a prime , $p \neq 2$ and r positive integer. By the Theorem 2.8, the cycle

(2, 3, 4,…., $l-1$, $l+1$, …., m – 1, m + 1 , n – 2, n – 1, n, $\boxed{l, 1, m}$, 2) …….. (3.2)

is a basic Hamilton cycle in M_n , where l and m are positive integers less than or equal to n which are not primes of a single prime. Clearly $l \neq n$ and $m \neq n$.

As in Theorem 3.4 one can see that the number of basic Hamilton cycles of the form 3.2, in the Mangoldt graph M_n is equal to ${}^{N}P_2$ Choosing any one these ${}^{N}P_2$ basic Hamilton cycles and replacing the triad (l, 1, m) in any one of the n – 3 places between

2,3 ; 3,4 ; …; $l-1$, $l+1$, ….; m – 1 , m + 1 ; ….; n – 2 , n – 1; n – 1, n, and n, 2 in this basic Hamilton cycle one gets the following n –3 basic Hamilton cycles, since $n \times 2 = p^r \times 2$, $p \neq 2$ and this not a power of a single prime so that there is an edge between n and 2.

(2, $\boxed{l, 1, m}$, 3, 4, …., $l-1$, $l+1$, …, m – 1, m + 1 , …, n – 2 , n – 1, n, 2),

(2, 3, $\boxed{l,1, m}$, 4, …., $l-1$, $l+1$, …, m – 1, m + 1 , …, n – 2, n – 1 , n, 2),

……… ………….

(2, 3, 4,….,l -1, $\boxed{l,1, m}$, l + 1 , …, m – 1, m + 1 , …, n – 2 , n – 1 , n, 2),

……….. …………...

(2, 3,4,…l – 1 , l + 1, …, m – 1, $\boxed{l,1, m}$, m + 1 , …, n – 2, n – 1, n, 2),

……… ………….

(2, 3, 4, …., l – 1 , l + 1 , …, m – 1, m + 1 , …, n – 2 , n – 1, n, $\boxed{l, 1, m}$, 2).

So the number of cycles is in M_n equal to (n – 3) x ${}^{N}P_2$.

Example 3.7 : Consider the Mangoldt graph M_{25} . Here n = 25 = 5^2 . Also $\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = 1$. The number N of positive integers less than 25 which are not a powers of a single prime and 1 is given by

N = 25 –(4 + 2 + 2 1 + 1 + 1 + 1 + 1 + 1 + 1) = 25 – 15 = 10 thus the number of basic Hamilton cycles in the Mangoldt graph M_{25} is equal to

(n – 3) x ${}^{N}P_2$ = (25 – 3) x ${}^{10}P_2$ = 22 x 10 x 9 = 1980.

Theorem 3.8 : Let n ≥ 10 be an integer which is not a power of a single prime. The number of basic Hamilton cycles in the Mangoldt graph M_n is equal to (n – 3) x ${}^{N-1}P_2$ + 2(n – 2) (N – 1).

Proof: Let n ≥ 10 be an integer which is not a power of a single prime.

Let l ≤ n and m ≤ n, l ≠ m be positive integers which are not powers of a single prime.

Case (1) : Let l ≤ n and m ≤ n .

In the Theorem 2.10, we have seen that (2, 3, 4,…., l – 1 , l + 1 , …., m – 1, m + 1 , n – 2, n – 1, n, $\boxed{l, 1, m}$,2) … (3.3)

is a basic Hamilton cycle in M_n ,

By the Lemma 3.3, the number of positive integers ≤ n which are not powers of a single prime and 1 is equal to N = n – ($\alpha_1 + \alpha_2 + \ldots + \alpha_k$ + 1) where α_i , 1 ≤ i ≤ k, are positive integers < n (since l ≠ n and m ≠ n) which are not powers of a single prime is equal to N – 1 . The number of basic Hamilton cycle of the form (3.3) are got by choosing all possible l and m in triad (l, 1, m) from these N – 1 positive integers < n which are not powers of a sing prime. Thus the number of basic Hamilton cycles of the form (3.3) in the Mangoldt graph M_n is equal to ${}^{(N-1)}P_2$. Choosing any one of these ${}^{(N-1)}P_2$ basic Hamilton cycles and replacing the triad (l, 1, m) in any one of the n – 3 places between 2,3 ; 3,4 ; …; l – 1 , l +1 , ….; m – 1 , m + 1 ; ….; n – 2 , n – 1; n – 1 , n and n, 2 we get n – 3 basic Hamilton cycles. Thus the number of basic Hamilton cycles in M_n is equal to (n – 3) x ${}^{(N-1)}P_2$.

Case (2) : Let l = n and m ≠ n.

Subcase (1) : Let n – 1 be not a power of 2.

By the Subcase (1) of Case (2) of the Theorem 2.10, the cycle

(2, 3, 4,.., m – 1, m + 1 , .., n – 2, n – 1, $\boxed{n, l, m}$,2) … (3.4)

is a basic Hamilton cycle in M_n, where m ≠ n is a positive integer which is not a power of single prime.

Since the number of positive integers m < n which are not powers of a single prime is equal to N – 1, the total number of basic Hamilton cycle of the form (3.4) in Mangoldt graph M_n is equal to N – 1. It is easy to see that (since n – 1 is not a power of 2) by replacing the triad (n, l, m) in any one of the n – 2 places between 2,3 ; 3,4 ; …; m – 1 , m + 1 ; ….; n – 2 , n – 1; and n – 1, 2 of any one the N – 1 basic Hamilton cycles of the form (3.4) one gets the following n – 2 basic Hamilton cycles so that the number of basic Hamilton cycles in M_n is equal to (n – 2) (N – 1).

Subcase (2) : Let n – 1 be a power of 2.

By the Subcase (2) of Case (2) of the Theorem 2.10, the cycle

(2,3,4,., m – 1, m + 1 , ., n – 3, n – 1, n – 2, $\boxed{n, l, m}$,2)…(3.5) is a basic Hamilton cycle in M_n, where m ≠ n is a not a power of a single prime and n – 1 is a power of 2. As in Subcase (1) the number of basic Hamilton cycles of the form (3.5) in M_n is equal to N – 1. By replacing the triad (n, l, m) in any one of the n – 2 places between 2,3 ; 3,4 ; …; m – 1 , m + 1 ; ….; n – 3 , n – 1; n – 1 , n – 2 and n – 2, 2 of in any of the N – 1 basic Hamilton cycles of the form (3.5) one gets the following n – 2 basic Hamilton cycles, since n – 1 is power of 2, (n – 2)2 = (2^s – 1)2 for some integer s > 1 and this is not a power of a single prime since 2^s – 1 is odd and 2 is even so that (n – 2, 2) is an edge in M_n. The number of basic Hamilton cycles in M_n is equal to (n – 2)(N – 1).

Case (3) : Let l ≠ n and m ≠ n.

As in the Case (2) one can see that the number of basic Hamilton cycles in M_n is equal to (n – 2) (N – 1).

From these three cases it follows that when n is not a power of a single prime the number of basic Hamilton cycles in the Mangoldt graph M_n is equal to

(n – 3) x ${}^{(N-1)}P_2$ + (n – 2) (N – 1) + (n – 2) (N – 1)

= (n – 3) x ${}^{(N-1)}P_2$ + 2(n – 2) (N – 1).

Example 3.9 : Consider the Mangoldt graph M_{15}. Here n = 15 Also $\alpha_1 = 3$, $\alpha_2 = 2$, α_3 , $\alpha_4 = \alpha_5 = \alpha_6 = 1$. The number of positive integers less than or equal to 50 which are not powers of a single prime and 1 is given by

N = 15 – (3 + 2 + 1 + 1 + 1 + 1 + 1 + 1) = 15 – 10 = 5 and the number of basic Hamilton cycles in the Mangoldt graph M_{15} is equal to (n – 3) x ${}^{(N-1)}P_2$ + 2 (n – 2) (N – 1) = (15– 3) x ${}^{(5-1)}P_2$ + 2(15 – 2)(5 -1) = 12 x 4 x 3 + 2 x 13 x 4 = 144 + 104 = 248.

Conclusion : The basic Hamilton Cycles Enumerated in section 3 are not disjoint and their number is too large. It will be interesting to find out the number of **disjoint** basic Hamilton Cycles in the Mangoldt Graph M_n for given integer n ≥ 1, in which case their number will be less and easy to handle.

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