On Fuzzy Contra $g^*\alpha$-Continuous Functions

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ABSTRACT
In this paper we introduce and study the new class of functions called fuzzy contra $g^*\alpha$-continuous and fuzzy almost contra $g^*\alpha$-continuous mappings on fuzzy topological spaces. We investigate some of their properties. Also we provide the relation between fuzzy contra $g^*\alpha$-continuous mappings and fuzzy almost contra $g^*\alpha$-continuous mappings.

General Terms Fuzzy topology, fuzzy generalized closed set, fuzzy $g\alpha$-closed, fuzzy $g^*\alpha$-closed set, fuzzy contra $\alpha$-continuous function fuzzy $g^*\alpha$-continuous.

Keywords
Fuzzy contra $g^*\alpha$-continuous function, fuzzy Contra $g^*\alpha$-irresolute function, fuzzy almost contra $g^*\alpha$-continuous functions.

1. INTRODUCTION

In this paper we introduce and study the new class of mappings called fuzzy contra $g^*\alpha$-continuous and fuzzy almost contra $g^*\alpha$-continuous functions in fuzzy topological spaces. Also we define the relation between of fuzzy contra $g^*\alpha$-continuous and fuzzy almost contra $g^*\alpha$-continuous mappings.

2. PRELIMINARY
Let $X$ be a non empty set. A collection $\tau$ of fuzzy sets in $X$ is called a fuzzy topology on $X$ if the whole fuzzy set and the empty fuzzy set 0 is the members of $\tau$ and $\tau$ is closed with respect to any union and finite intersection. The members of $\tau$ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set.

The closure of a fuzzy set $\lambda$ (denoted by $cl(\lambda)$ ) is the intersection of all fuzzy closed which contains $\lambda$. The interior of a fuzzy set $\lambda$ (denoted by $int(\lambda)$ ) is the union of all fuzzy open subsets of $\lambda$. A fuzzy set $\lambda$ in $X$ is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:
(i) semi-open fuzzy set [1] if $\lambda \subseteq cl(int(\lambda))$ and semi-closed fuzzy set if $int(cl(\lambda)) \subseteq \lambda$.
(ii) pre-open fuzzy set [4] if $\lambda \subseteq int(cl(\lambda))$ and pre-closed fuzzy set if $cl(int(\lambda)) \subseteq \lambda$.
(iii) $\alpha$-open fuzzy set [4] if $\lambda \subseteq int(cl(\lambda))$ and $\alpha$-closed fuzzy set if $cl(int(\lambda)) \subseteq \lambda$.
(iv) regular open fuzzy set [1] if $\lambda = int(cl(\lambda))$ and regular closed fuzzy set if $\lambda = cl(int(\lambda))$.

The $\alpha$-closure (resp. semi-closure, pre-closure) of a fuzzy set $\lambda$ in fuzzy topological space $(X, \tau)$ is intersection of all $\alpha$-closed (resp. semi-closed, pre-closed) fuzzy sets in $X$ containing $\lambda$ and is denoted by $\alpha = cl(\lambda)$ (resp. $scl(\lambda), pcl(\lambda)$).

Definition 2.2: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:
(i) generalized closed fuzzy set ($g\alpha$-closed) fuzzy set [2] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is open fuzzy set in $(X, \tau)$.
(ii) generalized $\alpha$-closed fuzzy set ($g\alpha\alpha$-closed) fuzzy set [2] if $acl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is open fuzzy set in $(X, \tau)$.
(iii) $g^*\alpha$-closed fuzzy set ($g^*\alpha$-closed) fuzzy set [8] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is open fuzzy set in $(X, \tau)$.
(iv) $g^*\alpha$-preclosed fuzzy set ($g^*\alpha\alpha$-closed) fuzzy set [3] if $pcl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is $g^*\alpha$-open fuzzy set in $(X, \tau)$.
(v) $g^*\alpha$-semiclosed fuzzy set ($g^*\alpha\alpha$-closed) fuzzy set [3] if $scl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is $g^*\alpha$-open fuzzy set in $(X, \tau)$.
(vi) $g^*\alpha$-alphaclosed fuzzy set ($g^*\alpha\alpha$-closed) fuzzy set [3] if $acl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and $\eta$ is $g^*\alpha$-open fuzzy set in $(X, \tau)$.

The complement of $g\alpha$-closed (resp. $g\alpha\alpha$-closed, $g^*\alpha\alpha$-closed) fuzzy sets are called fuzzy $g\alpha$-open (resp. $g\alpha\alpha$-open, $g^*\alpha\alpha$-open) fuzzy sets in fuzzy topological spaces.

Definition 2.3: A fuzzy topological space $(X, \tau)$ is called $T_{\alpha\alpha}$-space [6] if every $g\alpha\alpha$-closed fuzzy set is a closed fuzzy set in $X$.

Definition 2.4: A function $f$ from a fuzzy topological space $(X, \tau)$ to fuzzy topological space $(Y, \sigma)$ is called:
(i) fuzzy-contra continuous if $f^{-1}(\lambda)$ is fuzzy closed in $X$ for every fuzzy open set $\lambda$ of $Y$ [5].
3. FUZZY CONTRA $g\alpha$-CONTINUOUS FUNCTION

**Definition 3.1.** A function $f:X \to Y$ is called fuzzy contra $g\alpha$-continuous if $f^{-1}(\lambda) = \{x \in X | f(x) \in \lambda\}$ is closed in $X$ for every open set $\lambda \subset Y$.

**Theorem 3.2.** Every fuzzy contra $g\alpha$-continuous function is fuzzy contra $g\alpha'$-continuous function.

**Proof:** It follows from the fact that every fuzzy closed set is $g\alpha'$-closed set.

The converse of the above theorem need not be true as seen from the following examples.

**Example 3.3:** Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and $\lambda_1, \lambda_2$ be fuzzy sets in $X$ and $Y$, respectively. Let $\mu(x_1) = 0.5$, $\mu(x_2) = 0.3$, $\lambda(y_1) = 0.3$, and $\lambda(y_2) = 0.7$. Then $f$ is fuzzy contra $g\alpha$-continuous but not fuzzy contra $g\alpha'$-continuous.

**Theorem 3.4.** Every fuzzy contra $\alpha$-continuous mapping is fuzzy contra $g\alpha$-continuous function.

**Proof:** Straight forward and follows from the definitions.

The converse of the above theorem need not be true as seen from Example 3.2.

**Theorem 3.5.** Every fuzzy contra $g\alpha'$-continuous mapping is fuzzy contra $g\alpha$-continuous function as well as fuzzy $g\alpha'$-continuous.

**Proof:** Straight forward and follows from the definitions.

The converse of the above theorem need not be true as seen from Example 3.6 and Example 3.7.

**Example 3.6:** Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$. Let $\eta_1, \eta_2, \eta_3$ be fuzzy sets in $X$, $\mu$ be a fuzzy set in $Y$, defined as $\eta_1(x_1) = 0.2, \eta_1(x_2) = 0.3, \eta_2(x_1) = 0.3, \eta_2(x_2) = 0.5, \eta_3(x_1) = 0.6, \eta_3(x_2) = 0.7, \mu(y_1) = 0.3, \mu(y_2) = 0.5$. Let $\tau_{x_1} = \{0, \eta_1, \eta_2, \eta_3, 1\}$ and $\tau_{x_2} = \{0, \mu, 1\}$ be fuzzy topologies on sets $X$ and $Y$ respectively. Then $f$ is fuzzy contra $g\alpha$-continuous but not fuzzy contra $g\alpha'$-continuous.

**Example 3.7:** Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$. Let $\eta_1, \eta_2, \eta_3$ be fuzzy sets in $X$, $\mu$ be a fuzzy set in $Y$, defined as $\eta_1(x_1) = 0.1, \eta_1(x_2) = 0.2, \eta_2(x_1) = 0.4, \eta_2(x_2) = 0.6, \eta_3(x_1) = 0.7, \eta_3(x_2) = 0.7, \mu(y_1) = 0.2$ and $\mu(y_2) = 0.5$. Let $\tau_{x_1} = \{0, \eta_1, \eta_2, \eta_3, 1\}$ and $\tau_{x_2} = \{0, \mu, 1\}$ be fuzzy topologies on sets $X$ and $Y$ respectively. Let $f:X \to Y$ defined as $f(x_i) = y_i$, $i = 1, 2$. Then $f$ is fuzzy contra $g\alpha'$-continuous but not fuzzy contra $g\alpha$-continuous.

**Theorem 3.8.** If a function $f:(X, \tau) \to (Y, \sigma)$ is fuzzy contra $g\alpha'$-continuous and $(X, \tau)$ is fuzzy $T_\alpha^*$-space, than $f$ is fuzzy contra $\alpha$-continuous.

**Proof.** Let $\lambda$ be open fuzzy set in $X$. Then $f^{-1}(\lambda)$ is $g\alpha'$-closed fuzzy set in $X$. Since $X$ is fuzzy $T_\alpha^*$-space, $f^{-1}(\lambda)$ is closed fuzzy set in $X$. Thus $f$ is fuzzy contra $\alpha$-continuous function.

**Theorem 3.9.** If a function $f:(X, \tau) \to (Y, \sigma)$ is fuzzy contra $\alpha$-continuous and $(X, \tau)$ is fuzzy $T_\alpha^*$-space, than $f$ is fuzzy contra $g\alpha'$-continuous.

**Proof.** Let $\lambda$ be open fuzzy set in $Y$. Then $f^{-1}(\lambda)$ is $\alpha$-closed fuzzy set in $X$. Since $X$ is fuzzy $T_\alpha^*$-space, $f^{-1}(\lambda)$ is $g\alpha'$-closed fuzzy set in $X$. Thus $f$ is fuzzy contra $g\alpha'$-continuous function.

**Theorem 3.10.** If a function $f:(X, \tau) \to (Y, \sigma)$ is fuzzy contra $\alpha$-continuous and $(X, \tau)$ is fuzzy $T_\alpha^*$-space, than $f$ is fuzzy contra $\alpha$-continuous function.

**Proof.** Let $\lambda$ be open fuzzy set in $Y$. Then $f^{-1}(\lambda)$ is $\alpha$-closed fuzzy set in $X$. Since $X$ is fuzzy $T_\alpha^*$-space, $f^{-1}(\lambda)$ is closed fuzzy set in $X$. Every closed fuzzy set is $\alpha$-closed fuzzy set. Thus $f$ is fuzzy contra $\alpha$-continuous function.

**Theorem 3.11** Let $(X, \tau)$ and $(Y, \sigma)$ be two fuzzy topological spaces. The following statement are equivalent for a function $f:X \to Y$.

(i) $f$ is fuzzy contra $g\alpha'$-continuous.

(ii) $f^{-1}(\lambda)$ is $g\alpha'$-open fuzzy set in $X$ for each closed fuzzy set $\lambda$ in $Y$.

(iii) For each $x \in X$ and each closed fuzzy set $\lambda \in Y$ containing $f(x)$, there exist a $g\alpha'$-open fuzzy set $\eta$ in $X$ containing $x$ such that $f(\eta) \subset \lambda$.

(iv) For each $x \in X$ and open fuzzy set $\mu$ in $Y$ not containing $f(x)$, there exists a $g\alpha'$-closed fuzzy set $\theta$ in $X$ not containing $x$ such that $f^{-1}(\theta) \subset \mu$.

**Proof.**

(i) $\Rightarrow$ (ii). Let $\lambda$ be a closed fuzzy set in $(Y, \sigma)$. Then $1 - \lambda$ is fuzzy open. By (i), $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ is fuzzy $g\alpha'$-closed fuzzy set in $X$. So $f^{-1}(\lambda)$ is $g\alpha'$-open fuzzy set in $X$.

(ii) $\Rightarrow$ (i), proof as above.

(ii) $\Rightarrow$ (iii). Let $\lambda$ be any closed fuzzy set in $Y$ containing $f(x)$. By (ii), $f^{-1}(\lambda)$ is $g\alpha'$-open fuzzy set in $(X, \tau)$ and $x \in f^{-1}(\lambda)$. Take $\eta = f^{-1}(\lambda)$. Then $f(\eta) \subset \lambda$.

(iii) $\Rightarrow$ (ii). Let $\lambda$ be a closed fuzzy set in $Y$ and $x \in f^{-1}(\lambda)$. From (iii), there exists a $g\alpha'$-open fuzzy set $\eta$ in $X$ containing $x$ such that $f(\eta) \subset \lambda$. We have $f^{-1}(\lambda) = \bigcup_{\alpha \in f^{-1}(\lambda)} \eta$. Thus $f^{-1}(\lambda)$ is $g\alpha'$-open fuzzy set in $(X, \tau)$.

(iii) $\Rightarrow$ (iv). Let $\mu$ be any open fuzzy set in $Y$ not containing $f(x)$. Then $1 - \mu$ is a closed fuzzy set containing $f(x)$. By (iii) there exists a $g\alpha'$-open fuzzy set $\eta$ in $X$ containing $x$ such that $f(\eta) \subset 1 - \mu$. Hence $\eta \subset f^{-1}(1 - \mu) \subset 1 - f^{-1}(\mu)$ and then $f^{-1}(\mu) \subset 1 - \eta$. 


4. FUZZY ALMOST CONTRA \( g \)-CONTINUOUS FUNCTION

Definition 4.1. A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called fuzzy \( g \)-continuous if the fuzzy preimage of every \( \sigma \)-open subset of \( (Y, \sigma) \) is fuzzy \( g \)-open in \( (X, \tau) \) for every \( \sigma \)-open subset in \( (Y, \sigma) \).

Definition 4.2. Every fuzzy \( g \)-open set in \( Y \) is fuzzy contra \( g \)-continuous.

Definition 4.3. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a function. \( f \) is said to be fuzzy contra \( g \)-continuous if for every fuzzy \( \sigma \)-open set \( U \) in \( (Y, \sigma) \), \( f^{-1}(U) \) is fuzzy \( g \)-open in \( (X, \tau) \).

Theorem 4.4. If \( f \) is fuzzy \( g \)-continuous, then \( f \) is fuzzy contra \( g \)-continuous.

Proof. Let \( U \) be a fuzzy \( \sigma \)-open set in \( (Y, \sigma) \). Since \( f \) is fuzzy \( g \)-continuous, \( f^{-1}(U) \) is fuzzy \( g \)-open in \( (X, \tau) \) for every \( \sigma \)-open set \( U \) in \( (Y, \sigma) \). Hence, \( f \) is fuzzy contra \( g \)-continuous.

Example 4.1: Let \( X = \{0, 1\} \) and \( \tau = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) where \( Y = \{0, 1\} \) and \( \sigma = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\} \). Then \( f \) is fuzzy \( g \)-continuous but not fuzzy contra \( g \)-continuous.

Definition 4.5. A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called fuzzy \( g \)-almost contra \( g \)-continuous if \( f^{-1}(U) \) is fuzzy \( g \)-closed in \( (X, \tau) \) for every \( \sigma \)-open set \( U \) in \( (Y, \sigma) \).

Theorem 4.6. Every fuzzy \( g \)-closed set in \( Y \) is fuzzy \( g \)-contra \( g \)-continuous.

Proof. Let \( C \) be a fuzzy \( g \)-closed set in \( Y \). Since \( f \) is fuzzy \( g \)-continuous, \( f^{-1}(C) \) is fuzzy \( g \)-open in \( (X, \tau) \) for every \( \sigma \)-open subset \( C \) in \( (Y, \sigma) \).

5. CONCLUSION

We have introduced and studied new kind of mapping fuzzy contra \( g \)-continuous map. We have established some significant properties of fuzzy contra \( g \)-continuous maps.

REFERENCES


