

A New Approach to Semi-blind Channel Estimation for MIMO Wireless Communication System

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ABSTRACT

A new approach to Semi-Blind Channel Estimation (SBCE) technique for Multiple Input Multiple Output (MIMO) wireless communication system is proposed. It combines the Least Square (LS) and Minimum Mean Square Error (MMSE) Training Based Channel Estimation (TBCE) scheme with whitening rotation based orthogonal pilot maximum likelihood (OPML) semi-blind channel estimation (SBCE) scheme. In the new approach the whitening matrix is obtained from blind data and rotation matrix is obtained from LS estimated channel. Another modification suggested in this contribution is the use of whitening matrix for MMSE estimate. The channel correlation matrix required for MMSE estimation can be obtained based on latest channel statistics and hence is more reliable. At high SNR this scheme offers better performance than the OPML SBCE method. These advantages are achieved at the cost of negligible reduction in performance in low SNR regime.

General Terms

Channel Estimation, MIMO, Wireless Communication.

Keywords

Training Based Channel Estimation, Semi-blind Channel Estimation, Least Square, Minimum Mean Square Error, Multiple Input Multiple Output, Singular Value Decomposition, Spatial Diversity, Spatial Multiplexing

1. INTRODUCTION

MIMO wireless communication systems are proving to be useful for high data rate multimedia transmission. It was shown that, the use of multiple antennas at transmitter and receiver can improve the capacity [1][2][3]. The simple transmit diversity scheme suggested by Alamouti[4] and the space time coding suggested by V. Tarokh et al.[5] triggered research in this area. Several transmission schemes have been proposed that utilize the MIMO channel in different ways. These schemes can be categorized as spatial multiplexing, spatial diversity (space-time coding) and smart antennas & beamforming techniques [6].

The system's ability to achieve MIMO capacity depends on channel state information. Accurately estimating MIMO channel is much more challenging than SISO channel [7]. There are number of channel estimation schemes suggested in literature. These schemes can be categorized as Training based (TBCE), Blind (BCE) and Semi-Blind (SBCE). Training based schemes are capable of accurately estimating a MIMO channel, provided a large training overhead is made available. Hence there is considerable reduction in system throughput. The least-square (LS) and minimum-mean-square-error (MMSE) techniques are widely

used for channel estimation when training symbols are available. The LS method is simpler than MMSE but performance of MMSE scheme is better. MMSE method however, requires knowledge of channel correlation. Blind methods do not require the training overhead. However these methods not only impose high complexity and slow convergence, but also suffer from unavoidable estimation and decision ambiguities. Semi-blind methods offer attractive practical means of implementing MIMO systems. Semi-blind channel estimation schemes, use a few training symbols to provide the initial MIMO channel estimation and make use of blind information to further improve the estimation. Some SBCE schemes also exchange the information between the channel estimator and the data detector iteratively.

Several SBCE solutions have been proposed to minimize the computational cost, and hence the energy spent in channel estimation of MIMO systems. The SBCE schemes suggested in [8] [9] use few training symbols to provide initial estimate and then the data detector and estimator exchange the information iteratively. In [10] [11] and [12] the MIMO channel matrix is decomposed into whitening and rotation matrix. The whitening matrix is estimated using blind symbols and the rotation matrix is estimated using few orthogonal pilot symbols. This Orthogonal Pilot Maximum Likelihood (OPML) estimator shows a 1dB improvement of bit error rate (BER) compared to the conventional least squares (LS) training scheme if the same length of training sequence is used. Furthermore, SVD has to be applied twice to obtain the 'whitening' matrix and the rotation matrix. These operations lead to the increased computational complexity [12]. The authors feel that the semi-blind method with QR decomposition suggested in [12] is not mathematically correct and hence it is impossible to implement. Moreover the improvement suggested in [12] over the SVD-OPML method assumes knowledge of transmitted symbols X_b at the receiver which is practically infeasible. Because of the assumption of X_b at receiver the authors of [12] are successful in getting near optimal performance. A signal perturbation free whitening rotation based semi blind channel estimation is discussed in [14]. In [15] TBCE and SBCE, considering Perfect, LS, LMMSE, ML, and MAP estimators are studied in terms of BER and complexity. Subspace based semi-blind channel estimation is discussed in [17] [18]. A linear prediction based semi-blind estimation for FIR MIMO channel is proposed in [18]. Number of semi-blind channel estimation schemes are reported for OFDM and MIMO-OFDM systems as well [19]-[26].

This paper proposes a new approach to semi-blind channel estimation in which the whitening matrix of whitening rotation based schemes of [10], [11] and [12] can be used

along with the LS and MMSE channel estimation techniques. In this new approach the SVD has to be applied only once. Another advantage is that the MMSE technique which requires the channel correlation matrix can be obtained based on latest channel conditions. These advantages are achieved at the cost of negligible reduction in performance.

The rest of this paper is organized as follows. Section 2 describes the system model. The design of the proposed estimator is given in section 3. Section 4 has the simulation results and discussion on the results. Finally conclusion is given in section 5.

Throughout our discussions we adopt the following notational conventions. Boldface capitals and lower-case letters stand for matrices and vectors, respectively. \mathbf{I} denote the identity matrix. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^\dagger$ are transpose, conjugate transpose and Moore-Penrose pseudo inverse operators respectively, while $\|\cdot\|_F^2$ and $|\cdot|$ denote the Frobenius norm and magnitude operators respectively. Finally $E[\cdot]$ is the expectation operator

2. SYSTEM MODEL

Consider a MIMO system with M transmit and N receive antennas. Let us assume the channel to be flat fading with channel matrix $\mathbf{H} \in \mathbb{C}^{N \times M}$. Each element h_{ij} in the matrix, represents the flat-fading channel coefficient between the i th receive and j th transmit antenna. Denoting the complex received data by $\mathbf{Y} \in \mathbb{C}^{N \times 1}$ the equivalent base-band system can be modeled as [10],

$$\mathbf{Y}(k) = \mathbf{H}\mathbf{X}(k) + \mathbf{n}(k) \quad (1)$$

k is a time instant of transmission of symbol vector $\mathbf{X} \in \mathbb{C}^{M \times 1}$ and \mathbf{n} is the additive white Gaussian noise with zero mean and noise power σ_n^2 . Also, the sources are assumed to be spatially and temporally independent with identical source power σ_s^2 .

Assume that the channel remains constant for K symbol periods. In these K symbols, the first L symbols are used for training. Let these training symbol vectors be $\mathbf{X}_p = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \dots, \mathbf{x}_L]$ where $\mathbf{X}_p \in \mathbb{C}^{M \times L}$ and its corresponding output $\mathbf{Y}_p \in \mathbb{C}^{N \times L}$ is the received training symbol output vectors. The remaining $K - L$ symbol vectors are blind data symbols \mathbf{X}_b and their corresponding output \mathbf{Y}_b where $\mathbf{X}_b \in \mathbb{C}^{M \times K-L}$ and $\mathbf{Y}_b \in \mathbb{C}^{N \times K-L}$.

3. ESTIMATOR DESIGN

The matrix \mathbf{H} can be estimated using only training symbols. The LS estimation of \mathbf{H} is given by (2),

$$\hat{\mathbf{H}}_{ls} = \mathbf{Y}_p \mathbf{X}_p^\dagger \quad (2)$$

Where \mathbf{X}_p^\dagger denotes Moore-Penrose pseudo-inverse of \mathbf{X}_p .

The MMSE estimate of \mathbf{H} is given by (3),

$$\hat{\mathbf{H}}_{mmse} = \mathbf{R}_{hh} (\mathbf{R}_{hh} + \sigma_n^2 (\mathbf{X}\mathbf{X}^H)^{-1})^{-1} \hat{\mathbf{H}}_{ls} \quad (3)$$

\mathbf{R}_{hh} is channel correlation given as $E[\mathbf{H}\mathbf{H}^H]$. The MMSE estimator (3) performs better than LS estimator but is of considerable complexity since a matrix inversion is needed every time the training data in \mathbf{X} changes. We reduce the complexity of this estimator by averaging over the transmitted data, i.e. we replace the term $(\mathbf{X}\mathbf{X}^H)^{-1}$ in (3) with its expectation $E[(\mathbf{X}\mathbf{X}^H)^{-1}]$. Hence the simplified estimator will be [19],

$$\hat{\mathbf{H}}_{mmse} = \mathbf{R}_{hh} \left(\mathbf{R}_{hh} + \frac{\beta}{SNR} \mathbf{I} \right)^{-1} \hat{\mathbf{H}}_{ls} \quad (4)$$

Where $E[(\mathbf{X}\mathbf{X}^H)^{-1}] = E[1/x_k]^2 \mathbf{I}$ and average signal to noise ratio $SNR = E|x_k|^2 / \sigma_n^2$ and $\beta = E[1/x_k]^2 E|x_k|^2$.

Training based channel estimation techniques are easy to

implement, but because of the redundant training information, spectral efficiency suffers. Efficient usage of bandwidth can be achieved through blind methods but they have poor convergence rate and require long data records to be processed. Hence techniques which use a small amount of training symbols with blind data can provide attractive solution for channel estimation. One such technique called Orthogonal Pilot Maximum Likelihood Semi-blind Channel Estimation (OPML SBCE) is discussed in [10], [11] and [12].

The MIMO channel matrix can be decomposed as,

$$\mathbf{H} = \mathbf{W}\mathbf{Q}^H \quad (5)$$

Where the whitening matrix, $\mathbf{W} \in \mathbb{C}^{N \times M}$ and rotation matrix, $\mathbf{Q} \in \mathbb{C}^{M \times M}$. If we take SVD of channel matrix then, \mathbf{H} can be written as,

$$\mathbf{H} = \mathbf{P}\mathbf{\Sigma}\mathbf{V}^H \quad (6)$$

Where $\mathbf{P} \in \mathbb{C}^{N \times N}$ and $\mathbf{V} \in \mathbb{C}^{M \times M}$ are unitary matrices and $\mathbf{\Sigma} \in \mathbb{C}^{N \times M}$ is a diagonal matrix. Then, $\mathbf{W} = \mathbf{P}\mathbf{\Sigma}$ and $\mathbf{Q} = \mathbf{V}^H$. Hence $\mathbf{H}\mathbf{H}^H = \mathbf{W}\mathbf{W}^H$. The whitening matrix, \mathbf{W} can be estimated using blind data and \mathbf{Q} can be estimated using training symbols. The output correlation matrix \mathbf{R}_{yy} is given as,

$$\mathbf{R}_{yy} = E[\mathbf{Y}_b \mathbf{Y}_b^H] \approx (\mathbf{H}\mathbf{X}_b)(\mathbf{H}\mathbf{X}_b)^H + \sigma_n^2 \mathbf{I} \quad (7)$$

This can be further simplified as,

$$\mathbf{H}\mathbf{H}^H = \mathbf{W}\mathbf{W}^H = \frac{1}{\sigma_s^2} (\mathbf{R}_{yy} - \sigma_n^2 \mathbf{I}) \quad (8)$$

Here we have to make an important assumption about source power σ_s^2 and noise power σ_n^2 . These will be assumed to be known. It is not a practical assumption but then it is going to simplify our estimation task. Practically we have to estimate these values as well. Assuming σ_n^2 and σ_s^2 is known [10], then from Singular Value Decomposition (SVD) of $\frac{1}{\sigma_s^2} (\mathbf{R}_{yy} - \sigma_n^2 \mathbf{I})$ we can write,

$$\frac{1}{\sigma_s^2} (\mathbf{R}_{yy} - \sigma_n^2 \mathbf{I}) = \mathbf{U}\mathbf{\Sigma}^2\mathbf{U}^H. \quad (9)$$

Hence from (6), (8) and (9) we can write,

$$\mathbf{W} = \mathbf{U}\mathbf{\Sigma} \quad (10)$$

Thus \mathbf{W} is estimated blindly. The unitary matrix \mathbf{Q} is estimated using orthogonal pilots. Constrained ML estimator of \mathbf{Q} is obtained by minimizing the likelihood function,

$$\|\mathbf{Y}_p - \mathbf{W}\mathbf{Q}^H \mathbf{X}_p\|_F^2 \text{ Such that } \mathbf{Q}\mathbf{Q}^H = \mathbf{I} \quad (11)$$

It is shown in [11] that estimate of \mathbf{Q} is obtained as,

$$\hat{\mathbf{Q}} = \mathbf{V}_m \mathbf{U}_m^H \text{ Where } \mathbf{U}_m \mathbf{\Sigma}_m \mathbf{V}_m^H = \text{SVD}(\mathbf{W}^H \mathbf{Y}_p \mathbf{X}_p^H) \quad (12)$$

Thus it can be seen that the semi-blind procedure (OPML estimator) described above requires SVD to be calculated twice. This procedure is modified slightly as below in algorithms 1 and 2..

Algorithm 1: Semi-blind Channel Estimation with LS LS-SBCE ($\mathbf{X}_p, \mathbf{Y}_p, \mathbf{X}_b, \mathbf{Y}_b, \sigma_n^2, \sigma_s^2$)

1. Compute \mathbf{W} using (9) and (10)
 2. Estimate of $\mathbf{H}(\hat{\mathbf{H}}_{ls})$ is obtained using (2)
 3. Compute $\hat{\mathbf{Q}}^H = \mathbf{W}^\dagger \hat{\mathbf{H}}_{ls}$
 4. Finally obtain estimate of \mathbf{H} as $\hat{\mathbf{H}} = \mathbf{W}\hat{\mathbf{Q}}^H$
 5. end
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Algorithm 2: Semi-blind Channel Estimation with MMSE MMSE-SBCE ($\mathbf{X}_p, \mathbf{Y}_p, \mathbf{X}_b, \mathbf{Y}_b, \sigma_n^2, \sigma_s^2$)

1. Compute \mathbf{W} using (9) and (10)
 2. Estimate of $\mathbf{H}(\hat{\mathbf{H}}_{ls})$ is obtained using (2)
 3. Compute $\mathbf{R}_{hh} = \mathbf{W}\mathbf{W}^H$
 4. Estimate of $\mathbf{H}(\hat{\mathbf{H}}_{mmse})$ is obtained using (4)
 5. end
-

In this new approach, very few training symbols are used to obtain initial estimate of the channel matrix. The whitening matrix \mathbf{W} is obtained using blind output symbols. The rotation matrix \mathbf{Q} is however estimated from the initial estimate of the channel and \mathbf{W} to avoid second SVD. Note that the Moore-Penrose pseudo inverse in (2) needs to be calculated only once as the pilots are fixed whereas the SVD in (12) has to be calculated for every frame. The estimate obtained using second scheme (Algorithm 2) is better than the first. It is because the MMSE estimate has advantage over LS method of not enhancing the noise. Moreover, the MMSE estimate is obtained using the current statistics of the channel. Therefore the estimate is more reliable.

The Moore-Penrose pseudo inverse suggested in step 3 of algorithm 1 is used instead of second SVD of OPML SBCE. The complexity of calculation of Moore-Penrose inverse is discussed in [27][28][29]. Matlab provides the function *pinv* which is based on SVD. Fast computation of this operation is suggested in [28] and [29]. The alternative functions *ginv* and *geninv* are provided in these references. It can be verified that these functions provide better time complexity than *pinv*.

4. RESULTS & DISCUSSIONS

Extensive simulations are carried out in MATLAB 7.8 to test the performance of the proposed channel estimators and they are compared with the SVD-OPML estimators of [11] and [12]. The simulation is carried out for 2X4 and 4X8 MIMO systems under flat fading channel. The two types of MIMO systems viz. spatial diversity MIMO system and spatial multiplexing MIMO system with M-QAM modulation are used for this study. Performance of these systems under the assumption of channel knowledge is discussed by the authors of this paper in [30] [31]. The performance parameters used are Bit Error Rate (BER) and Mean Square Error (MSE).

4.1 Spatial Diversity MIMO System

Space Time Block Coded (STBC) symbols are transmitted on multiple antennas. For 2 transmitting antenna G2 (Alamouti) code is used while for 4 transmitting antenna G4 code is used [33]–[35].

Fig. 1 and 2 show the BER performance and MSE plot of 2X4 MIMO system using Alamouti coding [4] respectively. In simulation scenarios, 16-QAM data modulation is used with flat fading Rayleigh MIMO channel. In the simulation, a transmitted frame consists of 4 orthogonal pilots (L) and 100 blind symbols ($K - L$). The BER is calculated by averaging over 100 rounds of simulation with Perfect CE, LS TBCE, OPML SBCE and LS SBCE for different values of E_b/N_0 .

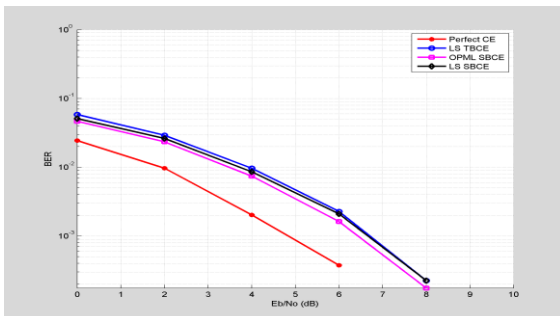


Fig. 1: BER Performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML SBCE and LS SBCE

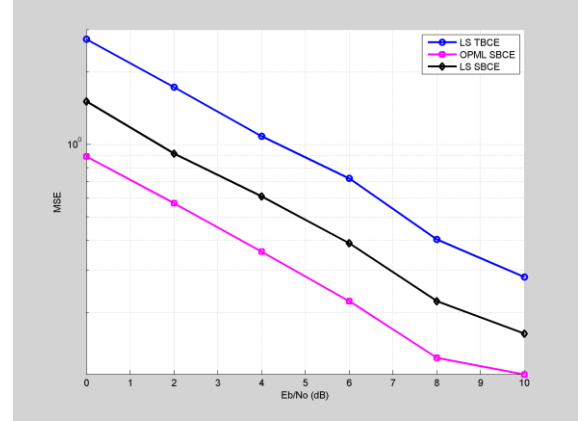


Fig. 2: MSE Performance for 2X4 MIMO system using Alamouti coding under LS TBCE and OPML SBCE and SBCE with LS

The BER plot shows that LS SBCE approach has negligible difference in performance compared to OPML SBCE method. The MSE plot however clearly shows that the OPML SBCE is better than LS SBCE. It is because MSE is calculated taking into account square of differences between all the channel gains involved.

Fig. 3 and 4 show the BER and MSE plots respectively of 2X4 MIMO system using Alamouti coding with Perfect CE, LS TBCE, OPML SBCE and MMSE SBCE for different values of E_b/N_0 .

The results shown in Fig. 3 and 4 depict that the proposed MMSE SBCE approach has negligible difference in BER performance compared to OPML SBCE method in low SNR regime. In high SNR regime MMSE performance approaches LS which is consistent with the theory [32].

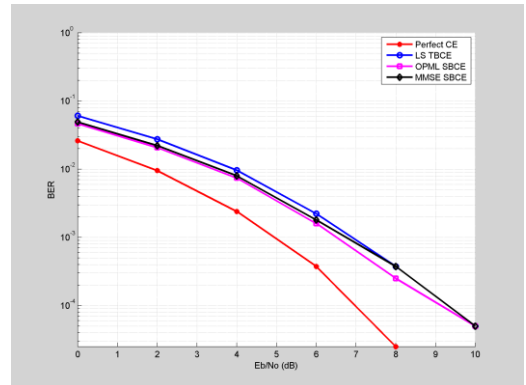


Fig. 3: BER performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML SBCE and MMSE SBCE

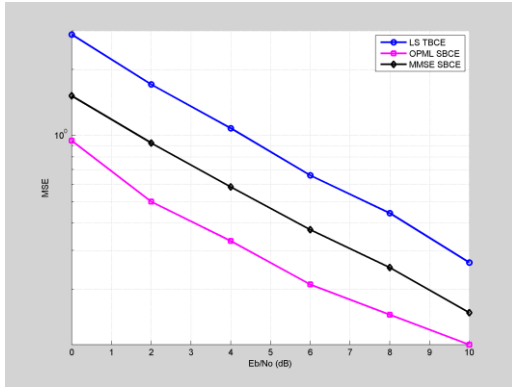


Fig. 4: MSE performance for 2X4 MIMO system using Alamouti coding under LS TBCE, OPML SBCE and MMSE SBCE

Fig. 5 and 6 show the BER and MSE performance respectively of 4X8 MIMO system using G4 STBC coding. In simulation scenarios, 64-QAM data modulation is used with flat fading Rayleigh MIMO channel. In the simulation, a transmitted frame consists of 8 orthogonal pilots (L) and 100 blind symbols ($K - L$). The BER is calculated by averaging over 100 rounds of simulation with Perfect CE, LS TBCE, OPML SBCE and the new SBCE with LS for different values of E_b/N_0

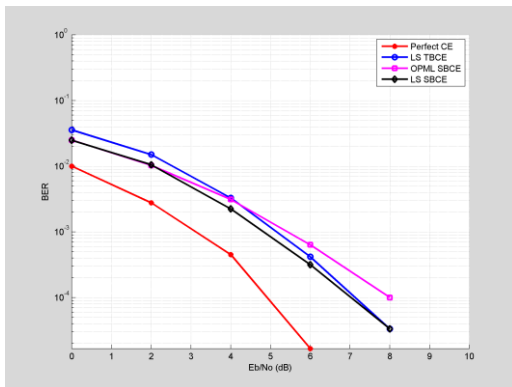


Fig. 5: BER performance of 4X8 MIMO system using G4 STBC under Perfect CE, LS TBCE and OPML SBCE and LS SBCE

The results shown in Fig. 5 depict that the proposed SBCE approach has negligible difference in BER performance compared to OPML SBCE method in low SNR regime. In high SNR range, the proposed scheme outperforms the OPML SBCE. It is observed that the performance of OPML SBCE degrades with higher order modulation. To improve the performance of OPML SBCE one needs to increase the data length. For example, we have to increase the blind symbols ($K - L$) from 100 to 1000 in our case. But then the channel state should remain constant over this period of time which practically may not be possible.

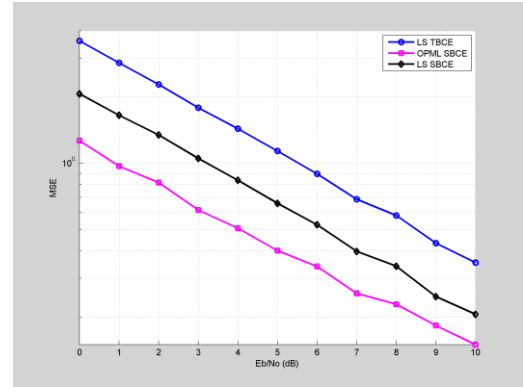


Fig. 6: MSE performance for 4X8 MIMO system using G4 STBC under LS TBCE, OPML SBCE and LS SBCE

Fig. 7 and 8 show the BER and MSE performance respectively of 4X8 MIMO system using G4 STBC with Perfect CE, MMSE TBCE, OPML SBCE and the new SBCE with MMSE for different values of E_b/N_0 .

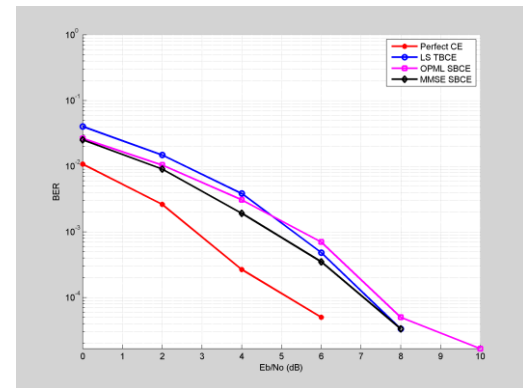


Fig. 7: BER performance of 4X8 MIMO system using G4 STBC under Perfect CE, LS TBCE and OPML SBCE and MMSE SBCE

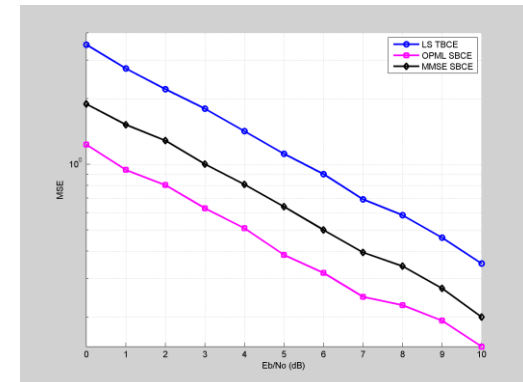


Fig. 8: MSE performance for 4X8 MIMO system using G4 STBC under LS TBCE, OPML SBCE and MMSE SBCE

The results shown in Fig. 7 and 8 depict that the proposed MMSE based SBCE approach performs far better than OPML SBCE method. The MSE plot however indicates that the OPML SBCE method is better but then the BER is more trusted performance measure than the MSE. It is because the MSE is calculated by taking sum of the square of the differences between the channel gains.

4.2 Spatial Multiplexing MIMO System

The proposed estimators are also tested and compared with OPML SBCE for spatial multiplexing MIMO system as well. Fig. 9 and 10 show the BER and MSE performance

respectively of 2X4 MIMO system using Spatial Multiplexing. In simulation scenarios, 4-QAM data modulation is used with flat fading Rayleigh MIMO channel. In the simulation, a transmitted frame consists of 4 orthogonal pilots (L) and 100 blind symbols ($K - L$). The BER is calculated by averaging over 100 rounds of simulation with Perfect CE, LS TBCE, OPML SBCE and LS SBCE and MMSE SBCE for different values of E_b/N_0 . The results reiterate the fact that for low order modulation the OPML SBCE scheme has a slight edge over the proposed schemes.

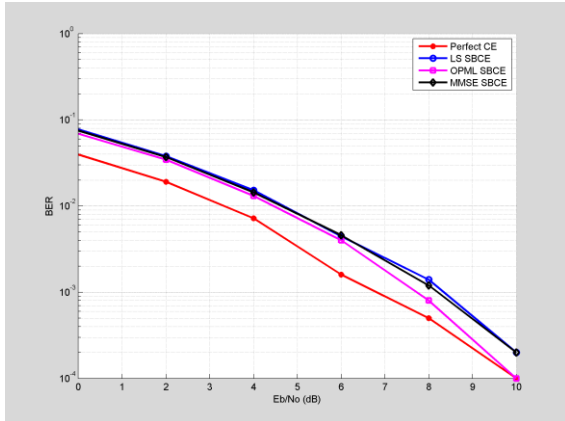


Fig. 9: BER performance of 2X4 MIMO system using Spatial Multiplexing under Perfect CE, LS SBCE and OPML SBCE and MMSE SBCE

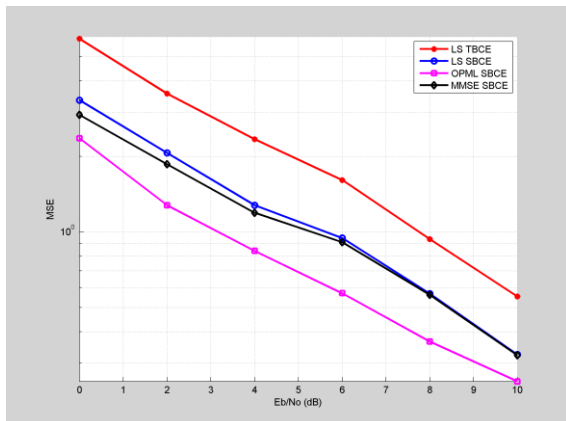


Fig. 10: MSE performance of 2X4 MIMO system using Spatial Multiplexing under LS TBCE, LS SBCE, OPML SBCE and MMSE SBCE

Performance of 4X8 MIMO system with higher order modulation was also studied. Fig. 11 and 12 show the BER and MSE performance respectively of 4X8 MIMO system using Spatial Multiplexing. 16 QAM modulation was used along with 8 pilots and 100 blind symbols in a frame.

The results shown in Fig. 11 and 12 depict that the proposed LS and MMSE based SBCE estimators perform far better than OPML SBCE method for spatial multiplexing MIMO systems which use higher order modulation schemes.

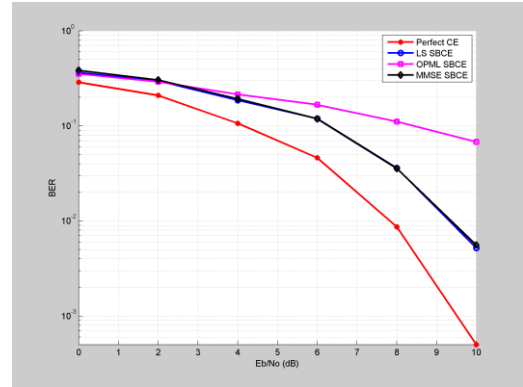


Fig. 11: BER performance of 4X8 MIMO system using Spatial Multiplexing under Perfect CE, LS SBCE and OPML SBCE and MMSE SBCE

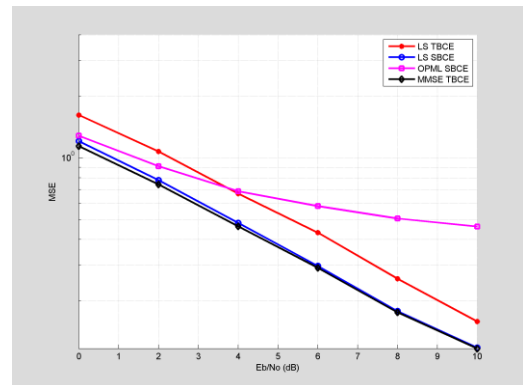


Fig. 12: MSE performance of 4X8 MIMO system using Spatial Multiplexing under LS TBCE, LS SBCE, OPML SBCE and MMSE SBCE

5. CONCLUSIONS

A new approach to semi-blind channel estimation for MIMO wireless communication system is proposed. The SVD based OPML SBCE method calculates the SVD twice. In the new approach, the second SVD is replaced by the Moore-Penrose inverse to calculate the rotation matrix. Another modification suggested takes advantage of MMSE based channel estimation which is based on current statistical information of the channel. The proposed scheme is tested using different MIMO systems and the two performance parameters viz. BER and MSE. The results obtained show that the performance of proposed schemes LS SBCE and MMSE SBCE is almost comparable with OPML SBCE scheme in the low SNR regime whereas their performance is better with higher SNR values. These schemes have far better performance if higher modulation order is used.

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