Hamiltonian Laceability in Line Graphs

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ABSTRACT
A Connected graph G is a Hamiltonian laceable if there exists in G a Hamiltonian path between every pair of vertices in G at an odd distance. G is a Hamiltonian-t-Laceable (Hamiltonian-t*-Laceable) if there exists in G a Hamiltonian path between every pair (at least one pair) of vertices at distance ‘t’ in G. 1 ≤ t ≤ diam G. In this paper we explore the Hamiltonian-t*-laceability number (λ*(t)) of graph L(G) i.e., Line Graph of G and also explore Hamiltonian-t*-Laceable of Line Graphs of Sunlet graph, Helm graph and Gear graph for t=1,2 and 3.

Keywords
Connected graph, Line graph, Sun let graph, Helm graph, Wheel graph, Gear graph and Hamiltonian-t-laceable graph.

1. INTRODUCTION
All graphs considered here are finite, simple, connected and undirected graph. Let \( (G = V(G), E(G)) \) be a graph.

\[ |V(G)| \text{ and } |E(G)| \] are called the order and the size of G respectively. The order of G denoted by \( O(G) \) is the cardinality of vertices of G. The distance between \( u \) and \( v \) denoted by \( d(u,v) \) is the length of the shortest \( u-v \) path in G.

G is a Hamiltonian path between every pair of the distinct vertices in it at an odd distance. G is a Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of the vertices \( u \) and \( v \) in G with the property \( d(u,v)=t \), where \( t \) is a positive integer, such that \( 1 \leq t \leq \text{diam } G \).

The Line graph \( L(G) \) of G has the edges of G as its vertices and two vertices of \( L(G) \) are adjacent if and only if they are adjacent in G. In [3],[5],[6] and [7] the authors have studied Hamiltonian-t-laceability and Hamiltonian-t*-laceability of various graph structures. In this paper we explore the Hamiltonian-t*-laceability number of Line graph \( L(G) \) and also Hamiltonian-t*-laceability of Line graph \( L(G) \) of the sun let graph, Helm graph and Gear graph.

DEFINITION 1
The Line graph \( L(G) \) of G is the graph of E in which \( x, y \in E \) are adjacent as vertices if and only if they are adjacent as edges in G. In Figure 1, we display the graph G and its Line graph L(G).

DEFINITION 2
The Sun let graph \( S_n \) is a graph with cycle where by each vertex of the cycle is attached to one pendant vertex. Each sun let graph contains \( r \)-vertices with \( r \)-edges.

In Figure 2, we display the Sun let graph \( S_n \).

DEFINITION 3
The wheel graph with \( n \) spokes, \( W_n \) is the graph that consists of an \( n \)-cycle and one additional vertex, say \( u \), which is adjacent to all the vertices of the cycle.

In Figure 3, we display the Wheel graph \( W_n \).
2. RESULTS

Theorem 2.1: The Line graph L(G), where G=S_n, the sun let graph is Hamiltonian-t*-laceable for t=1 and 2 if odd n ≥3, where 1 ≤ t ≤ diamG.

Proof: Consider the graph G=S_n, the Line graph L(S_n) denote the vertices L(G) by 

\[ a_1, b_1, a_2, b_2, a_3, b_3, \ldots - a_{n-1}, b_{n-1}, a_n, b_n \]

for t=1, 2

Case (i): For t=1

Let there exists a Hamiltonian path. Hence there exists a Hamiltonian path for t=1.

Case (ii): For t=2

Let there exists a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that \( d(a_1, b_1) = 1 \). Therefore G is a Hamiltonian-t*-laceable for t=2.
Lemma 2.1.1: The Line graph $L(G)$, where $G = S_n$, is a Hamiltonian-$t$-$\lambda$-laceability number if $(\lambda^*(t)) = 1$ for $t = 2$ if odd $n \geq 3$ and $t = 3$ if odd $n \geq 5$ where $1 \leq \text{diam}(G)$.

Proof: Consider the graph $G = S_n$, its line $L(S_n)$. Here we need to establish the following cases to show that, Hamiltonian-$t$-$\lambda$-laceability number if $(\lambda^*(t)) = 1$ for $t = 2$ if $n \geq 3$ and $t = 2$ and $3$ if $n \geq 5$

Case (i): For $t = 2$

In $L(S_n)$, we find that $d(a_1, b_2) = 2$ and the path

$P: (a_1, b_1) \cup (b_1, b_2) \cup (b_2, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup (b_{n-3}, a_{n-3}) \cup \ldots \cup (b_3, a_3) \cup (a_3, a_2) \cup (a_2, b_2)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_2) = 2$. Therefore $G$ is a Hamiltonian-$t$-$\lambda$-laceable for $t = 2$ and Laceyability number $(\lambda^*(t)) = 1$ for $t = 2$.

Figure 7: Hamiltonian path from the vertex $a_1$ to $a_2$ in Line graph $L(S_n)$

Case (ii): For $t = 3$ if odd $n \geq 5$

In $L(S_n)$, we find that $d(a_1, b_3) = 3$ and the path

$P: (a_1, b_1) \cup (b_1, a_2) \cup (a_2, b_2) \cup (b_2, a_3) \cup (a_3, a_4) \cup (a_4, b_4) \cup \ldots \cup (b_{n-3}, a_{n-3}) \cup (b_{n-2}, a_{n-2}) \cup (b_{n-1}, a_{n-1}) \cup (b_n, a_{n-1}) \cup (b_n, a_n) \cup (b_n, b_{n-1}) \cup (b_n, a_{n-2}) \cup (b_n, b_{n-2}) \cup (b_n, b_{n-3}) \cup (b_n, a_{n-3}) \cup \ldots \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_3) \cup (a_3, b_3)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_3) = 3$. Therefore $G$ is a Hamiltonian-$t$-$\lambda$-laceable for $t = 3$.

Figure 9: Hamiltonian path from the vertex $a_1$ to $b_3$ in Line graph $L(S_n)$

Theorem 2.2: The Line graph $L(G)$, where $G = S_n$, the sun let graph is Hamiltonian-$t$-$\lambda$-laceable for $t = 1, 2,$ and $3$ if even $n \geq 4$, where $1 \leq \text{diam}(G)$.

Proof: Consider the graph $G = S_n$, the Line graph $L(S_n)$ denote the vertices $L(G)$ by

$a_1, b_1, a_2, b_2, a_3, b_3, \ldots, a_{n-1}, b_{n-1}, a_n, b_n$ for $t = 1, 2,$ and $3$

Case (i): For $t = 1$

In $L(S_n)$, we find that $d(a_1, b_3) = 1$ and the path

$P: (a_1, b_1) \cup (b_1, a_2) \cup (a_2, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup (b_{n-3}, a_{n-3}) \cup \ldots \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_3) \cup (a_3, b_3)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_3) = 1$. Therefore $G$ is a Hamiltonian-$t$-$\lambda$-laceable for $t = 1$.

Figure 8: Hamiltonian path from the vertex $a_1$ to $b_3$ in Line graph $L(S_n)$
Figure 10: Hamiltonian path from the vertex \(a_1\) to \(b_1\) in Line graph \(L[S_n]\)

Case (ii): For \(t=2\)
In \(L(S_n)\), we find that \(d(a_1, a_2) = 2\) and the path
\[
P : (a_1, b_1) \cup (b_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup \ldots \cup (a_4, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2)
\]
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that \(d(a_1, a_2) = 2\). Therefore \(G\) is a Hamiltonian-\(t^*\)-laceable for \(t=2\).

Figure 11: Hamiltonian path from the vertex \(a_1\) to \(a_2\) in Line graph \(L[S_n]\)

Lemma 2.2.2: The Line graph \(L(G)\), where \(G=S_n\), is a Hamiltonian-\(t^*\)-laceability number, \((\lambda^*_t)\)

\[
= 1 \text{ for } t=2 \text{ and } 3 \text{ if even } n \geq 4, \text{ where } 1 \leq t \leq \text{ diam } G.
\]

Proof: Consider the graph \(G = S_n\), its line \(L(S_n)\). Here we need to establish the following cases to show that, Hamiltonian-\(t^*\)-laceability number if \((\lambda^*_t(t)) = 1\) for \(t=2\) and 3 if \(n \geq 4\)

Case (i): For \(t=2\)
In \(L(S_n)\), we find that \(d(a_1, b_2) = 2\) and the path
\[
P : (a_1, b_1) \cup (b_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup \ldots \cup (a_4, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2)
\]
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that \(d(a_1, b_2) = 2\). Therefore \(G\) is a Hamiltonian-\(t^*\)-laceable for \(t=2\) and Laceability number \((\lambda^*_t(t)) = 1\) for \(t=2\).

Figure 12: Hamiltonian path from the vertex \(a_1\) to \(a_2\) in Line graph \(L[S_n]\)

Case (ii): For \(t=3\)
In \(L(S_n)\), we find that \(d(a_1, b_3) = 3\) and the path
\[
P : (a_1, b_1) \cup (b_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup \ldots \cup (a_5, b_5) \cup (b_5, a_5) \cup (a_5, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2)
\]
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that \(d(a_1, b_3) = 3\). Therefore \(G\) is a Hamiltonian-\(t^*\)-Laceability number \((\lambda^*_t(t)) = 1\) for \(t=3\).
a_n, b_n, c_n. Hence we need to establish the following claims to show that G is a Hamiltonian-t*-laceable for t=1,2, 3 with diameter 3. In Figure 15, we display the Helm graph H_n.

Figure 15

Claim 2.3.1: For t=1
Case (i): If n is odd
In L(H_n), we find that \( d(a_1, c_1) = 1 \) and the path
\[
P: (a_1, b_1) \cup (b_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup \ldots \cup (a_6, b_3) \cup (b_3, a_3) \cup (a_3, a_2) \cup (b_2, a_2) \cup (a_2, a_1)
\]
is a Hamiltonian path.

Case (ii): If n is even
In L(H_n), we find that \( d(a_1, c_1) = 1 \) and the path
\[
P: (a_1, b_1) \cup (b_1, b_n) \cup (b_n, c_n) \cup (c_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, c_{n-2}) \cup (c_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup (b_{n-3}, c_{n-3}) \cup (c_{n-3}, a_{n-3}) \cup \ldots \cup (a_2, c_2) \cup (c_2, c_1) \cup (b_3, c_1) \cup (c_1, a_3) \cup (a_3, b_2) \cup (a_2, a_{n-1}) \cup (b_{n-1}, c_{n-1}) \cup (c_{n-1}, c_1)
\]
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that \( d(a_1, c_1) = 1 \). Therefore G is a Hamiltonian-t*-laceable for t=1.

Figure 16: Hamiltonian path from the vertex \( a_1 \) to \( c_1 \) in Line graph \( L[H_n] \)

3. Remark
If \( n \geq 4 \), the distance from \( d(a_1, a_2) = 3 \) is a Hamiltonian-t*-laceable for \( t=3 \) and its laceability number \( (\lambda^*_t(t)) = 1 \) for \( t=3 \), then the path
\[
P: (a_1, b_1) \cup (b_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup \ldots \cup (a_6, b_3) \cup (b_3, a_3) \cup (a_3, a_2) \cup (b_2, a_2) \cup (a_2, a_1)
\]
is a Hamiltonian path.

Figure 14: Hamiltonian path from the vertex \( a_1 \) to \( a_3 \) in Line graph \( L[S_n] \)

Theorem 2.3: The Line graph \( L(G) \), where \( G=H_n \), \( n \geq 3 \), the Helm graph is Hamiltonian-t*-laceable for \( t=1,2 \) and 3, with diameter 3.
Proof: Consider the graph \( G=H_n \), its Line graph is denoted by \( L(H_n) \) denote the vertices of \( L(G) \) by \( a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3, a_4, b_4, c_4, \ldots, a_{n-1}, b_{n-1}, c_{n-1} \).

Figure 13: Hamiltonian path from the vertex \( a_1 \) to \( b_3 \) in Line graph \( L[S_n] \)
$P: (a_1, b_n) \cup (b_n, a_n) \cup (a_n, c_n) \cup (c_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, c_{n-1}) \cup (c_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup \ldots \cup (b_4, a_4) \cup (a_4, c_4) \cup (c_4, b_2) \cup (b_2, a_2) \cup (a_2, c_2) \cup (c_2, b_1) \cup (b_1, c_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, c_1) = 1$. Therefore $G$ is a Hamiltonian-t*-Laceable for $t=1$.

![Image 17: Hamiltonian path from the vertex $a_1$ to $c_1$ in Line graph $L(H_n)$](image)

**Claim 2.3.2:** For $t=2$

**Case (iii): If $n$ is odd**

In $L(H_n)$, we find that $d(a_1, a_2) = 2$ and the path $P: (a_1, b_n) \cup (b_n, c_n) \cup (c_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, c_{n-2}) \cup (c_{n-2}, a_{n-2}) \cup \ldots \cup (c_4, a_4) \cup (a_4, b_2) \cup (b_2, a_2) \cup (a_2, c_2) \cup (c_2, b_1) \cup (b_1, a_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, a_2) = 2$. Therefore $G$ is a Hamiltonian-t*-Laceable for $t=2$.

![Image 18: Hamiltonian path from the vertex $a_1$ to $a_2$ in Line graph $L(H_5)$](image)

**Case (iv): If $n$ is even**

In $L(H_n)$, we find that $d(a_1, a_2) = 2$ and the path $P: (a_1, b_n) \cup (b_n, c_n) \cup (c_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, c_{n-2}) \cup (c_{n-2}, a_{n-2}) \cup \ldots \cup (c_4, a_4) \cup (a_4, b_2) \cup (b_2, a_2) \cup (a_2, c_2) \cup (c_2, b_1) \cup (b_1, a_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, a_2) = 2$. Therefore $G$ is a Hamiltonian-t*-Laceable for $t=2$.

![Image 19: Hamiltonian path from the vertex $a_1$ to $a_2$ in Line graph $L(H_6)$](image)

**Claim 3:** For $t=3$

**Case (v): If $n$ is odd**

In $L(H_n)$, we find that $d(a_1, a_3) = 3$ and the path $P: (a_1, b_n) \cup (b_n, c_n) \cup (c_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, c_{n-2}) \cup (c_{n-2}, a_{n-2}) \cup \ldots \cup (c_4, a_4) \cup (a_4, b_2) \cup (b_2, a_2) \cup (a_2, c_2) \cup (c_2, b_1) \cup (b_1, a_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, a_3) = 3$. Therefore $G$ is a Hamiltonian-t*-Laceable for $t=3$.

![Image 20: Hamiltonian path from the vertex $a_1$ to $a_3$ in Line graph $L(H_6)$](image)
(b_{n-2}, c_{n-2}) \cup (c_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup
\ldots \cup (c_{n-12}, a_{n-12}) \cup \ldots \cup (b_j, c_j) \cup
(c_j, c_2) \cup (c_2, a_2) \cup (a_2, b_2) \cup (b_2, a_3)

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that
d(a_1, a_3) = 3 \ d(a_1, a_3) = 3. Therefore G is a Hamiltonian-t*-Laceable for t=3.

**Figure 20:** Hamiltonian path from the vertex a_1 to b_2 in Line graph L[H_3]

**Case (vi): If n is even**

In L(H_n), we find that d(a_1, a_3) = 3 and the path

P : (a_1, b_n) \cup (b_n, a_n) \cup (a_n, c_n) \cup (c_n, b_{n-1}) \cup
(b_{n-1}, a_{n-1}) \cup (a_{n-1}, c_{n-1}) \cup (c_{n-1}, b_{n-2}) \cup
(b_{n-2}, a_{n-2}) \cup (a_{n-2}, c_{n-2}) \cup
\ldots \cup (b_2, a_4) \cup (a_4, c_4) \cup (c_4, b_4) \cup (b_4, a_3) \cup \ldots \cup
(c_2, b_1) \cup (b_1, c_1) \cup (c_1, a_1) \cup (a_1, a_3)

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that
d(a_1, a_3) = 3. Therefore G is a Hamiltonian-t*-Laceable for t=3.

**Figure 21:** Hamiltonian path from the vertex a_1 to a_3 in Line graph L[H_6]

**Theorem 2.4** The Line graph L(G), where G = G_n, n ≥ 4, the Gear graph is Hamiltonian-t*-laceable for t=1, 2 and 3, with diameter 3

Proof: Consider the graph G = G_n, its Line graph is denoted by L(G_n) denote the vertices of L(G) by

a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n. Hence we need to establish the following claims to show that G is a Hamiltonian-t*-laceable for t=1, 2 and 3 with diameter 3.

**Claim 1:** For t=1

**Case (i): If n is odd**

In L(G_n), we find that d(a_0, a_1) = 1 and the path

P : (a_0, a_{2n-2}) \cup (a_{2n-2}, a_{2n-4}) \cup (a_{2n-4}, a_{2n-4}) \cup
(a_{2n-9}, a_{3n-9}) \cup \ldots \cup (a_{16}, a_{15}) \cup
(a_{15}, a_{2n+5}) \cup \ldots \cup (a_{14}, a_{13}) \cup \ldots \cup
(a_{6}, a_{2n+2}) \cup (a_{2n+2}, a_3) \cup (a_5, a_4) \cup (a_3, a_2) \cup
(a_{2n-1}, a_2) \cup (a_2, a_1) is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that
d(a_0, a_1) = 1. Therefore G is a Hamiltonian-t*-Laceable for t=1.

**Figure 22:** Hamiltonian path from the vertex a_0 to a_1 in Line graph L[G_7]

**Case (ii): If n is even**

In L(G_n), we find that d(a_0, a_1) = 1 and the path

P : (a_0, a_{2n-2}) \cup (a_{2n-2}, a_{2n-1}) \cup (a_{2n-1}, a_{2n}) \cup
(a_{2n}, a_{2n+1}) \cup (a_{2n+1}, a_{2n+2}) \cup (a_{2n+2}, a_{2n+3}) \cup
\ldots \cup (a_{15}, a_{14}) \cup \ldots \cup (a_5, a_4) \cup
(a_4, a_5) \cup \ldots \cup (a_3, a_2) \cup
(a_2, a_3) is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that

d(a_0, a_1) = 1. Therefore G is a Hamiltonian-t*-Laceable for t=1.
4.1. Therefore G is a Hamiltonian graph.

Claim 3.4.1: For $t=2$

Case (i): If $n$ is odd

In $L(G_n)$, we find that $d(a_0, a_3) = 2$ and the path

$$P: (a_0, a_1) \cup (a_1, a_2) \cup (a_2, a_3) \cup (a_3, a_4) \cup (a_4, a_5) \cup (a_5, a_6) \cup (a_6, a_7) \cup (a_7, a_8) \cup (a_8, a_9) \cup (a_9, a_{10}) \cup (a_{10}, a_{11})$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0, a_2) = 2$. Therefore $G$ is a Hamiltonian-$t^*$-Laceable for $t=2$.

Figure 23: Hamiltonian path from the vertex $a_0$ to $a_3$ in Line graph $L[G_3]$

Figure 24: Hamiltonian path from the vertex $a_0$ to $a_2$ in Line graph $L[G_2]$

Case (ii): If $n$ is even

In $L(G_n)$, we find that $d(a_0, a_2) = 2$ and the path

$$P: (a_0, a_1) \cup (a_1, a_{2n-2}) \cup (a_{2n-2}, a_{2n-1}) \cup (a_{2n-1}, a_{2n}) \cup (a_{2n}, a_2)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0, a_2) = 2$. Therefore $G$ is a Hamiltonian-$t^*$-Laceable for $t=2$.

Claim 3.4.2: For $t=3$

Case (i): If $n$ is odd

In $L(G_n)$, we find that $d(a_0, a_3) = 3$ and the path

$$P: (a_0, a_1) \cup (a_1, a_2) \cup (a_2, a_3) \cup (a_3, a_4) \cup (a_4, a_5) \cup (a_5, a_6) \cup (a_6, a_7) \cup (a_7, a_8) \cup (a_8, a_9) \cup (a_9, a_{10}) \cup (a_{10}, a_{11}) \cup (a_{11}, a_{12})$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0, a_2) = 2$. Therefore $G$ is a Hamiltonian-$t^*$-Laceable for $t=3$.

Figure 25: Hamiltonian path from the vertex $a_0$ to $a_3$ in Line graph $L[G_3]$
4. CONCLUSION

In this present study, the concept of Hamiltonian-t*-laseability in line graphs and t*-laceability number (are investigated. In our further work, Laceability of total graphs of other kind is to be proposed.

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