An Inflation Induced Stock-Dependent Demand Inventory Model with Permissible delay in Payment

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ABSTRACT
This paper deals with an inflation induced stock dependent demand inventory model with permissible delay in payments. In real life situations, some products maintain freshness and quality for some time. This inventory model is developed for non-instantaneous deteriorating items. The purpose of this paper is to obtain the optimal policies for maximizing the total profits. Numerical examples are provided to demonstrate the developed model and also to provide the solution algorithm.

Keywords
Inflation, Inventory, Non-instantaneous deterioration, Permissible delay in payments, Purchasing cost, Sales revenue cost, Stock-dependent demand.

1. INTRODUCTION
In the modern age the display of the products/items in large quantities in the supermarkets attracts more and more customers and generates a higher demand. Therefore the effect of inflation and stock dependent demand cannot be ignored for obtaining the optimal inventory policy. It has been observed that most researchers on inventory models do not consider permissible delay in payment and inflation simultaneously. Inflation and permissible delay in payments play an important role in the optimal order policy and influences the demand of certain products.
First in **(1975) Buzacott** considered the EOQ inventory model with inflation. During the past few decades, many researchers have developed inventory models with permissible delay in payment. **Goyal (1985)** was the first to develope an EOQ model with a constant demand rate under the condition of permissible delay in payments. **Aggarwal and Jaggi (1995)** extended Goyal's model to consider a deterministic inventory model with constant rate of deterioration. **Hariga** and **M. Ben-daya (1996)** presented an optimal time-varying lot sizing inventory models under inflationary conditions.
Gupta and **Vrat (1986)** first discussed the inventory model for consumption environment to minimize the cost with the assumption that stock-dependent consumption rate is a function of the initial stock level. **Liao et al. (2001)** presented an inventory for initial stock dependent consumption rate and permissible delay in payment.
Jaggi et al. (2006) developed an inventory model in which units are deteriorate at constant rate and demand rate increase exponentially due to inflation over a finite planning horizon using discount cash flow approach. **Pal** and **Ghosh (2006)** developed an inventory model with shortage and quantity dependent permissible delay in payment. **Soni and Shah (2008)** discussed the optimal ordering policy for an inventory model with stock-dependent demand under progressive payment scheme. **Singh** and **Malik (2009)** developed a two warehouses model with inflation induced demand under the credit period. **Chang et al. (2010)** developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. **Khanra, Ghosh,** and **Chaudhuri (2011)** have discussed an EOQ model for a deteriorating item with time-dependent quadratic demand under permissible delay in payment. **Sana, S.S., (2012)** presented an EOQ inventory model for perishable items with stock-dependent demand.
In this paper, we have discussed an inventory model with inflation and stock-dependent demand. The optimal replenishment policy for non-instantaneous deteriorating items with inflation and stock-dependent demand is discussed in this study. The necessary and sufficient conditions of the existence and uniqueness of the optimal solutions are given.
This paper is organized as follow: section 2 presents the notations and assumptions. In section 3, the inventory model is formulated, finding the optimal policy for maximum profits and solution algorithm. In section 4, numerical examples are cited to illustrate the inventory model with sensitivity analysis.

2. NOTATIONS AND ASSUMPTIONS
To develop the inventory model we have used the following notations and assumptions:

- The demand rate at time *t* is $D(t) = a + bQ(t)$, Where *a*, *b* are positive constants and Q(t) is the inventory level at time *t*.
- Shortages are not permitted and lead time is zero.
- The retailer can accumulate revenue and earn interest after customers pay for the amount of purchasing cost to the retailer untill the completion of the permissible delay in payment period offered by the supplier.
- α is the deterioration rate
- C_o is the ordering cost per order
- C_h is the inventory holding cost per unit time
- C_p is the purchasing cost per unit
- C_d is the deteriorating cost per unit
- C_s is the sales revenue cost per unit
- r is the discount rate, representing the time value of money
- i is the inflation rate
- R is the net discount rate of inflation; R = r-i
- M is the permissible delay in payment offered by supplier in months
- I_p is the interest charges per $ per month
- I_e is the interest earned per $ in stocks per month
- Q_1 is the inventory level at time [0, t_1] in which the product has no deterioration.
- Q_2 is the inventory level at time [t_1, T] in which the product has deterioration. [$T=t_1+t_2$]
- t_1 is the length of fresh product time.

- t_2 is the length in which there is deterioration in product.
- TP is the total present value of profit per unit time of inventory system.

3. MATHEMATICAL MODEL

During the interval [0, t_1], the inventory level decreases due to stock-dependent demand rate. The inventory level drops to zero due to stock-dependent demand and deterioration during the time interval is [t_1, T]. $Q_1(t)$ denotes the inventory level at time $0 \le t \le t_1$ in which the product has no deterioration, $Q_2(t)$ is the inventory level at time $t_1 \le t \le t$ in which the product has deterioration. Therefore, the inventory level at any time t can be represented by the following differential equations:

$$\frac{dQ_1(t)}{dt} = -[a + bQ_1(t)] \qquad 0 \le t \le t_1 \qquad \text{.... (1)}$$

$$\frac{dQ_2(t)}{dt} + \alpha Q_2(t) = -[a + bQ_2(t)] \qquad t_1 \le t \le T \qquad \text{.... (2)}$$

With the boundary conditions showing $Q_1(0) = L$, $Q_2(T) = 0$ respectively, solving these differential equations, we get the inventory level as follows:

$$Q_1(t) = \frac{a}{b}\left(e^{-bt} - 1\right) + Le^{-bt}, \qquad 0 \le t \le t_1 \qquad \text{.... (3)}$$

$$Q_2(t) = \frac{a}{b+\alpha}\left(e^{(b+\alpha)(T-t)} - 1\right), \qquad t_1 \le t \le T \qquad \text{....(4)}$$

Considering continuity of Q(t) at $t=t_1$, it follows from Equations (3) and (4) that $Q_1(t_1) = Q_2(t_1)$

$$\Rightarrow L = \frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right) \qquad \text{.....(5)}$$

The total present value of profit per cycle consists of the following costs:

1) The ordering cost per cycle is OC= C_o. …(6)

2) The holding cost per cycle is given as

$$HC = C_h\left(\int_0^{t_1} e^{-Rt}\, Q_1(t)dt + \int_{t_1}^{T} e^{-Rt}\, Q_2(t)dt\right)$$

$$= C_h\left[\frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{bR(R+b)} + \frac{L}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \frac{a}{b+\alpha}\left\{\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right\}\right] \text{....(7)}$$

3) The deterioration cost per cycle is given as

$$DC = C_d\int_{t_1}^{T} \alpha\, e^{-Rt}\, Q_2(t)dt$$

$$= \alpha\, C_d\left[\frac{a}{b+\alpha}\left\{\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right\}\right] \qquad \text{...(8)}$$

4) The purchasing cost per cycle is given as

$$PC = C_p \times L = C_p\left[\frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right)\right] \qquad \text{...(9)}$$

5) The sales revenue cost per cycle is given as

$$SRC = C_s\int_0^{T} e^{-Rt}\, D(t)dt$$

$$= C_s\left[\frac{a}{R}\left(1 - e^{-RT}\right) + \frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{R(R+b)} + \frac{Lb}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \left(\frac{ab}{b+\alpha}\right)\left(\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right)\right] \text{...(10)}$$

In this paper we have considered permissible delay in payment in two periods: (based on the length of T and M)

Case-1: t_1≤M≤T, in this case, the interest payable is

$$IP_1 = C_p I_p \int_M^T Q_2(t)\,dt$$

$$= \frac{aC_p I_p}{(b+\alpha)^2}\left[e^{(b+\alpha)(T-M)} - I - (T-M)(b+\alpha)\right] \qquad \text{.... (11)}$$

The interest earned is

$$IE_1 = C_s I_e\left[\int_0^{t_1} t.D(t)\,dt + \int_{t_1}^{M} t.D(t)\;dt\right]$$

$$= C_s I_e\left[\frac{aM^2}{2} - \frac{at_1e^{-bt_1}}{b} - \frac{ae^{-bt_1}}{b^2} - \frac{at_1^2}{2} + \frac{1}{b^2} - L(e^{-bt_1-1}) + \frac{ab}{b+\alpha}\left\{-M\frac{e^{(b+\alpha)(T-M)}}{(b+\alpha)} - \frac{e^{(b+\alpha)(T-M)}}{(b+\alpha)^2} - \frac{M^2}{2} + t_1\frac{e^{(b+\alpha)t_2}}{(b+\alpha)} + \frac{e^{(b+\alpha)t_2}}{(b+\alpha)^2} + \frac{t_1{}^2}{2}\right\}\right] \qquad \text{.... (12)}$$

The total profit TP_1 per cycle per unit time is given by

$$TP_1 = \frac{1}{T}\left[SRC - OC - HC - DC - PC - IP_1 + IE_1\right] \qquad \text{... (13)}$$

The necessary and sufficient conditions for TP_1 (total present value of profit per unit time) is maximum if

$$\frac{dTP_1}{dt_2} = 0 \qquad \text{.... (14)}$$

and $$\frac{d^2TP_1}{dt_2^2} < 0, \qquad \text{.... (15)}$$

Case-I1: M≥T, in this case, the no interest charges are paid for the items, i.e., $IP_2 = 0$ .... (16)

The interest earned is

$$IE_2 = C_s I_e\left[\int_0^{t_1} t.D(t)\,dt + \int_{t_1}^{M} t.D(t)\;dt + D(t)T(M-T)\right]$$

$$= C_s I_e\left[\frac{aM^2}{2} - \frac{at_1e^{-bt_1}}{b} - \frac{ae^{-bt_1}}{b^2} - \frac{at_1^2}{2} + \frac{1}{b^2} - L(e^{-bt_1-1}) + \frac{ab}{b+\alpha}\left\{-M\frac{e^{(b+\alpha)(T-M)}}{(b+\alpha)} - \frac{e^{(b+\alpha)(T-M)}}{(b+\alpha)^2} - \frac{M^2}{2} + t_1\frac{e^{(b+\alpha)t_2}}{(b+\alpha)} + \frac{e^{(b+\alpha)t_2}}{(b+\alpha)^2} + \frac{t_1{}^2}{2}\right\} + aT(T-M)\right] \qquad \text{.... (17)}$$

The total profit TP_2 per cycle per unit time is given as

$$TP_2 = \frac{1}{T}\left[SRC - OC - HC - DC - PC - IP_2 + IE_2\right] \qquad \text{...(18)}$$

The necessary and sufficient conditions for TP_2 (total present value of profit per unit time) is maximum if

$$\frac{dTP_2}{dt_2} = 0 \qquad \text{.... (19)}$$

and $$\frac{d^2TP_2}{dt_2^2} < 0 \qquad \text{.... (20)}$$

Solution Algorithm for Proposed Model

Step.1. Input C_o, C_h, C_p, C_s, C_d, α, R, a, b, M, I_p, I_e, t_1.

Step.2. **Case-I:** From equation (14) compute t_2 and from Relation (13) compute TP_1.

Case-II: From equation (19) compute t_2^* and from Relation (18) compute TP_2.

Step.3. **Case-I:** Put the value of t_2 in equation (15) to check the optimal solution. If satisfied then go to stop otherwise go to step 1 for changing the parameters values.

Case-II: Put the value of t_2^* in equation (20) to check the optimal solution. If satisfied then go to stop otherwise go to step 1 for changing the parameters values.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

To illustrate the above results, we may consider the following examples:

Ex.1. A=800, R=0.01, C_s= 100 per unit, Cp = 40 per unit, M=2 month, I_p= 0.1 per month, I_e=0.08 per month, C_h= 0.40 per unit, C_d= 0.05 per unit, α=0.40 per unit, *a*= 200 and *b*=0.2 units. From Table 1.1, we observe that the system cost (TP_1) is Maximum when t_1=1/2 and t_2=5.177 (month).

Table 1.1 Variation of demand '*a*' according to t_2, L and TP_1

	Demand part *a*		
	200	300	400
t_2	5.177	5.170	5.167
L	7964	11900	15835
TP_1	4990	7538	10087

If the demand rates (*a*) increases, then the lower time t_2, the longer order quantity (L) and total profit (TP_1) increase.

Table 1.2. Variation of demand '*b*' according to t_2, L and TP_1

	Demand part *b*		
	0.18	0.19	0.20
t_2	4.436	4.679	5.177
L	467	5626	7964
TP_1	532	2711	4990

If the demand rate (*b*) increases, then the longer time t_2, the longer order quantity (L) and total profit (TP_1) increase.

Table 1.3. Variation of Deterioration rate α according to t_2, L and TP_1

	Deterioration rate α		
	0.40	0.45	0.50
t_2	5.177	4.112	3.535
L	7964	4689	3538
TP_1	4990	2621	845

If deterioration rate (α) increases, then the time t_2, order quantity (L) and total profit (TP_1) decrease.

Table 1.4. Variation of Sales revenue cost C_s according to t_2, L and TP_1

	Sales revenue cost C_s		
	90	95	100
t_2	3.765	4.274	5.177
L	3263	4522	7964
TP_1	367	2359	4990

If the sales revenue cost (C_s) increases, then the time t_2, order quantity (L) and total profit (TP_1) increase.

Table 1.5. Variation of Purchasing cost C_p according to t_2, L and TP_1

	Purchasing cost C_p		
	40	41	42
t_2	5.177	4.659	4.314
L	7964	5767	4640
TP_1	4990	3699	2629

If purchasing cost (C_p) increases, then the time t_2, order quantity (L) and total profit (TP_1) decrease.

Table 1.6. Variation of Holding cost C_h according to t_2, L and TP_1

	Holding cost C_h		
	.35	.40	.45
t_2	5.240	5.177	5.110
L	8281	7964	7675
TP_1	5113	4990	4871

If the holding cost (C_h) increases, then the time t_2, order quantity (L) and total profit (TP_1) decrease.

Table 1.7. Variation of Ordering cost C_o according to t_2, L and TP_1

	Ordering cost C_0		
	600	700	800
t_2	5.171	5.174	5.177
L	7933	7948	7964
TP	5025	5008	4990

If the ordering cost (A) increases, then it is quite natural that the total profit (TP_1) for this purpose decrease.

Table 1.8. Variation of Inflation rate *R* according to t_2, L and TP_1

	Inflation rate *R*		
	.010	.020	.030
t_2	5.177	4.785	4.497
L	7964	6242	5207
TP_1	4990	3928	3024

If the inflation rate (R) increases, then it is quite natural that the total profit (TP_1) for this purpose decrease.

The following graphs (Fig. 1 and 2) show the relation between total profit (TP_1) and time period t_1 and t_2.

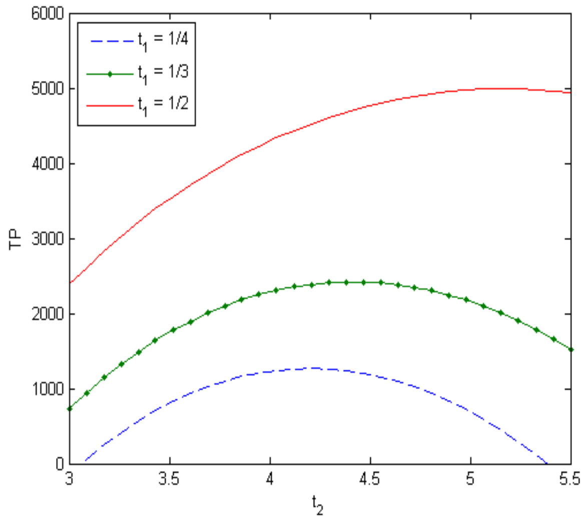

Fig 1: Total Profit TP_1 v/s t_2 for different t_1 values

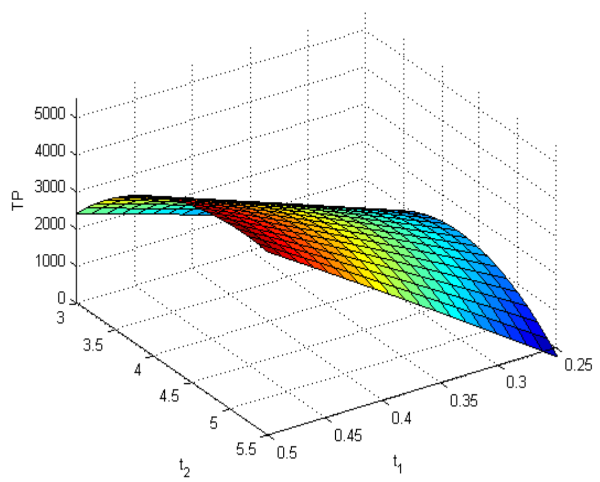

Fig 2: 3D view of Total Profit TP_1 v/s t_2 for different t_1 values

Therefore we see that in the above study the sensitivity analysis of the parameters present in this inventory model shows the total profit changes significantly with changes in the different parameters.

Ex.2. A=800, R=0.01, C_s= 100 per unit, Cp = 40 per unit, M=7.5 month, I_e=0.08 per month, C_h= 0.40 per unit, C_d= 0.05 per unit, α=0.40 per unit, *a*= 200 and *b*=0.2 units. From Table 2.1, we observe that the system cost (TP_2) is Maximum when t_1=1/2 and t_2^*=2.757 (month).

Table 2.2. Variation of demand 'a' according to t_2^*, L and TP_2

	Demand part *a*		
	200	300	400
t_2^*	2.757	2.671	2.632
L	1662	2348	3047
TP_2	19183	28868	38555

If the demand rates (*a*) increases, then the lower time t_2^*, the longer order quantity (L) and total profit (TP_2) increase.

Table 2.2. Variation of demand 'b' according to t_2^*, L and TP_2

	Demand part *b*		
	0.18	0.19	0.20
t_2^*	2.955	2.842	2.757
L	1821	1726	1662
TP_2	15559	17470	19183

If the demand rate (*b*) increases, then the lower time t_2^*, the lower order quantity (L) and total profit (TP_2) increase.

Table 2.3. Variation of Deterioration rate α according to t_2^*, L and TP_2

	Deterioration rate α		
	0.40	0.45	0.50
t_2^*	2.757	1.910	1.566
L	1662	941	734
TP_2	19183	18861	18742

If deterioration rate (α) increases, then the time t_2^*, order quantity (L) and total profit (TP_2) decrease.

Table 2.4. Variation of Sales revenue cost C_s according to t_2^*, L and TP_2

	Sales revenue cost C_s		
	90	95	100
t_2^*	1.846	2.106	2.757
L	851	1039	1662
TP_2	15590	17326	19183

If the sales revenue cost (C_s) increases, then the time t_2^*, order quantity (L) and total profit (TP_2) increase.

Table 2.5. Variation of Purchasing cost C_p according to t_2^*, L and TP_2

	Purchasing cost C_p		
	40	41	42
t_2^*	2.757	2.320	2.122
L	1662	1218	1052
TP_2	19183	18721	18306

If purchasing cost (C_p) increases, then the time t_2^*, order quantity (L) and total profit (TP_2) decrease.

Table 2.6. Variation of Holding cost C_h according to t_2^*, L and TP_2

	Holding cost C_h		
	.35	.40	.45
t_2^*	2.887	2.757	2.675
L	1819	1662	1570
TP	19218	19183	19151

If the holding cost (C_h) increases, then the time t_2^*, order quantity (L) and total profit (TP_2) decrease.

Table 2.7. Variation of Ordering cost C_o according to t_2^*, L and TP_2

	Ordering cost C_o		
	600	700	800
t_2^*	2.671	2.712	2.757
L	1566	1611	1662
TP_2	19245	19214	19183

If the ordering cost (A) increases, then it is quite natural that the total profit (TP) for this purpose decrease.

Table 2.8. Variation of Inflation rate *R* according to t_2, L and TP

	Inflation rate *R*		
	.010	.020	.030
t_2^*	2.757	2.310	2.101
L	1662	1209	1036
TP_2	19183	18798	18467

If the inflation rate (R) increases, then it is quite natural that the total profit (TP_2) for this purpose decrease.

The following graphs (Fig. 3 and 4) show the relation between total profit (TP_2) and time period t_1 and t_2^*.

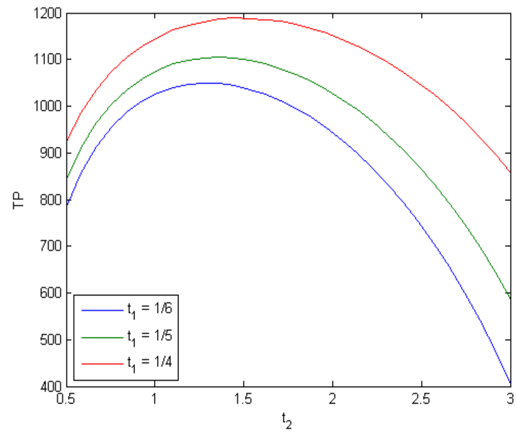

Fig 3: Total Profit TP_2 v/s t_2^* for different t_1 values

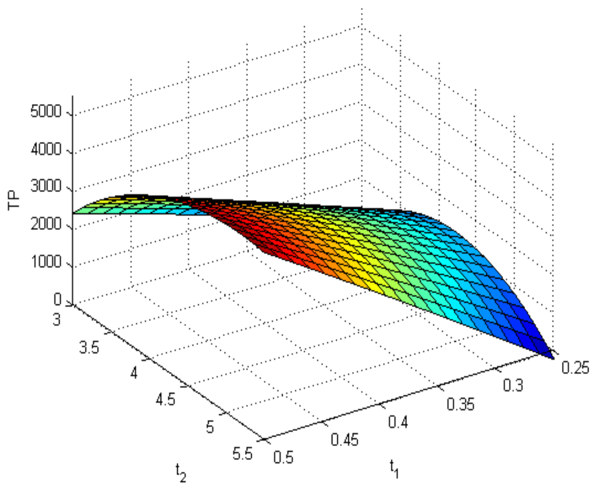

Fig 4: 3D view of Total Profit TP_2 v/s t_2^* for different t_1 values

It is observed that the sensitivity analysis of the parameters present in this inventory model shows that the total profit changes significantly with changes in the different parameters values.

5. CONCLUSION

This paper presents the concept of an inventory control system against the non-instantaneous deteriorating items with inflation and permissible delay in payments. Two numerical examples have been considered in this inventory model to illustrate the results and the significant features of the results are also discussed. A possible future research direction is the study of an inventory model for production rate, shortages, partial backlogging, two warehouses, linear demand and quadratic demand etc.

6. ACKNOWLEDGMENTS

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