Proposed Image Similarity Measurement Model based on Hypergraph

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ABSTRACT
In this article, we propose a new image similarity measurement model (SMM) based hypergraph which is easy to calculate and applicable to various image processing application. Hypergraphs are now used in many domains such as chemistry, engineering and image processing. We present an overview of a hypergraph-based Image representation and the Image Adaptive Neighborhood Hypergraph (IANH). With the IANH it is possible to build a new powerful similarity measurement model. Although the new model is mathematically defined and no human visual system model is explicitly employed, our experiments on various image distortion types indicate the efficient of proposed model.

General Terms
Hypergraphs, Image Similarity.

Keywords
Image Similarity; Image Processing; Similarity Measurement Model .

1. INTRODUCTION
Graphs are very powerful tools for describing many problems and structures in computer sciences but also in physic and mathematics. But graphs only describe some binary relations and are not always sufficient for modeling some complex problems or data. Hypergraph theory, originally developed by C. Berge [8] in 1960, is a generalization of graph theory. The idea consists in considering sets as generalized edges and then calling a hypergraph the family of these edges. This concept models more general types of relations than graph theory do. In the last decades, the theory of hyper graphs has proved to be of a major interest in applications to real-world problems. These mathematical frameworks can be used to model networks, data structures, process scheduling, computations, and a variety of other systems where complex relations between the objects in the system play a dominant role. To any digital image, a hypergraph, the Image Adaptive Neighborhood Hypergraph (IANH), can be associated and used for image processing. Many publications were written about this hypergraph model and many applications were found [1, 2, 4, 6]. This paper is present a new Similarity Measurement Model (SMM) based on this adaptive neighborhood hypergraph model. First, we give basic definitions about hypergraphs and the definition of the IANH. Then we present an algorithm building the IANH. Finally we illustrate applications of the IANH to the image Similarity Measurement. We give a powerful algorithm based on the adaptive neighborhood hypergraph associated to an image. A set of examples is shown to illustrate the effectiveness of the algorithm.

2. BACKGROUND AND DEFINITIONS
The general terminology concerning graphs and hypergraphs is similar to [8,7]. All graphs in this paper are, finite, undirected, connected with no isolated vertex and simple, i. e. graphs with no loops or multiple edges. We denote a graph G = (V ; E). Given a graph G, we denote by $\Gamma(x)$ the neighborhood of a vertex x, i. e. the set consisting of all vertices adjacent to x which is defined by $\Gamma(x) = \{y \in V, \{x, y\} \in E\}$.

A hypergraph H on a set X is a family $(E_i)_{i\in I}$ of non-empty subsets of X called hyperedges with;

$$\bigcup_{i\in I} E_i = X, I = \{1,2, \ldots\ldots, n\}, n \in N$$

Let us note H = (S; $(E_i)_{i\in I}$). For x ∈ S, a star of H (with center x) is the set of hyperedges which contains x, and is called H(x). The degree of x is the cardinality of the star H(x) denoted by dx = Card (H(x)).

Let H = (S; $E = (E_i)_{i\in I}$) be a hypergraph, the dual hypergraph H^* is the hypergraph such that the set of vertices is the set of hyperedges, and the set of hyperedges is the set of stars of H. We can represent a hypergraph as in figure 1-(a).

A hyperedge E is isolated if and only if:

$$\forall j \in I, j \neq i, \text{if } E_i \cap E_j \neq \emptyset \text{ then } E_j \subseteq E_i$$

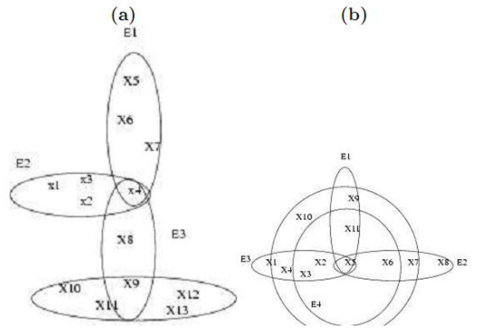
An important structure from a hypergraph is the notion of intersecting family. A family of hyperedges is an intersecting family if the hyperedges from this family intersect two by two. We can distinguish two types of intersecting families:

- Intersecting families with an empty intersection.

- Intersecting families with an non empty intersection.

A hypergraph has the HELLY property if each family of hyperedges intersecting two by two (intersecting family) has a non empty intersection (belongs to a star). As example in figure 1-(a) the hypergraph has the HELLY property. Figure 1-(b) shows these two types of intersecting hyperedges. To each graph one can associate a hypergraph. Indeed, let G = (X; E) be a graph, the hypergraph having the vertices of G as vertices and the neighborhood of these vertices as hyperedges (including these vertices) is called the neighborhood hypergraph of G and is denoted by:

$$H_G = (X, E_x = \{x\} \cup \Gamma(x)))$$



Figure(1): **(a) Example of hypergraph, the set of vertices is $\{x_1, x_2, \dots\dots x_{13}\}$ and the set of hyperedges is $\{E_1, E_2, E_3, E_4\}$. (b) We have two types of intersecting families the first is the star the second has an empty intersection.**

3. IMAGE ADAPTIVE HYPERGRAPH MODEL

First we recall some definitions about digital images. A distance d'on **X** defines a *grid* (a graph connected, regular, without both loop and multi-edge). A *digital image* (on a grid) is a two-dimensional discrete function that has been digitized both in spatial coordinates and in magnitude feature value. Throughout this paper a digital image will be represented by the application I : X $\subseteq Z^2 \rightarrow \mathcal{C} \subseteq Z^n$ with $n \geq 1$ where $\mathcal{C}$ identifies the *feature intensity level* and **X** identifies a set of points called *image points*. The couple (*x; I*(*x*)) is called a *pixel*. Let d be a distance on $\mathcal{C}$, we have a neighborhood relation on an image defined by:

$$\forall x \in X, \Gamma_{\alpha,\beta}(x) = \{x' \in X, x' \neq x | d(\mathfrak{I}(x) - \mathfrak{I}(x')) < \alpha \text{ and } d'(x, x') \leq \beta\} \quad (1)$$

The neighborhood of x on the grid will be denoted by $\Gamma_\beta(x)$.So to each image we can associate a hypergraph called *Image Adaptive Neighborhood Hypergraph* (IANH): $H_{\alpha,\beta} = (X, (\{x\} \cup \Gamma_{\alpha,\beta}(x))_{x \in X}$. The attribute α can be computed in an adaptive way depending on local properties of the image. If α is constant the hypergraph is called the *Image Neighborhood Hypergraph* (INH). Throughout this paper α will be estimated by the standard deviation of the pixels $\{x\} \cup \Gamma_\beta(x)$. The following algorithm summarize the technique of Image Adaptive Neighborhood Hypergraph:

Data structures used: For each $x, T_{\alpha,\beta}(x)$ is a table of tables of Booleans, so $E_{\alpha,\beta}$is $m_x \times m_y$ table of tables. The set X is a $m_x \times m_y$ table of Booleans**.**

Proposition 1. *Given β , the algorithm converges to a unique solution. Its com-plexity is in O(n) (n standing for the pixel number of the image). (For the proof report to [2]).*

Algorithm: Image Adaptive Neighborhood Hypergraph

Construction of the hypergraph $H_{\alpha,\beta}$

Data: Image I of size $m_x \times m_x$, and neighborhood order β

X = ∅;

For each pixel x **of** I, **do** ;

$\alpha =$ the standard deviation of the pixels $\{x\} \cup \Gamma_\beta(x)$.

$\Gamma_{\alpha,\beta}(x) = \emptyset$;

For each pixel y **of** $\Gamma_\beta(x)$, **do**

if $d(I(x) - I(y)) \leq \alpha$ **then**

$\Gamma_{\alpha,\beta}(x) = \Gamma_{\alpha,\beta}(x) \cup \{y\}$.

end if

end for

$X = X \cup \{x\}$; $E_{\alpha,\beta}(x) = \{\Gamma_{\alpha,\beta}(x) \cup \{x\}\}$.

end for

$H_{\alpha,\beta} = (X, (E_{\alpha,\beta}(x)))_{x \in X}$

End

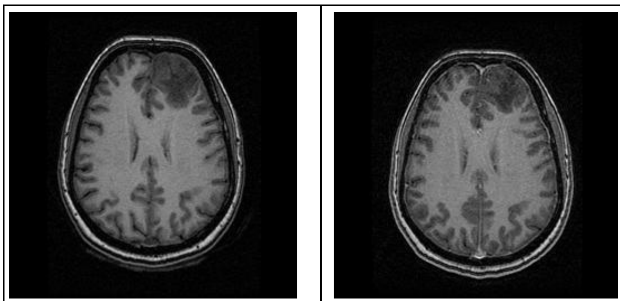
4. PROPOSED IMAGE SIMILARITY MEASUREMENT MODEL

In this section, we propose new tools for defining similarity measures based on statistical approach. Let x and y be the original and the test images with the dimension of $N \times M$. The proposed similarity measures model is defined as:

$$SMM = 1 - \frac{\partial(V_{H_{x,y}})}{N \times M} \quad (2)$$

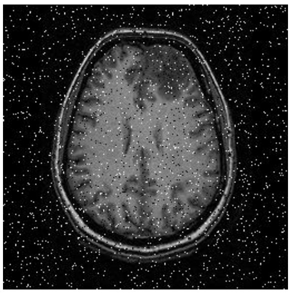
$$V_{H_{x,y}} = V_{H_x} - V_{H_y} \quad (3)$$

where N, M represent the dimension of image matrix, ∂ is the number of zero items of the vector $V_{H_{x,y}}$,and V_{H_x} ,V_{H_y} are the hypergraph vector of image x and image y respectively.

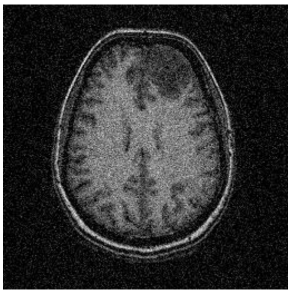


(A) (B)

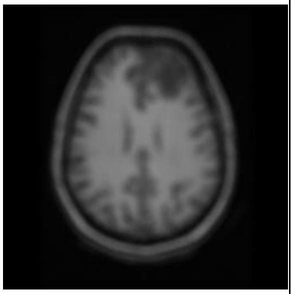
Figure (2): Evaluation of CT brain images section taken at two different times, We see an evolution of the pathological area in the top right, SMM= 0.6269.



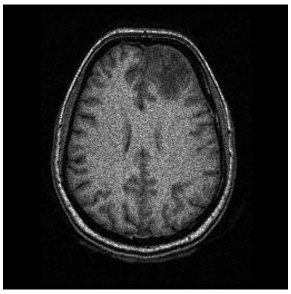
(C) Slat& pepper



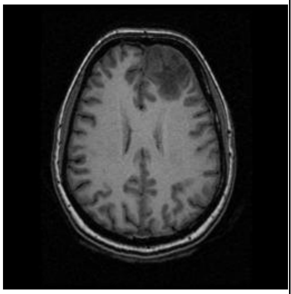
(D) Gaussian



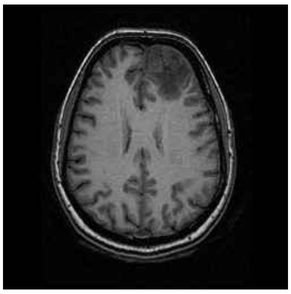
(E) Blurring



(F) Speckle



(G) Stretching



(H) Compression

Figure (3): The CT brain test image in (A) is distorted by a wide variety of corruptions: (C) impulsive salt-pepper noise, (D) additive Gaussian noise, (E) blurring (F) multiplicative speckle noise, (G) contrast stretching, (H) JPEG compression

Table 1: **Evaluation of "CT Brain Image with Different Types of Distortions**

Image	**Distortion Type**	**SSM**
Fig. 3 (C)	Impulsive Salt-pepper Noise	**0.8242**
Fig. 3 (D)	Additive Gaussian Noise	**0.3467**
Fig. 3 (E)	Blurring	**0.7037**
Fig. 3 (F)	Multiplicative Speckle Noise	**0.8331**
Fig. 3 (G)	Contrast stretching	**0.8778**
Fig. 3 (H)	JPEG Compression	**0.4715**

5. CONCLUSION

A new algorithm based on hypergraph for defining similarity measures model (SMM) is proposed in this paper. Our experimental results indicate the performance of proposed approach under different types of image distortions. So, a simple implementation of the new philosophy exhibits very promising results. The proposed SMM can be used to statistically distinguish any test images with different kinds of distortions.

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