An Interactive Method for Two-plants Production Inventory Control with Two-warehouse Facility under Imprecise Environment

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ABSTRACT

This paper develops an integrated production inventory model with an aim to minimize the cost of production per unit of the product and maximization of profit without compromising the quality of the product. In this model we consider two plants, two secondary warehouses (SWs) and two showrooms (SRs) those are adjacent to the respective plants. For the purpose, we assume to be erected by a firm two plants: Plant-I and Plant-II. Plant-I is situated in an urban area with a lower production capacity in comparison to its product demand where the demand meets through SR-I. On the other hand, Plant-II is situated in a rural area with higher production capacity in comparison to its product demand where the demand meets through SR-II. The excess production in Plant-II meets the current market demand in the area of Plant-I. Here, demand is assumed to be stock dependent in both the showrooms (SR-I and SR-II). Average profit in the integrated model is calculated and global optimum is obtained through a descriptive-cum-analytical review. The inventory parameters are taken as fuzzy numbers. The fuzzy numbers are first transformed into corresponding interval numbers and then follow the interval mathematics, the objective function for average profit is converted into respective multi-objective functions. Furthermore, the objective functions are being maximized and solved for a Pareto-optimum solution by interactive fuzzy decision-making procedure. The model also illustrates graphically and numerically.

General Terms:

Two-plants, Two-warehouse,

Keywords:

Inventory, Two-plants, Two-warehouse, Stock dependent demand, Fuzzy inventory cost parameters, Interactive decision making method.

1. INTRODUCTION

The works of Urban[34], Pal et.al.[30], Giri et.al.[11], Padmanabhan and Vrat [28], Sarkar et al[32], Giri and Chaudhuri[12] and others were developed for a single warehouse under the basic assumption that the available warehouse has unlimited storage capacity. However, this assumption is not realistic. Any warehouse has fi-

nite storage capacity. On the other hand, inventory management is generally attracted for large stock for several reasons - an attractive price discount for bulk purchase; the replenishment cost including transportation cost is higher than the inventory related cost; the demand of an item is very high and so on. Therefore, due to space limitation of showroom (SR), one (or sometimes more than one) warehouse(s)are hired on rental basis to store the excess items.

The secondary warehouse (SW) may be located away from or nearer to SR. The actual service to the customer is done at SR only. As the holding cost in SW is greater than in SR, the stocks of SW are emptied first transporting the stocks from SW to SR by continuous or bulk release patterns in order to reduce the holding cost. Hartely [17]first introduced the basic two warehouse problem in his book "Operations Research - A Managerial Emphasis". In his analysis, he ignored the cost of transportation for transferring the items from SW to SR and proposed a heuristic procedure for determining the optimal order quantity. After Hartely[17], a number of research papers have been published by the different authors. Among them, the works done by Sarma[33], Dave[8],Goswami and Chaudhuri[13], Pakkala and Achary[29], Bhunia and Maiti[3,5] , Benkherouf[2], Zhou[37] and Kar et. al.[19] are worth mentioning. However, all these models were based on an assumption that the rented warehouse has unlimited (infinite) storage capacity. Chung and Huang[7]proposed a two-warehouse inventory model for deteriorating items under a permissible delay in payments, but they assumed that the deterioration rate of the two warehouses were the same. Dey et al.[10] considered a finite time horizon inventory problem for a deteriorating item having two separate warehouses with interval-valued lead-time under inflation and a time value of money. Niu and Xie[27] modified Pakkala and Achary's[29]model (last-in-first-out) where inventory in the SW was stored last but would be consumed before those in the SR. Hsieh et al.[18] developed a deterministic inventory model for deteriorating items with two warehouses allowing for shortages and assuming the inventory costs in the SW to be higher than those in the SR. Lee and Hsu [20] extended the Lee and Ma[21],'s model using an approach which permitted variable production cycle times instead of equal production cycle times. Chung et al.[6] extended a two-warehouse inventory model with an imperfect quality production processes. Liao and Huang[24] studied an order-level inventory model for deteriorating items with two-storage facilities and a permissible delay in payment. Hariga[16] proposed an EOQ

model with multiple storage facilities where both owned and rented warehouses had limited stock capacity. Liang and Zhou[23] investigated a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payments. They assumed the rented warehouse had higher unit holding costs than the own warehouse but offered better preservation resulting in a lower rate of deterioration for the goods than in the own warehouse. In reality there are many situations in business sector where the demand rate is not constant but varies. It may depend on time, initial or on hand inventory levels, selling price, advertisement expenditure, the frequency of advertisement etc. There are certain types of items (like consumer goods, fashionable items etc.) for which, according to market research, customers are motivated by the display of the items in the showrooms i.e., the demand rate is dependent on the displayed inventory level. For these items, the consumption goes up if the inventory level is high and vice versa. Many researchers like Levin et al.[22] have observed that the presence of greater quantity of the same item tends to attract more customers. This implies that holding higher inventory level will probably make the retailer sell more items. Under this situation, the demand rate should depend on the inventory level. Such type of demand was considered by Urban[34], Gupta and Vrat[15], Mondal and Phaujdar [25,26], Urban [34,35], Bhuina and Maiti[4] etc. Zhou and Yang[38] presented a two-warehouse inventory model with stock-level-dependent demand rate.

In this study, we considered a production inventory model of two production plants (plant-I and plant-II), two secondary warehouses (SW-I and SW-II) and two showroom-cum-retail outlets (SR-I and SR-II). Production plant-I is situated in an urban area where the density of population is considerably high and thereby there is a high market demand of the product produced in plant-I. The cost of the factors of production in plant-I viz., raw-materials, labour, power, water supply, rent of the factory building etc. is very high and thereby the cost of the production per unit of the output is also very high. In this plant, the rate of production is less in comparison to the market demand of the output produced in plant-I. The plant-I has been erected in the area concerned, where there is a scarcity of the factors of production albeit the locational and other advantages for industrial output are prevalent in the area concerned. In order to minimize the total cost of production per unit of the output produced in both the plants and at the same time to meet the high market demand of the product exits in the area of plant-I, another plant-II has been erected in a rural area where the factors of production required in plant-I is less costly albeit the market demand of the output in the area of plant-II is less than that of the market demand in the area of plant-I. So, the cost of production per unit of output produced in Plant-II is less than that of the cost of production per unit of output produced in plant-I. It is mentioned here that the quality standard of the products produced in both the plants are maintained.

There is a large volume of literature on the 'two warehouse inventory model'. The literature suggests that the holding cost of secondary warehouse per unit is more than that of the holding cost per unit of showroom cum-outlet. Furthermore, the literature suggests that the holding cost per unit in the secondary warehouse is high due to the preservation cost for maintaining the quality of the product and other costs related to handling large quantity of the product in the secondary warehouse. In the present model it is presumed that the holding cost of secondary warehouses is less than the holding cost of showrooms cum-outlets inasmuch as the nature of the output/product produced in both the plants are non-deteriorating and without having any preservation cost. But it is important to mention here that both the secondary warehouses are located adjacent to the corresponding showroom-cum-retail outlets and the transportation cost from both the warehouses to showrooms is insignificant, and hence avoided. Furthermore, the product stored in SW-II is transferred to SW-I through bulk-release-pattern and corresponding transportation cost between two warehouses has been taken into account in the model. This apart, output transferred from both the plants to the showrooms-cum-retail outlets in a continuous-release-pattern. The customer service made from the both the showrooms and the products of both the plants transformed on a regular basis / constantly from both the warehouses to both the showrooms to fulfil the demand (as the demand of the product is stock-dependent) even though the holding cost per unit is higher than in showrooms than that of warehouses.

Average profit of the integrated model has been calculated and global optimum was obtained analytically. Here inventory cost parameters are taken imprecise i.e, fuzzy in nature. The said parameters are expressed by fuzzy numbers, Which are then converted into appropriate interval numbers following Grzegorzewski[14] and using the concepts of interval arithmetic, we have constructed an equivalent multi-objective deterministic model corresponding to the original problem. This equivalent problem has been solved by using interactive fuzzy decision making procedure. The Optimum and Pareto-optimum solutions are derived by using a gradient non-linear optimization technique-Generalized Reduced Gradient (GRG) method.

2. THE NEAREST INTERVAL APPROXIMATION OF A FUZZY NUMBER

According to Grzegorzewski[14], the nearest interval approximation of a Triangular fuzzy number(TFN) $\tilde{A} = (a_1, a_2, a_3)$ is $\left(\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}\right)$ and the nearest interval approximation of a Parabolic fuzzy number(PFN) $\tilde{A} = (a_1, a_2, a_3)$ is $\left(\frac{2a_1 + a_2}{3}, \frac{a_2 + 2a_3}{3}\right)$.

3. ASSUMPTIONS AND NOTATIONS

Following are the assumptions and notations to develope the model:

3.1 Assumptions

(i) Model is developed for single item product.

(ii) Lead time is negligible.

(iii) Demand of customers in the showrooms SR-I and SR-II is stock dependent.

(iv) Shortage are allowed only in SR-I.

(v) Idle costs are taken into account.

(vi) Production capacities of two plants are different.

3.2 Notations

$q_{1s}(t)$ = Inventory level of SR-I at time t.

$q_{1r}(t)$ = Inventory level of SW-I at time t.

$q_{2s}(t)$ = Inventory level of SR-II at time t.

$q_{2r}(t)$ = Inventory level of SW-II at time t.

P_1 = Constant production rate of plant-I
$(P_1 < a_1, a_1 > 0)$.

P_2 = Constant production rate of plant-II.

W_1 = Capacity of SR-I.

W_2 = Capacity of SR-II.

$D_1(t)$ = Demand rate of SR-I,where

$$D_1(t) = \begin{cases} a_1 & \text{when } 0 \le t < t_1 \\ a_1 + b_1 W_1 & \text{when } t_1 \le t < t_2 + k' \\ a_1 + b_1 q_{1s}(t) & \text{when } t_2 + k' \le t \le T_{p_1} \end{cases}$$

a_1, b_1 are constant.

$D_2(t)$ = Demand rate of SR-II, where

$$D_2(t) = \begin{cases} a_2 & \text{when } 0 \le t < t_1' \\ a_2 + b_2 W_2 & \text{when } t_1' \le t < t_3 \\ a_2 + b_2 q_{2s}(t) & \text{when } t_3 \le t \le T_{p_2} \end{cases}$$

a_2, b_2 are constant.

S_1 = Shortage amount in SR-I at time t_1.

c_{1s} = Shortage cost per unit per unit time in SR-I of plant-I.

h_{1s} = Holding cost per unit per unit time of SR-I.

h_{1r} = Holding cost per unit per unit time of SW-I.

h_{2s} = Holding cost per unit per unit time of SR-II.

h_{2r} = Holding cost per unit per unit time of SW-II.

c_{p_1} = Production cost per unit item for plant-I.

c_{p_2} = Production cost per unit item for plant-II.

s_1 = Selling price per unit item in SR-I.

s_2 = Selling price per unit item in SR-II.

p_1' = Transfer rate of produce items from SW-I to SR-I during (t_1, t_{p_1}).

p_1'' = Transfer rate of produce items from SW-I to SR-I during $(t_{p_1}, t_2 + k')$.

k = Time duration of shifting the amount kp_1' from SW-II to SW-I during (t_1, t_{p_1}).

k' = Time duration of shifting the amount $k'p_1''$ from SW-II to SW-I during (t_{p_1}, t_2).

t_1 = Time where SR-I received of amount$(S_1 + W_1)$.

t_{p_1} = Time where plant-I stop its production.

t_{p_2} = Time where plant-II stop its production.

T_{p_1} = Time where items are exhausted from SR-I.

T_{p_2} = Time where items are exhausted from SR-II.

t_2 = Time of shifting last lot from SW-II to SW-I.

t_3 = Time where items are exhausted from SW-II.

A_{1s} = Ordering cost for SR-I.

A_{2s} = Ordering cost for SR-II.

A_{1r} = Ordering cost for SW-I.

A_{2r} = Ordering cost for SW-II.

id_{1s} = Idle cost per unit time of SR-I.

id_{2s} = Idle cost per unit time of SR-II.

id_{1r} = Idle cost per unit time of SW-I.

id_{2r} = Idle cost per unit time of SW-II.

n = Number of times to shifting the amount $p_1'k$ in each time from SW-II to SW-I during $[t_1, t_{p_1}]$.

r = Number of times to shifting the amount $p_1''k'$ in each time from SW-II to SW-I during $[t_{p_1}, t_{p_2}]$.

$m - r$ = Number of times to shifting the amount $p_1''k'$ in each time from SW-II to SW-I during $[t_{p_2}, t_2]$.

4. MODEL DESCRIPTION AND DIAGRAMMATIC REPRESENTATION

The production plants (i.e plant-I and plant-II), starts their production at time $t = 0$ and produced items are selling through the showrooms(i.e SR-I and SR-II). In plant-I, the production rate is lower than the demand rate of SR-I and thus shortages occurs at SR-I for time period $(0, t_1)$. In plant-II, the production rate is higher than demand rate of SR-II, so excess items are stored in the secondary ware house SW-II at the rate $P_2 - D_2$ in continuous release pattern. At time t_1, for one time, the items of amount $(S_1 + W_1 + kp_1')$ are transferred from SW-II to fulfill the shortage amounts S_1 at SR-I, to filled up the showroom SR-I by the amount W_1 and rest amount kp_1' is stored in the secondary ware house SW-I. Items are then transferred from SW-I to SR-I in continuous release pattern at the rates p_1' and p_1'' during the period (t_1, t_{p_1}) and $(t_{p_1}, t_2 + k')$ respectively to filled-up showroom SR-I continuously, as the demand rate at SR-I is stock depended. The items are transferred from SW-II to SW-I in bulk release pattern of amount kp_1' with time interval k and of amount $k'p_1''$ with time interval k' for n and m times during the periods (t_1, t_{p_1}) and (t_{p_1}, t_2) respectively. The amount kp_1' is transferred r times among m times with time interval k' for the period (t_{p_1}, t_{p_2}). As demand is stock dependent at SR-II, so the items are continuously transferred with rate D_2 to filled-up the showroom SR-II from the production plant-II and SW-II during $(0, t_{p_2})$ and (t_{p_2}, t_3) respectively.

Mathematical formulation for different showrooms and secondary warehouses are depicted in different sub-section. The block diagrams of production inventory model is given in Fig.-1.
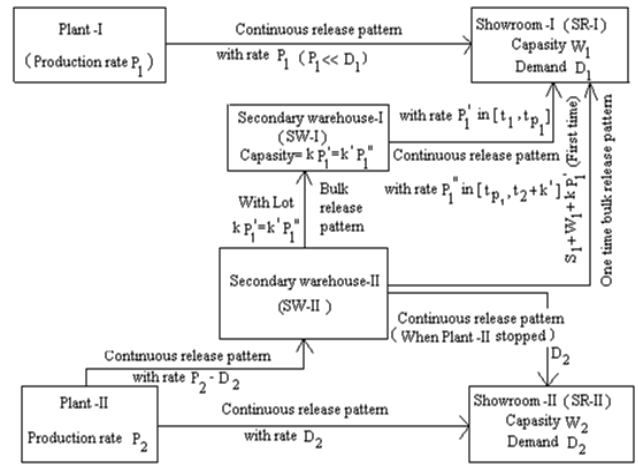

Fig. 1. Block diagram of the production inventory model

5. MATHEMATICAL FORMULATION OF THE MODEL

5.1 For Showroom-I(SR-I)

Differential equation for the SR-I in $[0, T_{p_1}]$ is given by

$$\frac{dq_{1s}}{dt} = \begin{cases} P_1 - a_1, & 0 \le t < t_1 \\ 0, & t_1 \le t < t_2 + k' \\ -[a_1 + b_1 q_{1s}(t)], & t_2 + k' \le t \le T_{p_1} \end{cases} \quad (1)$$

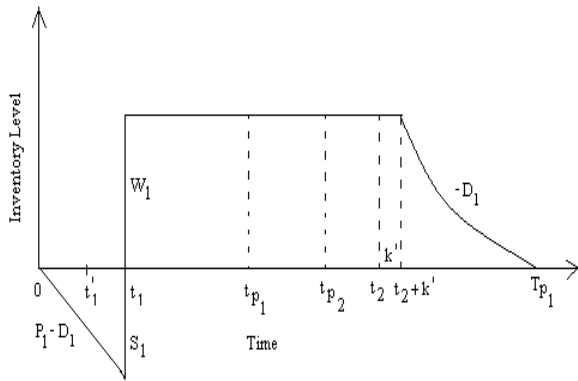with boundary condition $q_{1s}(t) = 0$ at $t = 0, T_{p_1}$, $q_{1s}(t_1 - 0) =$

Fig. 2. Inventory level of SR-I

$-S_1$
and continuity condition $q_{1s}(t) = W_1$ during $[t_1, t_2 + k']$.
Solving the differential equation (1) with boundary conditions, we have

$$q_{1s}(t) = \begin{cases} (P_1 - a_1)t, & 0 \le t < t_1 \\ W_1, & t_1 \le t < t_2 + k' \\ \frac{a_1}{b_1}\left(e^{b_1(T_{p_1}-t)} - 1\right), & t_2 + k' \le t \le T_{p_1} \end{cases}$$

Here $\quad S_1 = A + Bk \quad$ where A and B are given by

$$A = (a_1 - P_1)\left[\frac{t_1'(P_2 - (a_2 + b_2 W_2)) + W_1}{(P_2 - (a_2 + b_2 W_2)) - (a_1 - P_1)}\right]$$

$$\text{and, } B = (a_1 - P_1)\left[\frac{P_1'}{(P_2 - (a_2 + b_2 W_2)) - (a_1 - P_1)}\right]$$

Also, $P_1 + p_1' = a_1 + b_1 W_1$, $p_1'' = a_1 + b_1 W_1$ and $T_{p_1} = t_2 + k' + \frac{1}{b_1}log(1 + \frac{b_1 W_1}{a_1})$

$S_{1s} = $ Inventory Shortage cost of SR-I

$$= c_{1s}\left[-\int_0^{t_1}(P_1 - a_1)tdt\right] = -c_{1s}\left[(P_1 - a_1)\frac{t_1^2}{2}\right]$$

$H_{1s} = $ Inventory holding cost H_{1s} of SR-I

$$= h_{1s}\left[W_1(t_2 + k' - t_1) + \int_{t_2+k'}^{T_{p_1}}\frac{a_1}{b_1}\left(e^{b_1(T_{p_1}-t)} - 1\right)dt\right]$$

$$= h_{1s}\left[W_1(t_2 + k' - t_1) + \frac{W_1}{b_1} - \frac{a_1}{b_1^2}log(1 + \frac{b_1 W_1}{a_1})\right]$$

$Id_{1s} = $ The idle cost of SR-I $= id_{1s}(T_{p_2} - T_{p_1})$

5.2 For Showroom-II (SR-II)

Differential equation for the SR-II in $[0, T_{p_2}]$ is given by

$$\frac{dq_{2s}}{dt} = \begin{cases} P_2 - (a_2 + b_2 + W_2 q_{2s}(t)), & 0 \le t < t_1' \\ 0, & t_1' \le t < t_3 \\ -(a_2 + b_2 + W_2 q_{2s}(t)), & t_3 \le t \le T_{p_2} \end{cases} \quad (2)$$

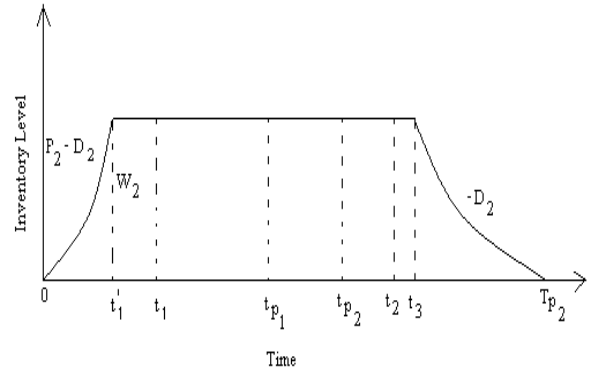with boundary condition $q_{2s}(t) = 0$, at $t = 0, T_{p_2}$, and continuity

Fig. 3. Inventory level of SR-II

condition $q_{2s}(t) = W_2$ in $[t_1', t_3]$.
Solving the differential equation(2) with boundary conditions, we have

$$q_{2s}(t) = \begin{cases} \frac{(P_2 - a_2)}{b_2}(1 - e^{-b_2 t}), & 0 \le t < t_1' \\ W_2, & t_1' \le t < t_3 \\ \frac{a_2}{b_2}\left(e^{b_2(T_{p_2}-t)} - 1\right), & t_3 \le t \le T_{p_2} \end{cases}$$

$$\text{with } t_1' = -\frac{1}{b_2}log(1 - \frac{b_2 W_2}{a_2})$$

$$\text{and, } T_{p_2} = t_3 + \frac{1}{b_2}log(1 + \frac{b_2 W_2}{a_2}) = U + Vk$$

where U and V are given by

$$U = (1 + \frac{P_2}{a_2 + b_2 W_2})$$
$$\left[\frac{(a_1 - P_1)\left[\frac{t_1'(P_2-(a_2+b_2 W_2))+W_1}{(P_2-(a_2+b_2 W_2))-(a_1-P_1)}\right] + W_1}{P_2 - a_2 - b_2 W_2}\right]$$
$$+ \frac{1}{b_2}log(1 + \frac{b_2 W_2}{a_2}) > 0$$

$$V = (1 + \frac{P_2}{a_2 + b_2 W_2})\frac{p_1'}{(P_2 - a_2 - b_2 W_2) - (a_1 - P_1)}$$
$$+ n(\frac{P_2 + a_2 + b_2 W_2}{a_2 + b_2 W_2}) + \frac{m(a_2 + b_2 W_2) + rP_2}{a_2 + b_2 W_2}\frac{p_1'}{p_1''}$$
$$- \frac{m + n}{a_2 + b_2 W_2}p_1''$$

$H_{2s} = $ Inventory holding cost of SR-II

$$= h_{2s}\left[W_1(t_3 - t_1) + \frac{(P_2 - a_2)}{b_2}\int_0^{t_1'}(1 - e^{-b_2 t})dt\right.$$

$$+ \frac{a_2}{b_2} \int_{t_3}^{T_{p_2}} \left(e^{b_2(T_{p_2}-t)} - 1 \right) dt \Big]$$

$$= h_{2s} \left[\frac{P_2 - a_2}{b_2} \left[t_1^{'} + \frac{e^{-b_2 t_1^{'}}}{b_2} \right] W_2(t_3 - t_1) \right.$$

$$+ \left. \frac{W_2}{b_2} - \frac{a_2}{b_2^2} log(1 + \frac{b_2 W_2}{a_2}) \right]$$

5.3 For Secondary Warehouse-I (SW-I)

Differential equation for the SW-I in $[t_1, t_2 + k^{'}]$ is given by

$$\frac{dq_{1r}}{dt} = \begin{cases} -p_1^{'}, & t_1 \leq t < t_{p_1} \\ -p_1^{''}, & t_{p_1} \leq t \leq t_2 + k^{'} \end{cases} \quad (3)$$
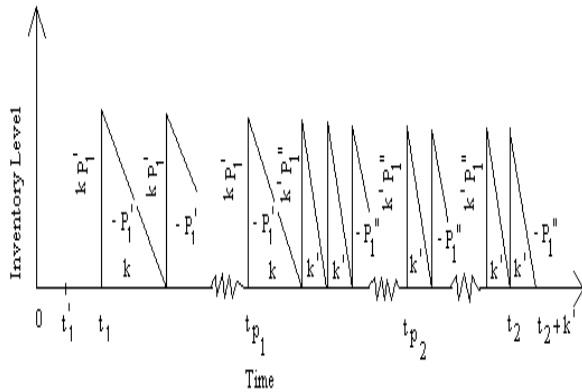
with boundary conditions

Fig. 4.   Inventory level of SW-I

$$q_{1r}(t) = \begin{cases} p_1^{'}k, & at\, t = t_1 + ik, i = 0,1,2,.,.,n \\ 0, & at\, t = t_1 + ik - 0, i = 0,1,2.,.,.,n \\ p_1^{''}k^{'}, & at\, t = t_{p_1} + jk^{'}, j = 0,1,2,.,.,.,m \\ 0 & at\, t = t_{p_1} + jk^{'} - 0, j = 0,1,2,.,.,.,m \end{cases}$$

where $t_{p_1} = t_1 + nk$, $t_2 = t_{p_1} + mk^{'}$ and $p_1^{'}k = p_1^{''}k^{'}$.
Solving the differential equation(3) with boundary conditions, we have

$$q_{1r}(t) = \begin{cases} p_1^{'}\left[t_1 + ik - t\right], & t_1 + (i-1)k \leq t < t_1 + ik \\ & i = 1,2,.,.,.,n \\ p_1^{''}\left[t_{p_1} + jk^{'} - t\right], & t_{p_1} + (j-1) \leq t < t_{p_1} + jk^{'} \\ & j = 1,2,.,.,.,m \\ p_1^{''}\left[k^{'} + t_2 - t\right], & t_2 \leq t < t_2 + k^{'} \end{cases}$$

$$H_{1r} = \text{Inventory holding cost of SW-I}$$

$$= h_{1r} \left[\sum_{1}^{n} \int_{t_1+(i-1)k}^{t_1+ik} q_{1r}(t)dt \right.$$

$$+ \left. \sum_{1}^{m+1} \int_{t_{p_1}+(j-1)k^{'}}^{t_{p_1}+jk^{'}} q_{1r}(t)dt \right]$$

$$= h_{1r} \left[\frac{n}{2}p_1^{'}k^2 + \frac{m+1}{2}p_1^{''}k^{'2} \right]$$

$$Id_{1r} = \text{The idle cost of SW-I}$$

$$= id_{1r}(t_1 + T_{p_2} - t_2 - k^{'})$$

5.4 For Secondary Warehouse-II (SW-II)

Differential equation for the SW-II in $[0, t_3]$ is given by

$$\frac{dq_{2r}}{dt} = \begin{cases} P_2 - (a_2 + b_2 W_2), & t_1^{'} \leq t < t_{p_2} \\ -(a_2 + b_2 W_2), & t_{p_2} \leq t \leq t_3 \end{cases} \quad (4)$$
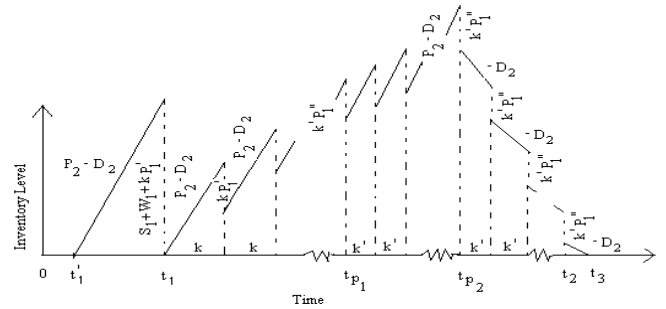
with boundary conditions

Fig. 5.   Inventory level of SW-II

$$q_{2r}(t) = \begin{cases} 0, & at\, t = t_1^{'}, t_1 \\ S_1 + W_1 + kp_1^{'}, & at\, t = t_1 - 0 \\ \left[P_2 - p_1^{'} - (a_2 + b_2 W_2)\right]ik, \\ \quad at\, t = t_1 + ik,\ i = 0,1,2...n \\ \left[P_2 - p_1^{''} - (a_2 + b_2 W_2)\right]jk^{'}, \\ \quad at\, t = t_{p_1} + jk^{'},\ j = 0,1,2...r \\ \left[P_2 - (a_2 + b_2 W_2)\right]t_{p_2} - jp_1^{''}k^{'}, \\ \quad at\, t = t_{p_1} + jk^{'},\ \jmath = r+1, r+2...m \\ 0 & at\, t = t_3 \end{cases}$$

where $t_{p_2} = t_{p_1} + rk^{'}$, $t_2 = t_{p_1} + mk^{'}$ and $t_3 = t_1 + nk + mk^{'} + \frac{P_2}{a_2+b_2 W_2}(t_1 + nk + rk^{'}) - \frac{m+n}{a_2+b_2 W_2}p_1^{'}k$
Solving the differential equation(4) with boundary conditions, we have

$$q_{2r}(t) = \begin{cases} \left[P_2 - (a_2 + b_2 W_2)\right](t - t_1^{'}), & t_1^{'} \leq t < t_1 \\ \left[P_2 - (a_2 + b_2 W_2)\right](t - t_1^{'}) - (i-1)p_1^{'}k, \\ t_1 + (i-1)k \leq t < t_1 + ik,\ i = 1,2...n \\ \left[P_2 - (a_2 + b_2 W_2)\right](t - t_1) - nkp_1^{'} - (j-1)p_1^{''}k^{'}, \\ t_{p_1} + (j-1)k^{'} \leq t < t_{p_1} + jk^{'},\ j = 1,2...r \\ P_2 t_{p_2} - (a_2 + b_2 W_2)t - (n-r)kp_1^{'} - (j-1)p_1^{''}k^{'}, \\ t_{p_2} + (j-1)k^{'} \leq t < t_{p_2} + jk^{'},\ j = 1,2...m-r \\ (a_2 + b_2 W_2)(t_3 - t), & t_2 \leq t < t_3 \end{cases}$$

$$H_{2r} = \text{Inventory holding cost of SW-II}$$

$$= h_{2r} \left[\int_{t_1^{'}}^{t_1} q_{2r}(t)dt + \sum_{1}^{n} \int_{t_1+(i-1)k}^{t_1+ik} q_{2r}(t)dt \right.$$

$$+ \sum_1^r \int_{t_{p_1}+(j-1)k'}^{t_{p_1}+jk'} q_{2r}(t)dt + \sum_1^{m-r} \int_{t_{p_2}+(j-1)k'}^{t_{p_2}+jk'} q_{2r}(t)dt$$

$$+ \int_{t_2}^{t_3} q_{2r}(t)dt \Big]$$

$$= h_{2r}\Big[\frac{P_2-(a_2-b_2W_2)}{2}(t_1-t_1')^2$$

$$+ \frac{P_2-(a_2-b_2W_2)}{2}n^2k^2 - \frac{n(n+1)}{2}p_1'k^2$$

$$+ \frac{P_2-(a_2-b_2W_2)}{2}\big[2nrkk'+r^2k'^2\big] - nrp_1''k'^2$$

$$- \frac{r(r+1)}{2}p_1''k'^2 + nP_2(t_1+nk+rk')k'$$

$$- \frac{(a_2-b_2W_2)}{2}\big[(m-r)^2k^2+2(m-r)(t_1+nk+rk')k'\big]$$

$$- \frac{(m-r)(m+n-1)}{2}p_1'k^2 + \frac{(a_2-b_2W_2)}{2}(t_2-t_3)^2 \Big]$$

Id_{2r} = The idle cost of SW-II = $id_{2r}(T_{p_2}-t_3)$

C_t = total transportation cost

  = $c_t[S_1+W_1+(m+n+1)kp_1']$

6. INTEGRATED MODEL:

6.1 In Crisp Environment

Total profit of the production inventory system TP is given by

TP = Total profit of the production inventory system

  = Revenue from sales -production cost -S_{1s} -H_{1s} -H_{2s}

  -H_{1r} -H_{2r} -Id_{1s} -Id_{2s} -Id_{1r} -Id_{2r} -C_t -A_{1s} -A_{2s} -A_{1r} -A_{2r}

  = Xk^2+Yk+Z where X , Y and Z are given by

$$X = -\Big[c_{1s}(a_1-p_1)\Big[\frac{Bp_1'}{(P_2-a_2-b_2W_2)^2}\Big]\frac{p_1'}{P_2-a_2-b_2W_2}$$

$$+ c_{1s}(a_1-p_1)\Big[\frac{B^2}{(P_2-a_2-b_2W_2)^2}\Big] + h_{1r}\frac{B}{P_2-a_2-b_2W_2}$$

$$\Big[nP_2\frac{p_1'}{p_1''} - (a_2+b_2W_2)(m-r)\Big] + h_{1r}\Big[\frac{n}{2}p_1' + \frac{m+1}{2}\frac{p_1'^2}{p_1''}\Big]$$

$$+ h_{2r}\Big[\frac{p_1'^2}{2(P_2-a_2-b_2W_2)} + \frac{P_2-a_2-b_2W_2}{2}(n^2+r^2$$

$$+ \frac{2nrp_1'}{p_1''}) - \frac{n(n-1)}{2}p_1' - r(n+r-1)\frac{p_1'^2}{p_1''} + nP_2(n\frac{p_1'}{p_1''}$$

$$+ r\frac{p_1'^2}{p_1''^2}) - \frac{a_2+b_2W_2}{2}\Big((m-r)^2+2(m-r)(n\frac{p_1'}{p_1''}+r\frac{p_1'^2}{p_1''^2})$$

$$- \frac{(m-r)(m+n-1)}{2}p_1'^2\Big)\frac{p_1'}{P_2-a_2-b_2W_2}\Big(nP_2\frac{p_1'}{p_1''}$$

$$- (a_2+b_2W_2)(m-r)\Big)\Big] + c_{s1}\frac{(a_1-P_1)p_1'^2}{2(P_2-a_2-b_2W_2)^2}\Big] < 0,$$

$$Y = (s_1-s_2)B + \Big[\frac{B}{P_2-a_2-b_2W_2}\Big]$$

$$\Big[(s_1-c_{p_1})P_1+(s_2-c_{p_2})P_2\Big]$$

$$- 2c_{1s}(a_1-P_1)B\big[(P_2-a_2-b_2W_2)t_1'$$

$$+ \frac{A+W_1}{(P_2-a_2-b_2W_2)^2}\big] + (n+\frac{p_1'}{P_2-a_2-b_2W_2})$$

$$\big[(s_1-c_{p_1})P_1+(s_2-c_{p_2})P_2\big] + r(s_2-c_{p_1})$$

$$P_2\frac{p_1'}{p_1''} + (s_1-s_2)(m+n+1)p_1'$$

$$- c_{s1}(a_1-p_1)\big[t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2}\big]$$

$$\frac{p_1'}{P_2-a_2-b_2W_2} - h_{1s}W_1\big[(n+1)+\frac{mp_1'}{p_1''}\big]$$

$$- h_{2s}\big[W_2(1+\frac{P_2}{a_2-b_2W_2})(\frac{B}{P_2-a_2-b_2W_2})$$

$$- \frac{a_2}{b_2^2}log(1+\frac{b_2W_2}{a_2})\big] - h_{2s}W_2\big[n$$

$$+ \frac{p_1'}{P_2-a_2-b_2W_2} + \frac{mp_1'}{p_1''} + \frac{P_2}{a_2+b_2W_2}$$

$$(n+\frac{p_1'}{P_2-a_2-b_2W_2} + \frac{rp_1'}{p_1''}) - \frac{(m+n)p_1'}{a_2-b_2W_2}\big]$$

$$- h_{1r}(t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2})\big[nP_2\frac{p_1'}{p_1''} - (a_2+b_2W_2)$$

$$(m-r)\big] - h_{2r}\frac{P_2}{2}\big[n+\frac{p_1'}{P_2-a_2-b_2W_2} + \frac{rp_1'}{p_1''}$$

$$- \frac{m+n}{2}p_1'\big] - id_{1s}\frac{rp_1'}{p_1''} - (id_{1r}+h_{2r}\frac{P_2}{2})$$

$$(\frac{B}{P_2-a_2-b_2W_2}) - id_{1r}\big[\frac{p_1'}{P_2-a_2-b_2W_2}$$

$$- \frac{(m-r+1)p_1'}{p_1''}\big] - c_t(m+m+1)p_1'$$

$$\text{and, } Z = (s_1-s_2)(A+W_1) + \big[t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2}\big]$$

$$\big[(s_1-c_{p_1})P_1+(s_2-c_{p_2})P_2\big] - c_{1s}(a_1-P_1)$$

$$\big[t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2}\big]^2 - c_{1s}(a_1-P_1)$$

$$\big[\frac{(P_2-a_2-b_2W_2)t_1'+A+W_1}{P_2-a_2-b_2W_2}\big]^2$$

$$- h_{1s}\big[\frac{W_1}{b_1} - \frac{a_1}{b_1^2}log(1+\frac{b_1W_1}{a_1})\big]$$

$$- h_{2s}\big[\frac{P_2-a_2}{b_2}(t_1'+\frac{1}{b_2}\frac{P_2-a_2-b_2W_2}{P_2-a_2})$$

$$+ W_2(1+\frac{P_2}{a_2-b_2W_2})(t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2})$$

$$- \frac{a_2}{b_2^2}log(1+\frac{b_2W_2}{a_2})\big] - (id_{1r}+h_{2r}\frac{P_2}{2})$$

$$(t_1'+\frac{A+W_1}{P_2-a_2-b_2W_2}) - id_{2r}\big[\frac{1}{b_2}log(1+\frac{b_2W_2}{a_2})$$

$$- c_t(A+W_1) - A_{1s}-A_{2s}-A_{1r}-A_{2r}\big]$$

The average profit ATP(k) is given by

$$ATP(k) \;=\; \frac{TP}{T_{p_2}} = \frac{Xk^2 + Yk + Z}{U + Vk} \qquad (5)$$

For optimum value of ATP(k),we must have

$$\frac{d(ATP)}{dk} = 0$$
$$\Rightarrow \quad VXk^2 + 2UXk + (UY - VZ) = 0$$
$$\Rightarrow \quad k = -\frac{U}{V} \pm \sqrt{\frac{U^2}{V^2} - \frac{UY - VZ}{VX}}$$

$$\text{Now} \quad \frac{d^2(ATP)}{dk^2}\bigg]_{k=k^*} < 0$$
$$\text{for} \quad U^2 X - V(UY - VZ) < 0$$
$$\text{and} \quad UY - VZ > 0 \qquad (6)$$

Where k^* is the corresponding optimal value of k and given by

$$k^* = -\frac{U}{V} + \sqrt{\frac{U^2}{V^2} - \frac{UY - VZ}{VX}} \qquad (7)$$

Therefore ATP is concave for $k = k^*$ if the inequations in (6) holds. Substituting the value of k^* in (5) we get the optimum value $ATP^*(k)$ of $ATP(k)$.

6.2 In Fuzzy Environment

Considering all holding costs, shortage costs, set-up costs, and transportation cost as fuzzy numbers and then from equation (5), average total profit of the system is given by

$$A\tilde{T}P(k) \;=\; \frac{TP}{T_{p_2}} = \frac{\tilde{X}k^2 + \tilde{Y}k + \tilde{Y}}{U + Vk} \qquad (8)$$

where $\tilde{X}$,$\tilde{Y}$ and $\tilde{Z}$ are given by

$$
\tilde{X} = -\Big[\tilde{c_{1s}}(a_1 - p_1)\Big[\frac{Bp_1'}{(P_2 - a_2 - b_2 W_2)^2}\Big]\frac{p_1'}{P_2 - a_2 - b_2 W_2}
$$
$$
+ \tilde{c_{1s}}(a_1 - p_1)\Big[\frac{B^2}{(P_2 - a_2 - b_2 W_2)^2}\Big] + \tilde{h_{1r}}\frac{B}{P_2 - a_2 - b_2 W_2}
$$
$$
\Big[nP_2\frac{p_1'}{p_1''} - (a_2 + b_2 W_2)(m - r)\Big] + \tilde{h_{1r}}\Big[\frac{n}{2}p_1' + \frac{m+1}{2}\frac{p_1'^2}{p_1''}\Big]
$$
$$
+ \tilde{h_{2r}}\Big[\frac{p_1'^2}{2(P_2 - a_2 - b_2 W_2)} + \frac{P_2 - a_2 - b_2 W_2}{2}(n^2 + r^2
$$
$$
+ \frac{2nrp_1'}{p_1'}) - \frac{n(n-1)}{2}p_1' - r(n+r-1)\frac{p_1'^2}{p_1''} + nP_2(n\frac{p_1'}{p_1''}
$$
$$
+ r\frac{p_1'^2}{p_1''^2}) - \frac{a_2 + b_2 W_2}{2}\Big((m-r)^2 + 2(m-r)(n\frac{p_1'}{p_1''} + r\frac{p_1'^2}{p_1''^2})
$$
$$
- \frac{(m-r)(m+n-1)}{2}p_1'^2\Big)\frac{p_1'}{P_2 - a_2 - b_2 W_2}\Big(nP_2\frac{p_1'}{p_1''}
$$
$$
- (a_2 + b_2 W_2)(m - r)\Big)\Big] + \tilde{c_{1s}}\frac{(a_1 - P_1)p_1'^2}{2(P_2 - a_2 - b_2 W_2)^2}\Big],
$$

$$
\tilde{Y} = (s_1 - s_2)B + \Big[\frac{B}{P_2 - a_2 - b_2 W_2}\Big]\Big[(s_1 - c_{p_1})P_1
$$
$$
+ (s_2 - c_{p_2})P_2\Big] - 2\tilde{c_{1s}}(a_1 - P_1)B\Big[(P_2 - a_2 - b_2 W_2)t_1'
$$

$$
+ \frac{A + W_1}{(P_2 - a_2 - b_2 W_2)^2}\Big] + (n + \frac{p_1'}{P_2 - a_2 - b_2 W_2})\Big[(s_1 - c_{p_1})P_1
$$
$$
+ (s_2 - c_{p_2})P_2\Big] + r(s_2 - c_{p_1})P_2\frac{p_1'}{p_1''} + (s_1 - s_2)
$$
$$
(m + n + 1)p_1' - \tilde{c_{1s}}(a_1 - p_1)\Big[t_1'
$$
$$
+ \frac{A + W_1}{P_2 - a_2 - b_2 W_2}\Big]\frac{p_1'}{P_2 - a_2 - b_2 W_2}
$$
$$
- \tilde{h_{1s}}W_1\Big[(n+1) + \frac{mp_1'}{p_1''}\Big] - \tilde{h_{2s}}\Big[W_2(1
$$

$$
+ \frac{P_2}{a_2 - b_2 W_2})(\frac{B}{P_2 - a_2 - b_2 W_2}) - \frac{a_2}{b_2^2}log(1 +
$$
$$
\frac{b_2 W_2}{a_2})\Big] - \tilde{h_{2s}}W_2\Big[n + \frac{p_1'}{P_2 - a_2 - b_2 W_2} + \frac{mp_1'}{p_1''}
$$
$$
+ \frac{P_2}{a_2 + b_2 W_2}(n + \frac{p_1'}{p_2 - a_2 - b_2 W_2} + \frac{rp_1'}{p_1''})
$$
$$
- \frac{(m + n)p_1'}{a_2 - b_2 W_2}\Big] - \tilde{h_{1r}}(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2})
$$
$$
\Big[nP_2\frac{p_1'}{p_1''} - (a_2 + b_2 W_2)(m - r)\Big] - \tilde{h_{2r}}\frac{P_2}{2}\Big[n +
$$
$$
\frac{p_1'}{P_2 - a_2 - b_2 W_2} + \frac{rp_1'}{P_1''} - \frac{m + n}{2}p_1'\Big] - i\tilde{d_{1s}}\frac{rp_1'}{p_1''}
$$
$$
- (i\tilde{d_{1r}} + \tilde{h_{2r}}\frac{P_2}{2})(\frac{B}{P_2 - a_2 - b_2 W_2})
$$
$$
- i\tilde{d_{1r}}\Big[\frac{p_1'}{P_2 - a_2 - b_2 W_2} - \frac{(m - r + 1)p_1'}{p_1''}\Big]
$$
$$
- \tilde{c_t}(m + m + 1)p_1'
$$

$$
\text{and,} \quad \tilde{Z} = (s_1 - s_2)(A + W_1) + \Big[t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}\Big]
$$
$$
\Big[(s_1 - c_{p_1})P_1 + (s_2 - c_{p_2})P_2\Big] - \tilde{c_{1s}}(a_1 - P_1)
$$
$$
\Big[t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}\Big]^2 - \tilde{c_{1s}}(a_1 - P_1)
$$
$$
\Big[\frac{(P_2 - a_2 - b_2 W_2)t_1' + A + W_1}{P_2 - a_2 - b_2 W_2}\Big]^2
$$
$$
- \tilde{h_{1s}}\Big[\frac{W_1}{b_1} - \frac{a_1}{b_1^2}log(1 + \frac{b_1 W_1}{a_1})\Big] - \tilde{h_{2s}}\Big[\frac{P_2 - a_2}{b_2}
$$
$$
(t_1' + \frac{1}{b_2}\frac{P_2 - a_2 - b_2 W_2}{P_2 - a_2}) + W_2(1 +
$$
$$
\frac{P_2}{a_2 - b_2 W_2})(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2})
$$
$$
- \frac{a_2}{b_2^2}log(1 + \frac{b_2 W_2}{a_2})\Big] - (i\tilde{d_{1r}} + \tilde{h_{2r}}\frac{P_2}{2})
$$
$$
(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}) - i\tilde{d_{2r}}\Big[\frac{1}{b_2}log(1 +
$$
$$
\frac{b_2 W_2}{a_2}) - \tilde{c_t}(A + W_1) - \tilde{A_{1s}} - \tilde{A_{2s}}
$$
$$
- \tilde{A_{1r}} - \tilde{A_{2r}}\Big]
$$

6.2.1 Deterministic representation of the proposed model.

Following Grzegorzewski[14], the fuzzy numbers are now trans-

formed to interval numbers and the expression (8) is expressed as

$$A\tilde{T}P(k) = \left[F_L, F_R\right] \qquad (9)$$

(For detail calculations of F_L and F_R , see Appendix-A)
According to Dey et.al[10], the interval problem (9) can be represented as

$$Maximize\{F_L, F_C\} \qquad (10)$$

where $F_C = (F_L + F_R)/2$.

7. INTERACTIVE APPROACH

Now considering the imprecise nature of DM's judgement, DM may have different fuzzy or imprecise goals for each of the objective functions and hence interactive approach is used for the man-machine interaction. To derive the membership functions μ_{F_L} and μ_{F_C} for the objective functions F_L and F_C respectively from DM's viewpoint, we first calculate individual minimum(i.e. F_L^{min}, F_C^{min}) and individual maximum(i.e. F_L^{max}, F_C^{max}) by a non-linear optimization technique. With the help of individual minimum and maximum, the DM can select any one from among the following three types of membership functions

(i) Linear membership functions.

(ii) Quadratic membership functions.

(ii) Exponential membership functions.

The membership functions μ_{F_L} and μ_{F_C} for the corresponding objective functions F_L and F_C can be written as

$$\mu_{F_k} = \begin{cases} 1, & if \quad F_k \le F_k^0, \\ d_k, & if \quad F_k^0 \le F_k \le F_k^1, \\ 0, & if \quad F_k^1 \le F_k \end{cases} \qquad (11)$$

where F_k^0 and F_k^1 are to be chosen such that $F_k^{min} \le F_k^0 \le F_k^1 \le F_k^{max}$ and d_k is a strictly monotonic decreasing continuous function of F_k which may be linear or non-linear.

7.1 Description of the Membership functions

7.1.1 **Linear membership function (Type-I)**. For each objective function, the corresponding Linear membership functions are as follows:

$$\mu_{F_k} = \begin{cases} 1, & if \quad F_k \le F_k^0, \\ 1 - \dfrac{F_k^1 - F_k}{P_k}, & if \quad F_k^0 \le F_k \le F_k^1, \\ 0 & if \quad F_k^1 \le F_k \end{cases} \quad (12)$$

where F_k^0 and F_k^1 are to be chosen such that $F_k^{min} \le F_k^0 \le F_k^1 \le F_k^{max}$ and $P_k = F_k^1 - F_k^0$ is the tolerance of k-th objective function F_k

7.1.2 **Quadratic membership function (Type-II)**. For each objective function, the corresponding quadratic membership functions are as follows:

$$\mu_{F_k} = \begin{cases} 1, & if \ F_k \le F_k^0, \\ 1 - \left(\dfrac{F_k^1 - F_k}{P_k}\right)^2, & if F_k^0 \le F_k \le F_k^1, \\ 0, & if \quad F_k^1 \le F_k \end{cases} \quad (13)$$

where F_k^0 and F_k^1 are to be chosen such that $F_k^{min} \le F_k^0 \le F_k^1 \le F_k^{max}$ and $P_k = F_k^1 - F_k^0$ is the tolerance of k-th objective function F_k.

7.2 Fuzzy decision method

After determining the different linear/nnon-linear membership functions(MF) for each of the objective functions, following Bellman and Zadeh[1] and Zimmermannn[36], the given problem(10)can be formulated as

$$\begin{aligned} &Maximize \qquad\qquad \lambda \\ &Subject\ to\ \ \lambda \le \mu_{F_L}, \quad \lambda \le \mu_{F_C} \\ &\qquad\qquad\qquad 0 \le \lambda \le 1. \end{aligned} \qquad (14)$$

with the help of two different types of membership functions given by (12) and (13),the above problem can be restated for a particular choice of DM as

$$\begin{aligned} &Maximize \qquad \lambda \\ &Subject\ to\ \ \lambda \le 1 - \frac{F_L^1 - F_L}{P_L}, \\ &if\ the\ MF\ of\ first\ objective\ \in\ Type-I, \\ &\lambda \le 1 - \left(\frac{F_C^1 - F_C}{P_C}\right)^2, \\ &if\ the\ MF\ of\ first\ objective\ \in\ Type-II, \\ &\qquad 0 \le \lambda \le 1. \end{aligned} \qquad (15)$$

Here DM selects the above membership functions for the corresponding objective functions. Then the above problem can be solved by a non-linear optimization technique and optimal solution of λ, say λ^* is obtained.
Now after obtaining λ^*, the DM selects the most important objective function from among the objective functions F_L and F_C. Here F_L is selected as DM would like to maximize his/her worst case. Then the problem becomes ($\lambda = \lambda^*$)

$$\begin{aligned} &Maximize \qquad\qquad F_L \\ &Subject\ to\ \ F_L \ge m_L, \quad F_C \ge m_C \\ &\qquad\qquad\qquad 0 \le \lambda \le 1. \end{aligned} \qquad (16)$$

Where $m_L = F_L^1 - (F_L^1 - F_L^0)(1 - \lambda^*)$,
if the MF of first objective $\in$ Type $- I$,
$m_C = F_C^1 - (F_C^1 - F_C^0)(1 - \lambda^*)^{\frac{1}{2}}$,
if the MF of first objective $\in$ Type $- II$,
$0 \le \lambda \le 1$.

7.3 Pareto-optimal solution

Now, after deriving the optimum decision variables, Pareto-optimality test is performed according to Sakawa[31]. Let the decision variable k^* and optimum values, $F_L^* = F_L(k^*)$ and $F_C^* = F_C(k^*)$ are obtained from (16). With these values, the following problem is solving using a non-linear optimization technique:

$$\begin{aligned} &Minimize \qquad V = (\epsilon_L + \epsilon_C) \\ &\qquad Subject\ to\ \ F_L - \epsilon_L = F_L^*, \\ &\qquad\qquad\qquad F_C - \epsilon_C = F_C^* \\ &\epsilon_L, \epsilon_C \ge 0, \quad 0 \le \lambda \le 1. \end{aligned} \qquad (17)$$

The optimal solutions of (17), say, k, $\bar{F}_L$ and $\bar{F}_C$ are called strong Pareto-optimal solutions of the problem(10) provided V is very small; otherwise it is weak Pareto-optimal.

8. NUMERICAL EXAMPLE

To illustrate the proposed inventory model, following crisp input data are considered in table-1. We consider case-I, when fuzzy parameters are TFN and case-II when fuzzy parameter are PFN.The nearest interval approximation according to Grzegorzewski[14] in both cases(i.e, case-I and case-II)are given in Table-2.

Following (14) and (15), the problem (9) is solved and the results are presented in Table-3 and Table-4.

Let, with the above values, the membership functions of the objective functions be formed of the types as per Table 5.

Let, at the beginning, analysis is performed to find optimum λ with the membership functions F_L as linear(Type-I) and F_C as quadratic (Type-II). The optimum value of λ is presented in Table-6.

With this value of λ^*, the objective function F_L is optimized and the optimum results are given in Table-7.

Now, the results obtained from Table-7 are tested for pareto-optimality and the pareto-optimal results are given in Table-8.

9. DISCUSSION:

In Table 8, the values of V are quite small in both cases and hence, the optimum results in table 7 are strong Pareto-optimum and can be accepted.Still, if the decision-maker/practitioner is not satisfied with the outputs, he/she may perform the above analysis again re-choosing the membership functions for F_L and F_C as linear, quadratic and exponential(say). If the second time analysis does not also give the desired result, the DM may perform the analysis with the other possible different combinations(in this case,3^2 times) of the membership functions and can select the most suitable optimum solution for his/her firm/factory for implementation.Furthermore we observed that the average profit is greater in case-II than Case-I.

10. CONCLUSION

This production inventory model, which comprises of two production plants, two secondary warehouses and two showrooms, is aiming at minimizing the overall production cost per unit of product. In order to attain it we derived a closed form of solution of the model. Here, two plant concepts were used in the sense that a manufacturer / firm wanted to minimize the overall production cost by erecting two plants in two different regions / areas. In a nut shell, the study suggests that the production cost per unit of the product will minimize and thereby the total net profit will maximize by erecting two plants out of which one plant has substantially lower production cost per unit than the other.

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Table 1. Input data for different crisps parameter of the model.

Parameter	P_1	P_2	s_1	s_2	c_{p_1}	c_{p_2}	a_1	b_1	W_1	a_2	b_2	W_2	n	r
value	100	200	90	70	75	50	110	.5	75	75	.5	50	11	18

Table 2. Input data for different TFN/PFN parameter of the model.

Parameter	Fuzzy value	Case-I Nearest interval for TFN	Case-II Nearest interval for PFN
$\tilde{c_t}$	[24, 30, 36]	$[c_{tL}, c_{tR}] = [27, 33]$	$[c_{tL}, c_{tR}] = [25.66, 32]$
$\tilde{c_{1s}}$	[.04, .1, .16]	$[c_{s_{1L}}, c_{s_{1R}}] = [.07, .13]$	$[c_{s_{1L}}, c_{s_{1R}}] = [.06, .12]$
$\tilde{h_{1s}}$	[.24, .3, .36]	$[h_{1s_L}, h_{1s_R}] = [.27, .33]$	$[h_{1s_L}, h_{1s_R}] = [.256, .32]$
$\tilde{h_{2s}}$	[.14, .2, .26]	$[h_{2s_L}, h_{2s_R}] = [.17, .23]$	$[h_{2s_L}, h_{2s_R}] = [.16, .22]$
$\tilde{h_{1r}}$	[.14, .2, .26]	$[h_{1r_L}, h_{1r_R}] = [.17, .23]$	$[h_{1r_L}, h_{1r_R}] = [.16, .22]$
$\tilde{h_{2r}}$	[.04, .1, .16]	$[h_{2r_L}, h_{2r_R}] = [.07, .13]$	$[h_{2r_L}, h_{2r_R}] = [.06, .12]$
$\tilde{a_{1s}}$	[380, 400, 420]	$[a_{1s_L}, a_{1s_R}] = [390, 410]$	$[a_{1s_L}, a_{1s_R}] = [386.66, 406.66]$
$\tilde{a_{2s}}$	[280, 300, 320]	$[a_{2s_L}, a_{2s_R}] = [290.310]$	$[a_{2s_L}, a_{2s_R}] = [286.66, 306.66]$
$\tilde{a_{1r}}$	[180, 200, 220]	$[a_{1r_L}, a_{1r_R}] = [190, 210]$	$[a_{1r_L}, a_{1r_R}] = [186.66, 206.66]$
$\tilde{a_{2r}}$	[80, 100, 120]	$[a_{2r_L}, a_{2r_R}] = [90, 110]$	$[a_{2r_L}, a_{2r_R}] = [86.66, 106.66]$
$\tilde{id_{1s}}$	[.4, .6, .8]	$[id_{1s_L}, id_{1s_R}] = [.5, .7]$	$[id_{1s_L}, id_{1s_R}] = [.46, .66]$
$\tilde{id_{2s}}$	[.1, .3, .5]	$[id_{2s_L}, id_{2s_R}] = [.2, .4]$	$[id_{2s_L}, id_{2s_R}] = [.16, .36]$
$\tilde{id_{1r}}$	[.16, .2, .26]	$[id_{1r_L}, id_{1r_R}] = [.18, .23]$	$[id_{1r_L}, id_{1r_R}] = [.17, .22]$
$\tilde{id_{2r}}$	[.04, .1, .16]	$[id_{2r_L}, id_{2r_R}] = [.07, .13]$	$[id_{2r_L}, id_{2r_R}] = [.06, .12]$

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11.1 Appendix-A

$$F_L = \frac{X_L k^2 + Y_L k + Y_L}{U + V k}$$
$$and, \quad F_R = \frac{X_R k^2 + Y_R k + Y_R}{U + V k}$$

Table 3. Individual minimum and maximum of objective function.

cases	Case − I		Case − II	
Objective functions	Minimum	Maximum	Minimum	Maximum
F_L	3561.014	3622.093	3642.440	3707.427
F_C	3807.755	3881.665	3905.213	3983.940

Table 4. Input data for F_k^0, F_k^1.

cases	Case − I		Case − II	
Objective functions	F_k^0	F_k^1	F_k^0	F_k^1
F_L	3561.015	3622.092	3642.441	3707.426
F_C	3807.756	3881.664	3905.214	3983.939

where X_L, X_R, Y_L, Y_R, Z_L and Z_R are given by

$$X_L = -\Big[c_{1s_R}(a_1 - p_1)\Big[\frac{Bp_1'}{(P_2 - a_2 - b_2W_2)^2}\Big]\frac{p_1'}{P_2 - a_2 - b_2W_2}$$

$$+ c_{1s_R}(a_1 - p_1)\Big[\frac{B^2}{(P_2 - a_2 - b_2W_2)^2}\Big] + h_{1_{rR}}\frac{B}{P_2 - a_2 - b_2W_2}$$

$$\Big[nP_2\frac{p_1'}{p_1''} - (a_2 + b_2W_2)(m - r)\Big] + h_{1_{rR}}\Big[\frac{n}{2}p_1' + \frac{m+1}{2}\frac{p_1'^2}{p_1''}\Big]$$

$$+ h_{2_{rR}}\Big[\frac{p_1'^2}{2(P_2 - a_2 - b_2W_2)} + \frac{P_2 - a_2 - b_2W_2}{2}(n^2 + r^2$$

$$+ \frac{2nrp_1'}{P_1'}) - \frac{n(n-1)}{2}p_1' - r(n+r-1)\frac{p_1'^2}{p_1''} + nP_2(n\frac{p_1'}{p_1''} + r\frac{p_1'^2}{p_1''^2})$$

$$- \frac{a_2 + b_2W_2}{2}\Big((m-r)^2 + 2(m-r)(n\frac{p_1'}{p_1''} + r\frac{p_1'^2}{p_1''^2})$$

$$- \frac{(m-r)(m+n-1)}{2}p_1'^2\Big)\frac{p_1'}{P_2 - a_2 - b_2W_2}\Big(nP_2\frac{p_1'}{p_1''}$$

$$- (a_2 + b_2W_2)(m - r)\Big)\Big] + c_{1s_R}\frac{(a_1 - P_1)p_1'^2}{2(P_2 - a_2 - b_2W_2)^2}\Big],$$

$$X_R = -\Big[c_{1s_L}(a_1 - p_1)\Big[\frac{Bp_1'}{(P_2 - a_2 - b_2W_2)^2}\Big]\frac{p_1'}{P_2 - a_2 - b_2W_2}$$

$$+ c_{1s_L}(a_1 - p_1)\Big[\frac{B^2}{(P_2 - a_2 - b_2W_2)^2}\Big] + h_{1_{rL}}\frac{B}{P_2 - a_2 - b_2W_2}$$

$$\Big[nP_2\frac{p_1'}{p_1''} - (a_2 + b_2W_2)(m - r)\Big] + h_{1_{rL}}\Big[\frac{n}{2}p_1' + \frac{m+1}{2}\frac{p_1'^2}{p_1''}\Big]$$

$$+ h_{2_{rL}}\Big[\frac{p_1'^2}{2(P_2 - a_2 - b_2W_2)} + \frac{P_2 - a_2 - b_2W_2}{2}(n^2 + r^2$$

$$+ \frac{2nrp_1'}{p_1'}) - \frac{n(n-1)}{2}p_1' - r(n+r-1)\frac{p_1'^2}{p_1''} + nP_2(n\frac{p_1'}{p_1''}$$

$$+ r\frac{p_1'^2}{p_1''^2}) - \frac{a_2 + b_2W_2}{2}\Big((m-r)^2 + 2(m-r)(n\frac{p_1'}{p_1''} + r\frac{p_1'^2}{p_1''^2})$$

$$- \frac{(m-r)(m+n-1)}{2}p_1'^2\Big)\frac{p_1'}{P_2 - a_2 - b_2W_2}\Big(nP_2\frac{p_1'}{p_1''}$$

$$- (a_2 + b_2W_2)(m - r)\Big)\Big] - c_{1s_L}\frac{(a_1 - P_1)p_1'^2}{2(P_2 - a_2 - b_2W_2)^2}\Big],$$

$$Y_L = (s_1 - s_2)B + \Big[\frac{B}{p_2 - a_2 - b_2W_2}\Big]\Big[(s_1 - c_{p_1})P_1$$

$$+ (s_2 - c_{p_2})P_2\Big] - 2c_{1s_R}(a_1 - P_1)B\Big[(P_2 - a_2 - b_2W_2)t_1'$$

$$+ \frac{A + W_1}{(p_2 - a_2 - b_2W_2)^2}\Big] + (n + \frac{p_1'}{P_2 - a_2 - b_2W_2})$$

$$\Big[(s_1 - c_{p_1})P_1 + (s_2 - c_{p_2})P_2\Big] + r(s_2 - c_{p_1})P_2\frac{p_1'}{p_1''}$$

$$+ (s_1 - s_2)(m + n + 1)p_1' - c_{1s_R}(a_1 - p_1)\Big[t_1'$$

$$+ \frac{A + W_1}{P_2 - a_2 - b_2W_2}\Big]\frac{p_1'}{P_2 - a_2 - b_2W_2} - h_{1sR}W_1\Big[(n+1)$$

$$+ \frac{mP_1'}{p_1''}\Big] - h_{2sR}\Big[W_2(1 + \frac{P_2}{a_2 - b_2W_2})(\frac{B}{P_2 - a_2 - b_2W_2})$$

$$- \frac{a_2}{b_2^2}log(1 + \frac{b_2W_2}{a_2})\Big] - h_{2sR}W_2\Big[n + \frac{p_1'}{P_2 - a_2 - b_2W_2} + \frac{mp_1'}{p_1''}$$

$$+ \frac{P_2}{a_2 + b_2W_2}(n + \frac{p_1'}{P_2 - a_2 - b_2W_2} + \frac{rp_1'}{p_1''}) - \frac{(m+n)p_1'}{a_2 - b_2W_2}\Big]$$

$$- h_{1rR}(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2W_2})\Big[nP_2\frac{p_1'}{p_1''} - (a_2 + b_2W_2)(m - r)\Big]$$

$$- h_{2rR}\frac{P_2}{2}\Big[n + \frac{p_1'}{P_2 - a_2 - b_2W_2} + \frac{rp_1'}{p_1''} - \frac{m+n}{2}p_1'\Big]$$

$$- id_{1sR}\frac{rp_1'}{p_1''} - (id_{1rR} + h_{2rR}\frac{P_2}{2})(\frac{B}{P_2 - a_2 - b_2W_2}) - id_{1rR}$$

$$\Big[\frac{p_1'}{P_2 - a_2 - b_2W_2} - \frac{(m - r + 1)p_1'}{p_1''}\Big] - c_{tR}(m + m + 1)p_1'$$

$$Y_R = (s_1 - s_2)B + \Big[\frac{B}{p_2 - a_2 - b_2W_2}\Big]\Big[(s_1 - c_{p_1})P_1$$

$$+ (s_2 - c_{p_2})P_2\Big] - 2c_{1s_L}(a_1 - P_1)B\Big[(P_2 - a_2 - b_2W_2)t_1'$$

$$+ \frac{A + W_1}{(P_2 - a_2 - b_2W_2)^2}\Big] + (n + \frac{p_1'}{P_2 - a_2 - b_2W_2})$$

$$\Big[(s_1 - c_{p_1})P_1 + (s_2 - c_{p_2})P_2\Big] + r(s_2 - c_{p_1})P_2\frac{p_1'}{p_1'}$$

$$+ (s_1 - s_2)(m + n + 1)p_1' - c_{1s_L}(a_1 - p_1)\Big[t_1'$$

$$+ \frac{A + W_1}{P_2 - a_2 - b_2W_2}\Big]\frac{p_1'}{P_2 - a_2 - b_2W_2}$$

$$- h_{1sL}W_1\Big[(n+1) + \frac{mp_1'}{p_1''}\Big] - h_{2sL}\Big[W_2(1 + \frac{P_2}{a_2 - b_2W_2})$$

$$(\frac{B}{P_2 - a_2 - b_2 W_2}) - \frac{a_2}{b_2^2} log(1 + \frac{b_2 W_2}{a_2})] - h_{2sL} W_2$$

$$[n + \frac{p_1'}{P_2 - a_2 - b_2 W_2} + \frac{m p_1'}{p_1''} + \frac{P_2}{a_2 + b_2 W_2}(n$$

$$+ \frac{p_1'}{P_2 - a_2 - b_2 W_2} + \frac{r p_1'}{p_1''}) - \frac{(m+n) p_1'}{a_2 - b_2 W_2}]$$

$$- h_{1rL}(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2})[n P_2 \frac{p_1'}{p_1''} - (a_2 + b_2 W_2)$$

$$(m - r)] - h_{2rL} \frac{P_2}{2}[n + \frac{p_1'}{P_2 - a_2 - b_2 W_2} + \frac{r p_1'}{p_1''}$$

$$- \frac{m+n}{2} p_1'] - id_{1sL} \frac{r p_1'}{p_1''} - (id_{1rL} + h_{2rL} \frac{P_2}{2})$$

$$(\frac{B}{P_2 - a_2 - b_2 W_2}) - id_{1rL}[\frac{p_1'}{P_2 - a_2 - b_2 W_2}$$

$$- \frac{(m - r + 1) p_1'}{p_1''}] - c_{tL}(m + m + 1) p_1',$$

$$- (id_{1rL} + h_{2rL} \frac{P_2}{2})(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2})$$

$$- id_{2rL}[\frac{1}{b_2} log(1 + \frac{b_2 W_2}{a_2}) - c_{tL}(A + W_1)$$

$$- A_{1sL} - A_{2sL} - A_{1rL} - A_{2rL}]$$

$$Z_L = (s_1 - s_2)(A + W_1) + [t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}]$$

$$[(s_1 - c_{p_1}) P_1 + (s_2 - c_{p_2}) P_2] - c_{1s_R}(a_1 - P_1)$$

$$[t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}]^2 - c_{1s_R}(a_1 - P_1)$$

$$[\frac{(P_2 - a_2 - b_2 W_2) t_1' + A + W_1}{P_2 - a_2 - b_2 W_2}]^2 - h_{1sR}[\frac{W_1}{b_1}$$

$$- \frac{a_1}{b_1^2} log(1 + \frac{b_1 W_1}{a_1})] - h_{2sR}[\frac{P_2 - a_2}{b_2}(t_1'$$

$$+ \frac{1}{b_2} \frac{P_2 - a_2 - b_2 W_2}{P_2 - a_2}) + W_2(1 + \frac{P_2}{a_2 - b_2 W_2})$$

$$(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}) - \frac{a_2}{b_2^2} log(1 + \frac{b_2 W_2}{a_2})]$$

$$- (id_{1rR} + h_{2rR} \frac{P_2}{2})(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2})$$

$$- id_{2rR}[\frac{1}{b_2} log(1 + \frac{b_2 W_2}{a_2}) - c_{tR}(A + W_1)$$

$$- A_{1sR} - A_{2sR} - A_{1rR} - A_{2rR}]$$

$$and, \; Z_R = (s_1 - s_2)(A + W_1) + [t_1' + \frac{A + W_1}{p_2 - a_2 - b_2 W_2}]$$

$$[(s_1 - c_{p_1}) P_1 + (s_2 - c_{p_2}) P_2] - c_{1s_L}(a_1 - P_1)$$

$$[t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}]^2 - c_{1s_L}(a_1 - P_1)$$

$$[\frac{(P_2 - a_2 - b_2 W_2) t_1' + A + W_1}{P_2 - a_2 - b_2 W_2}]^2 - h_{1sL}[\frac{W_1}{b_1}$$

$$- \frac{a_1}{b_1^2} log(1 + \frac{b_1 W_1}{a_1})] - h_{2sL}[\frac{P_2 - a_2}{b_2}(t_1'$$

$$+ \frac{1}{b_2} \frac{P_2 - a_2 - b_2 W_2}{P_2 - a_2}) + W_2(1 + \frac{P_2}{a_2 - b_2 W_2})$$

$$(t_1' + \frac{A + W_1}{P_2 - a_2 - b_2 W_2}) - \frac{a_2}{b_2^2} log(1 + \frac{b_2 W_2}{a_2})]$$

Table-5. Types of MF for objective functions.

$Objective\ function$	$Types\ of\ membership\ functions$
F_L	$Types-I$
F_C	$Types-II$

Table-6: Optimal values of λ

$Maximum\ \lambda$	$Case-I$	$Case-II$
λ^*	.9975313	.9973593

Table-7: Optimal results when F_L is chosen as the most important objective function

$Parameter$	k^*	F_L^*	F_C^*	F_R^*
Case-I	1.084373	3621.941	3877.992	3922.341
Case-II	1.172279	3707.254	3979.894	4023.375

Table-8: Pareto-optimal results

$Parameter$	k^*	F_L^*	F_C^*	V^*
Case-I	1.084409	3621.942	3877.993	.000857474
Case-II	1.172386	3707.255	3979.897	.000847669