International Journal of Computer Applications (0975 – 8887)
Volume 96– No.12, June 2014

Fractional Evolution Integrodifferential Systems with Nonlocal Conditions in Banach Spaces

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ABSTRACT
This paper is concerned with the proof for the existence and uniqueness of local mild and classical solutions of a class of nonlinear fractional evolution integrodifferential systems with nonlocal conditions in Banach spaces based on the theory of resolvent operators, the fractional powers of operators, fixed point technique and the Gelfand-Shilov principle.

General Terms
2000 Mathematics Subject Classification: 34G20, 34K05, 26A33, 35A07, 35B65

Keywords
Fractional parabolic equation, fractional powers, mild solution, classical solution, local existence, resolvent operators

1. INTRODUCTION
The main objective of the study is to obtain the local mild and classical solutions for the nonlocal fractional Cauchy problem of the form

$$\begin{cases} \frac{d^{\alpha}u(t)}{dt^{\alpha}} + A(t)u(t) = f\left(t, u(t), \int_{t_0}^{t} k\big(t, s, u(s)\big)ds\right) \\ \qquad + \int_{t_0}^{t} B(t-s)g(s, u(s)ds, t \in [0, b], \\ \qquad u(t_0) + h(u) = u_0. \end{cases} \tag{1.1}$$

in a Banach space X, where $0 < \alpha \leq 1$, $0 \leq t_0 < t$. Let J denote the closure of the interval $[t_0, T), t_0 < T \leq \infty$. Under the assumptions that $-A(t)$ is a closed linear operator defined on a dense domain $D(A)$ in X into X such that $D(A)$ is independent of t and $-A(t)$ generates an evolution operator in the Banach space X, the function B is real valued and locally integrable on $[t_0, \infty) \times X$ into X and let $h: C(J, X) \to \overline{D(A)}$.

The Fractional differential equations have been studied with more interest by many authors [8, 12, 14, 15, 16, 17, 18, 20, 22, 25] in the recent times as they effectively describe many phenomena arising in engineering, physics, economy and science including several applications in the fields of viscoelasticity, electrochemistry, electro-magnetic, and so on. The existence of solutions to evolution equations with nonlocal conditions in Banach space was established first by Byszewski [9]. Many kinds of nonlinear evolution equations were developed by the authors [17, 18, 22, 25]. Deng [13] examined that using the nonlocal condition for the diffusion phenomenon of a small amount of gas in a transparent tube in place of the usual local Cauchy problem. Many authors [3, 4, 5, 6, 7, 10, 21, 28] formulated several works related to the first order differential equations with initial conditions.

The results are the generalizations of the research works of the authors Debbouche [11], Bahuguna [2], El-Borai [17, 18], Pazy [26] and Yan [29]. We study to investigate the regularity of the mild solution of the considered problem and to show under the additional condition of Hölder continuity on B that this mild solution is originally the classical solution.

2. PRELIMINARIES
By means of Gelfand and Shilov principle [20], we define the fractional integral of order $\alpha > 0$ as

$$I_a^{\alpha}u(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-s)^{\alpha-1} f(s)ds.$$

and the fractional derivative of the function f of order $0 < \alpha < 1$ as

$${}_aD_t^{\alpha}u(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_a^t (t-s)^{-\alpha}u(s)ds.$$

where f is an abstract continuous on the interval $[a, b]$ and $\Gamma(\alpha)$ is the Gamma function refer to [12].

Definition 2.1 By a classical solution of (1.1) on J, we mean a function u with values in X such that:

(1) u is continuous function on $[t_0, T)$ and $u(t) \in D(A)$,

(2) $\frac{d^{\alpha}u}{dt^{\alpha}}$ exists and is continuous on $[t_0, T), 0 < \alpha < 1$, and u satisfies on $(t_0, T)$and the nonlocal condition (1.1).

By a local classical solution of (1.1) on J, we mean that there exist a T_0, $t_0 < T_0 < T$ and a function u defined from $J_0 = [t_0, T_0]$ into X such that u is a classical solution of (1.1).

Let E be the Banach space formed from $D(A)$ with the graph norm. Since $-A(t)$ is aclosed operator, it follows that $-A(t)$ is in the set of bounded operators from E to X.

Definition 2.2 A resolvent operator for problem (1.1) is a bounded operator valued function $R(t, s) \in B(X)$, $0 \leq s \leq t < T$, the space of bounded linear operators on X having the following properties refer to [8, 29]:

(i) $R(t, s)$ is strongly continuous in s and t,

$R(s, s) = I, 0 \leq s < T, \|R(t, s)\| \leq Me^{\beta(t-s)}$

for some constant M and β.

(ii) $R(t, s)E \subset E, R(t, s)$ is strongly continuous in s and t on E.

(iii) For $x \in X, R(t, s)x$ is continuously differentiable in $s \in [0, T)$ and

$$\frac{\partial R}{\partial s}(t,s)x = R(t,s)A(s)x.$$

(iv) For $x \in X,\ s \in [0,T), R(t,s)x$ is continuously differentiable in $t \in [s,T)$ and

$$\frac{\partial R}{\partial s}(t,s)x = -A(t)R(t,s)x.$$

with $\frac{\partial R}{\partial s}(t,s)x$ and $\frac{\partial R}{\partial t}(t,s)x$ are strongly continuous on $0 \le s \le t < T$.

Here $R(t,s)$ can be extracted from the evolution operator of the generator $-A(t)$. The resolvent operator is similar to the evolution operator for nonautonomous differential equations in a Banach space.

Definition 2.3 A continuous function $u: J \to X$ is said to be a mild solution of problem (1.1) if for all $u_0 \in X$, it satisfies the integral equation

$$u(t) = R(t,t_0)[u_0 - h(u)] + \frac{1}{\Gamma\alpha}\int_{t_0}^{t}(t-s)^{\alpha-1}R(t,s)$$
$$\times [f\left(s, u(s), \int_{t_0}^{s} k(t,\tau,u(\tau))d\tau\right)$$
$$+ \int_{u_0}^{s} B(s-\tau)g(\tau,u(\tau))d\tau]ds \qquad (2.1)$$

By a local mild solution of (1.1) on J, we mean that there exist a $T_0, t_0 < T_0 < T$, and a function u defined from $J_0 = [t_0, T_0]$ into X such that u is a mild solution of (1.1) refer to [8,22,25].

We define the fractional power $A^{-q}(t)$ by

$$A^{-q}(t) = \frac{1}{\Gamma\alpha}\int_0^\infty x^{q-1}R(x,t)dx, q > 0.$$

For $0 < q \le 1, A^q(t)$ (A^q in short form) is a closed linear operator whose domain $D(A^q) \supset D(A)$ is dense in X. This implies that $D(A^q)$ endowed with the graph norm

$$\|u\|_{D(A)} = \|u\| + \|A^q u\|, u \in D(A^q)$$

is a Banach space as clearly $A^q = A^{-(-q)}$ because A^{-q} is one to one. Since $0 \in \rho(A), A^q$is invertible, and is graph norm is equivalent to the norm

$$\|u\|_q = \|A^q u\|.$$

Thus $D(A^q)$ equipped with norm $\|\cdot\|_q$ is a Banach space denoted by X_q refer to [1, 2]. To establish the main results, the assumptions on the maps f, k and g are as follows:

(F) Let U be an open subset of $[0,\infty) \times X_q \times X_q$, and for every $(t,x,y) \in U$ there exist a neighbourhood $V \subset U$ of (t,x,y) and constants $L > 0, 0 < \mu < 1$ such that

$$\|f(s_1,u_1,v_1) - f(s_2,u_2,v_2)\|$$
$$\le L[|s_1 - s_2|^\mu + \|u_1 - u_2\|_q + \|v_1 - v_2\|_q],$$

for all (s_1,u_1,v_1) and (s_2,u_2,v_2) in V.

(G) Let D be an open subset of $[0,\infty) \times [0,\infty) \times X_q$, and for every $(t,s,x) \in D$ there exist a neighbourhood $E \subset D$ of (t,s,x) and constants $L_1 > 0, 0 < \mu < 1$ such that

$$\|k(t_1,s_1,u_1) - k(t_2,s_2,u_2)\|$$
$$\le L_1[|t_1 - t_2|^\mu + \|s_1 - s_2\|_q + \|u_1 - u_2\|_q],$$

for all (t_1,s_1,u_1) and (t_2,s_2,u_2) in E.

(H) Let P be an open subset of $[0,\infty) \times [0,\infty) \times X_q$, and for every $(t,x) \in P$ there exist a neighborhood $W \subset P$ of (t,x) and constants $L_2 > 0, 0 < \mu < 1$ such that

$$\|g(t_1,x) - g(t_2,y)\| \le L_2[|t_1 - t_2|^\mu + \|x - y\|_q],$$

for all (t_1,x) and (t_2,y) in W.

3. LOCAL MILD SOLUTIONS

Assume that $-A$ is invertible and $t_0 < T < \infty$ and refer to [19, 23, 24, 26, 27], we deduce the following to prove the local existence of (1.1).

Lemma 3.1 Let $A(t)$ be the infinitesimal generator of a resolvent operator $R(t,s)$. we denote by $\rho[A(t)]$ the resolvent set of $A(t)$, then

(a) $R(t,s): X \to D(A^q)$ for every $0 \le s \le t < T$ and $q > 0$,

(b) For every $u \in D(A^q)$, we have

$$R(t,s)A^q(t)u = A^q(t)R(t,s)u,$$

(c) The operator $A^q R(t,s)$ is bounded and

$$\|A^q R(t,s)\| \le M_{q,\beta}(t-s)^{-q}.$$

Let $Y = C([t_0,t_1]; X_q)$ be endowed with the supremum norm

$$\|y\|_\infty = \sup_{t_0 \le t \le t_1}\|y(t)\|_q, y \in Y.$$

Then Y is a Banach space.

The function $h: Y \to X_q$ is continuous and there exists a number b such that $\|R(t,t_0)\| < 1/_{2b}$ and

$\|h(x) - h(y)\|_q \le b\|x - y\|_\infty$ for all $x, y \in Y$.

Note that, if $z \in Y$, then $A^{-q}z \in Y$.

Theorem 3.2 Suppose that the operator $-A(t)$ generates the resolvent operator $R(t,s)$ with $\|R(t,s)\| \le Me^{\beta(t-s)}$and that $0 \in \rho[-A(t)]$. If the maps f, k and g satisfy (F),(G),(H) and the real-valued map B is integrable on J, then (1.1) has a unique local mild solution for every $u_0 \in X_q$.

Proof: We fix a point (t_0,u_0) in the open subset U of $[0,\infty) \times X_q$ and choose $t_0' > t_0$ and $\epsilon > 0$ such that (F) holds for the functions f and g on the set

$$V = \{(t,x) \in U: t_0 \le t_0', \|x - u_0\|_q \le \epsilon\}. \qquad (3.1)$$

Let

$$N_1 = \sup_{t_0 \le t \le t_1'}\left\|f\left(t, u_0, \int_{t_0}^{t} k(t,s,u_0)ds\right)\right\|,$$
$$N_2 = \sup_{t_0 \le t \le t_1'}\|g(t,u_0)\|$$

and let

$$\lambda = \sup_{x \in Y}\|h(x)\|_q$$

and choose $t_1 > t_0$ such that

$$\|R(t,t_0) - I\|[\|u_0\|_q + \lambda] \le \frac{\epsilon}{2}, t_0 \le t \le t_1.$$

and

$$t_1 - t_0 < \min\left\{t_1' - t_0, [\tfrac{\epsilon}{2} M_{q,\beta}^{-1}\Gamma(\alpha)(\alpha - q)\{(L\epsilon + L_1\epsilon J + N_1) + a_T(L_2\epsilon + N_2)\}^{-1}]^{\frac{1}{\alpha-q}}\right\}, \quad (3.2)$$

where $$a_T = \int_0^T |B(s)|\, ds. \quad (3.3)$$

We define a map on Y by $\phi y = \tilde{y}$, where $\tilde{y}$ is given by

$$\tilde{y}(t) = R(t,t_0)A^q[u_0 - h(A^{-q}y)] + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}A^qR(t,s) \times [f\left(s, A^{-q}y(s), \int_{t_0}^{s} k(t,\tau,A^{-q}y(\tau))d\tau\right) + \int_{u_0}^{s} B(s-\tau)g\big(\tau, A^{-q}y(\tau)\big)d\tau]ds.$$

For every $y \in Y$, $\phi y(t_0) = A^q[u_0 - h(A^{-q}y)]$, and for $t_0 \le s \le t \le t_1$ we have

$$\begin{aligned}
&\phi y(t) - \phi y(s) \\
&= [R(t,t_0) - R(s,t_0)]A^q[u_0 - h(A^{-q}y)] \\
&+ \frac{1}{\Gamma(\alpha)}\int_{s}^{t}(t-s)^{\alpha-1}A^qR(t,\tau) \\
&\times [\, f\left(\tau, A^{-q}y(\tau), \int_{t_0}^{\tau} k(\tau,\mu,A^{-q}y(\mu))d\mu\right) \\
&+ \int_{t_0}^{\tau} B(\tau-\eta)g\big(\eta, A^{-q}y(\eta)\big)d\eta]d\tau \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{s}(t-\tau)^{\alpha-1}A^q[R(t,\tau) - R(s,\tau)] \\
&\times [\, f\left(\tau, A^{-q}y(\tau), \int_{t_0}^{\tau} k(\tau,\mu,A^{-q}y(\mu))d\mu\right) \\
&+ \int_{t_0}^{\tau} B(\tau-\eta)g\big(\eta, A^{-q}y(\eta)\big)d\eta]d\tau.
\end{aligned}$$

It follows from (F), (G) and (H) on the functions f, k and g, Lemma 3.1.c and (3.2) that $\phi: Y \to Y$. Let S be the nonempty closed and bounded set given by

$$S = \{y \in Y: y(t_0) = A^q[u_0 - h(A^{-q}y)], \|y(t) - A^q[u_0 - h(A^{-q}y)]\| \le \epsilon\}. \quad (3.4)$$

Then for $y \in S$, we have

$$\begin{aligned}
&\|\phi y(t) - A^q[u_0 - h(A^{-q}y)]\| \\
&\le \|R(t,t_0) - I\|\|A^q[u_0 - h(A^{-q}y)]\| \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\| \\
&\times \left\| f\left(s, A^{-q}y(s), \int_{t_0}^{s} k(t,\tau,u(\tau))d\tau\right) - f\left(s, u_0, \int_{t_0}^{s} k(t,\tau,u_0)d\tau\right)\right\| ds \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\| \\
&\times \int_{u_0}^{\tau} |B(s-t)|\big\|g\big(\tau, A^{-q}y(\tau)\big) - g(\tau, u_0)\big\|d\tau\, ds \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\| \\
&\times \left\| f\left(s, u_0, \int_{t_0}^{s} k(t,\tau,u_0)d\tau\right)\right\| ds \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\| \\
&\times \int_{t_0}^{\tau} |B(s-t)|\|g(\tau,u_0)\|d\tau\, ds.
\end{aligned}$$

Using Lemma 3.1.c, (3.3) and (3.4) we get

$$\begin{aligned}
&\|\phi y(t) - A^q[u_0 - h(A^{-q}y)]\| \\
&\le \tfrac{\epsilon}{2} + \tfrac{M_{q,\beta}}{\Gamma(\alpha)}(\alpha - q)^{-1}\{(L\epsilon + L_1\epsilon J + N_1) + a_T(L_2\epsilon + N_2)\} \\
&\quad \times (t_1 - t_0)^{\alpha-q} \le \epsilon \qquad (3.5)
\end{aligned}$$

Then $\phi: S \to S$. To show that ϕ is a strict contraction on S and to ensure the existence of a unique continuous function satisfying (1.1), Let y and z be two elements in S, then

$$\begin{aligned}
&\|\phi y(t) - \phi z(t)\| = \|\tilde{y}(t) - \tilde{z}(t)\| \\
&\le \|R(t,t_0)\|\|h(A^{-q}z) - h(A^{-q}y)\|_q \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1} \\
&\times \|A^qR(t,s)\| \left\| f\left(s, A^{-q}y(s), \int_{t_0}^{s} k(t,\tau,A^{-q}y(\tau))d\tau\right) - f\left(s, A^{-q}z(s), \int_{t_0}^{s} k(t,\tau,A^{-q}z(\tau))d\tau\right)\right\| ds \\
&+ \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\| \\
&\times \left[\int_{u_0}^{\tau} |B(s-t)|\big\|g\big(\tau, A^{-q}y(\tau)\big) - g\big(\tau, A^{-q}z(\tau)\big)\big\|d\tau\right] ds.
\end{aligned}$$

Using assumption (F), (G),(H) on f, k and g, (3.5), Lemma 3.1.c and (3.4) respectively, we get

$$\begin{aligned}
&\|\phi y(t) - \phi z(t)\| \\
&\le b\|R(t,t_0)\|\|y - z\|_\infty \\
&\quad + \frac{1}{\Gamma(\alpha)}[(L + L_1 J + L_2 a_T)\int_{t_0}^{t}(t-s)^{\alpha-1}\|A^qR(t,s)\|ds] \times \|y - z\|_Y \\
&\le b\|R(t,t_0)\|\|y - z\|_Y \\
&\quad + \frac{1}{\Gamma(\alpha)}(L + L_1 J + L_2 a_T)M_{q,\beta}(\alpha - q)^{-1}(t_1 - t_0)^{\alpha-q} \times \|y - z\|_Y
\end{aligned}$$

$$< \frac{1}{2}\|y-z\|_Y + \frac{1}{\epsilon\Gamma(\alpha)}[(L\epsilon + L_1\epsilon J + N_1) + a_T(L_2\epsilon + N_2)] \times M_{q,\beta}(\alpha - q)^{-1}(t_1 - t_0)^{\alpha - q}\|y-z\|_Y$$

$$< \frac{1}{2}\|y-z\|_Y + \frac{1}{2}\|y-z\|_Y.$$

Thus ϕ is a strict contraction map from S into S and therefore by the Banach contraction principle there exists a unique fixed point y in S such that

$$\phi y = y = \tilde{y}. \tag{3.6}$$

Let $u = A^{-q}y$. Using lemma 3.1.b, we have

$$u(t) = R(t,t_0)[u_0 - h(u)] + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}R(t,s)[f\left(s, u(s), \int_{t_0}^{s}k\big(t,\tau,u(\tau)\big)d\tau\right) + \int_{t_0}^{s}B(s-\tau)g\big(\tau,u(\tau)\big)d\tau]ds \tag{3.7}$$

for every $t \in [t_0, t_1]$. Hence u is a unique local mild solution of (1.1). □

4. LOCAL CLASSICAL SOLUTIONS

In this Section, we establish the regularity of the mild solutions of (1.1). Let J denote the closure of the interval $[t_0, T), t_0 < T \le \infty$. In addition to the hypotheses mentioned in the earlier sections, we assume the condition (H) on the kernel B as that

(H) There exist constants $L_0 \ge 0$ and $0 < p \le 1$ such that

$|B(t_1) - B(t_2)| \le L_0|t_1 - t_2|^p$, for all $t_1, t_2 \in J$.

Theorem 4.1 Suppose that $-A(t)$ generates the resolvent operator $R(t,s)$ such that $\|R(t,s)\| \le Me^{\beta(t-s)}$ and$0 \in \rho[-A(t)]$. Further, suppose that the maps f and g satisfy (F) and the kernel B satisfies (H). Then (1.1) has aunique local classical solution for $u_0 \in X_q$.

Proof: From Theorem 3.2, it follows that there exist T_0, $t_0 < T_0 < T$ and a function u such that u is a unique mild solution of (1.1) on solution $J_o = [t_0, T)$ given by (3.7). Let $v(t) = A^q u(t)$. Then

$$v(t) = R(t,t_0)A^q[u_0 - h(u)] + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}A^q R(t,s)[\tilde{f}(s) + \int_{t_0}^{s}B(s-\tau)\tilde{g}(\tau)d\tau]ds, \tag{4.1}$$

where $\tilde{f}(t) = f\big(t, A^{-q}v(t)\big), \tilde{g}(t) = g\big(t, A^{-q}v(t)\big)$. Since $u(t)$ is continuous on J_o and the maps f and g satisfy (F), it follows that $\tilde{f}$ and $\tilde{g}$ are continuous, and therefore bounded on J_o. Let

$$M_1 = \sup_{t\in J_o}\|\tilde{f}(t)\|$$

and

$$M_2 = \sup_{t\in J_o}\|\tilde{g}(t)\|$$

Using the method as in [14, Theorem 3.2], we can prove that $v(t)$ is locally Hölder continuous on J_o. Then there exist a constant C such that for every $t_0' > t_0$, we have

$$\|v(t_1) - v(t_2)\| \le C|t_1 - t_2|^p, \tag{4.2}$$

for all $t_0 < t_0' < t_1, t_2 < T_0$. Now, assumption (F) with (4.2) implies that there exist constants $k_1, k_2 \ge 0$ and $0 < \gamma, \eta < 1$ such that for all $t_0 < t_0' < t_1, t_2 < T_0$, we have

$$\|\tilde{f}(t_1) - \tilde{f}(t_2)\| \le k_1|t_1 - t_2|^\gamma,$$

$$\|\tilde{g}(t_1) - \tilde{g}(t_2)\| \le k_2|t_1 - t_2|^\eta,$$

which shows that $\tilde{f}$ and $\tilde{g}$ are locally Hölder continuous on J_o.

Let

$$\omega(t) = \tilde{f}(t) + \int_{t_0}^{t}B(s-\tau)\tilde{g}(\tau)d\tau.$$

It is clear that $\omega(t)$ is locally Hölder continuous on J_o.

For $t_2 < t_1$, we have

$$\|\omega(t_1) - \omega(t_2)\| \le C^*|t_1 - t_2|^\beta,$$

For some constants $C^* \ge 0$ and $0 < \beta < 1$. Consider the Cauchy problem

$$\frac{d^\alpha u(t)}{dt^\alpha} + A(t)v(t) = \omega(t), t > t_0, \tag{4.3}$$

and $$v(t_0) = u_0 - h(u). \tag{4.4}$$

From Pazy [26], the problem (4.3), (4.4) has a unique solution v on J_o into X given by

$$v(t) = R(t,t_0)[u_0 - h(u)] + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}R(t,s)\omega(s)ds, \tag{4.5}$$

for $t > t_0$, Each term on the right-hand side belongs to $D(A)$, hence belongs to $D(A^q)$.

Applying A^q on both sides of (4.5) and using the uniqueness of $v(t)$, we have that $A^q v(t) = u(t)$. It follows that u is the classical solution of (1.1) on J_o. Thus u is the unique local classical solution of (1.1) on J. □

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