Applications of $\lambda$-Closed Sets in Intuitionistic Fuzzy Topological Space

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ABSTRACT
In this paper we study the relationship between $\lambda$-closed sets and some other intuitionistic fuzzy sets already exists. We also define intuitionistic fuzzy $\lambda$-irresolute map and study some of its properties.

KEYWORDS
Intuitionistic fuzzy topology, intuitionistic fuzzy $\lambda$-closed sets, intuitionistic fuzzy $\lambda$-open sets, and intuitionistic fuzzy $\lambda$-irresolute maps.

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1. INTRODUCTION
After the introduction of fuzzy sets by L.A Zadeh [17] in 1965, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1983. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topology in 1997. This approach provides a wide field for investigation in the area of fuzzy topology and its application. The aim of this paper is to study the relations between intuitionistic fuzzy $\lambda$-closed sets and the other intuitionistic fuzzy sets already exists. Moreover, we investigate intuitionistic fuzzy $\lambda$-irresolute map and study some of its properties.

2. PRELIMINARIES
Definition 2.1: [1] Let $X$ be a nonempty set. An intuitionistic fuzzy set (IFS in short) $A$ in $X$ is an object having the form $A = \{ <x, \mu_A(x), \nu_A(x) : x \in X \}$, where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\nu_A(x)$ of each element $x \in X$ to the set $A$ respectively and $0 \leq \mu_A(x) \leq 1$ and $0 \leq \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1]: Let $A$ and $B$ be intuitionistic fuzzy sets of the form

$A = \{ <x, \mu_A(x), \nu_A(x) : x \in X \}$, and form $B = \{ <x, \mu_B(x), \nu_B(x) : x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ <x, \mu_A(x), \mu_A(x) > x \in X \}$
(d) $A \cap B = \{ <x, \mu_B(x) \wedge \mu_B(x), \nu_B(x) \vee \nu_B(x) > x \in X \}$

(ε) $A \cup B = \{ <x, \mu_B(x) \vee \mu_B(x), \nu_B(x) \wedge \nu_B(x) > x \in X \}$.

The intuitionistic fuzzy sets $\emptyset = \{ <x, 0,1 > x \in X \}$ and $I = \{ <x, 0,1 > x \in X \}$ are respectively the empty set and whole set of $X$.

Definition 2.3: [1]: Let $(\alpha, \beta) \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$ is defined to be IFS of $X$ given by

$p_{(\alpha, \beta)} = \{ (\alpha, \beta) \} \times (0,1)$ otherwise

Definition 2.4: [5]: An intuitionistic fuzzy topology (IFT) on $X$ is a family of IFSs which satisfying the following axioms.

(i) $0, 1 \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $G_i \in \tau$ for any family $\{ G_i \} \subseteq \tau$

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS) and each intuitionistic fuzzy set in $\tau$ is known as an intuitionistic fuzzy open set (IFOS) for short in $X$.

The complement $A$ of an IFOS in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS) in $(X, \tau)$.

Definition 2.5: [5]: Let $(X, \tau)$ be an intuitionistic fuzzy topology and $A = \{ <x, \mu_A(x), \nu_A(x) : x \in X \}$, be an intuitionistic fuzzy set in $X$. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$Int(A) = \cup \{ G / G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq A \}$

$Cl(A) = \cap \{ K / K \text{ is an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K \}$

Remark 2.6: [5]: For any intuitionistic fuzzy set $A$ in $(X, \tau)$, we have

(i) $cl(A^c) = [int(A)]^c$,
(ii) $int(A^c) = [cl(A)]^c$,
(iii) $A$ is an intuitionistic fuzzy closed set in $X$ if and only if $A = Cl(A)$.
A is an intuitionistic fuzzy open set in $X \Leftrightarrow \text{int}(A) = A$

**Definition 2.7** ([6]: An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}$ in an intuitionistic fuzzy topology space $(X, \tau)$ is called

(i) Intuitionistic fuzzy semi closed if $\text{int}(\text{cl}(A)) \subseteq A$.

(ii) Intuitionistic fuzzy pre closed if $\text{cl}(\text{int}(A)) \subseteq A$.

**Definition 2.8**: An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called

(i) intuitionistic fuzzy generalized closed set [15] (intuitionistic fuzzy g–closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is intuitionistic fuzzy semi open

(ii) intuitionistic fuzzy g–open set [14], if the complement of an intuitionistic fuzzy g–closed set is called intuitionistic fuzzy g–open set.

(iii) intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) if there exists an intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) such that $U \subseteq A \subseteq \text{cl}(U)$ (resp. int$(U)$ $\subseteq A \subseteq U$).

**Remark 2.9**: ([15]: Every intuitionistic fuzzy closed set (intuitionistic fuzzy g–closed) is intuitionistic fuzzy g–open set) but the converse may not be true.

**Definition 2.10**: An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called

(i) an intuitionistic fuzzy w-closed [14] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is intuitionistic fuzzy semi open.

(ii) an intuitionistic fuzzy w–closed set [16] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is intuitionistic fuzzy regular semi open.

(iii) an intuitionistic fuzzy rg–closed set [16] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is intuitionistic fuzzy regular open.

(iv) an intuitionistic fuzzy generalized $\alpha$–closed set [8] (IFGCS) if $\text{nc}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is IFOS in $(X, \tau)$

(v) an intuitionistic fuzzy $\alpha$–generalized closed set [12] (IFGCS) if $\text{nc}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is IFOS in $(X, \tau)$

**Definition 2.11** ([12]): An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called

(i) intuitionistic fuzzy $\alpha$–open set (IF $\alpha$OS in short) if $A \subseteq \text{int}(\text{cl}(A))$

(ii) intuitionistic fuzzy $\alpha$–closed set (IF $\alpha$CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

**Definition 2.12** ([8]): An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called intuitionistic fuzzy alpha generalised closed set (IFGACS in short) if $\text{cl}(A) \subseteq U$ and $U$ is an IFOS in $(X, \tau)$.

**Definition 2.13** ([5]): Let $X$ and $Y$ be nonempty sets and $f: X \to Y$ is a function.

(a) If $B = \{<y, \mu_B(y), \nu_B(y)> : y \in Y\}$ is an intuitionistic fuzzy set in $Y$, then the pre image of $B$ under $f$ denoted by $f^{-1}(B)$, is defined by $f^{-1}(B) = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}$

(b) $\text{If } A = \{<x, \mu_A(x), \nu_A(x)> / x \in X\}$ is an intuitionistic fuzzy set in $X$, then the image of $A$ under $f$ denoted by $f(A)$ is the intuitionistic fuzzy set in $Y$ defined by $f(A) = \{<y, \mu_A(y), \nu_A(y)> : y \in Y\}$ where $f(\mu_A) = 1 - f(1-\nu_A)$.

**Definition 2.14**: ([10]): An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topology space $(X, \tau)$ is called an

(i) intuitionistic fuzzy $\lambda$–closed set (IF $\lambda$–CS) if $A \supseteq \text{cl}(U)$ whenever $A \subseteq U$ and $U$ is intuitionistic fuzzy open set in $X$

(ii) intuitionistic fuzzy $\lambda$–open set (IF $\lambda$–OS) if the complement $A^c$ of an intuitionistic fuzzy $\lambda$–closed set

**Definition 2.15**: ([13]): Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFCS $(Y, \sigma)$. Then $f$ is said to be an

(i) intuitionistic fuzzy open mapping (IF open mapping) if $f(A)$ is an IFOS in $Y$ for every IFOS $A$ in $X$.

(ii) intuitionistic fuzzy closed mapping (IF closed mapping) if $f(A)$ is an IFCS in $Y$ for every IFCS $A$ in $X$.

**Definition 2.16**: ([11]): A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy $\lambda$–continuous if the inverse image of every intuitionistic fuzzy closed set of $Y$ is intuitionistic fuzzy $\lambda$–closed in $X$.

**Definition 2.17**: ([11]): A topological space $(X, \tau)$ is called intuitionistic fuzzy $\lambda$–CS in space

(1) whenever an intuitionistic fuzzy $\lambda$–closed set is intuitionistic fuzzy closed in $X$.

3. APPLICATIONS OF INTUITIONISTIC FUZZY $\lambda$–CLOSED SET.

In this section we study the relations between Intuitionistic fuzzy $\lambda$–closed sets and some other Intuitionistic fuzzy sets already exist.

**Definition 3.1**: Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the intuitionistic fuzzy $\lambda$–interior and intuitionistic fuzzy $\lambda$–closure of $A$ are defined as follows:

$\lambda$–int$(A) = \cup\{G \mid G$ is an IF$\alpha$–CS in $X$ and $G \subseteq A\}$

$\lambda$–cl$(A) = \cap\{K \mid K$ is an IFCS in $X$ and $A \subseteq K\}$.

**Theorem 3.2**: Every IF preclosed set is IF $\lambda$–closed set.

**Proof**: Let $A$ be a IF preclosed set. Let $G$ be an IF open set such that $A \cap G$. Then $\text{cl}(\text{int}(A)) \supseteq \text{cl}(\text{int}(G)) \supseteq \text{cl}(G)$. Therefore $\text{cl}(\text{int}(A)) \supseteq \text{cl}(G)$ since $A$ is IF preclosed set. Thus $A \supseteq \text{cl}(\text{int}(A)) \supseteq \text{cl}(G)$ therefore $A$ is IF $\lambda$–closed set.
Remark 3.3: The converse above theorem need not be true as seen from the following example.

Example 3.4: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.3, 0.6>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.5, 0.5>, <b, 0.8, 0.2>\}$ is IF $\lambda$-closed set but not IF $\omega$-closed set.

Remark 3.5: IF $\lambda$-closed sets and IF $\omega$-closed sets are independent to each other example.

Example 3.6: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.5, 0.2>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.5, 0.5>, <b, 0.5, 0.4>\}$ is not intuitionistic fuzzy IF $\lambda$-closed set but not IF $\omega$-closed set.

Example 3.7: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.4, 0.6>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.5, 0.5>, <b, 0.5, 0.5>\}$ is IF $\omega$-closed set but not IF $\lambda$-closed set.

Remark 3.8: IF $\lambda$-closed sets and IF $\omega$-closed sets are independent to each other example.

Example 3.9: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.5, 0.2>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.5, 0.5>, <b, 0.5, 0.4>\}$ is not intuitionistic fuzzy IF $\lambda$-closed set but not IF $\omega$-closed set.

Example 3.10: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.7, 0.2>, <b, 0.6, 0.3>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.7, 0.2>, <b, 0.6, 0.3>\}$ is IF $\omega$-closed set but not IF $\lambda$-closed set.

Remark 3.11: IF $\lambda$-closed set and IF $\omega$-closed set are independent to each other example.

Example 3.12: Let $X = \{a, b\}$ and $\tau = \{\emptyset, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.5, 0.2>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.5, 0.5>, <b, 0.5, 0.4>\}$ is not intuitionistic fuzzy IF $\lambda$-closed set but not IF $\omega$-closed set.
Example 3.13: Let $X = \{ a,b,c,d \}$ and $\tau = \{ \emptyset, \{ a \}, \{ b \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Let $U = \{ < a, 0.7, 0.1 >, < b, 0.6, 0.3 >, < c, 0.3, 0.7 >, < d, 0.1 > \}$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.7, 0.2 >, < d, 0.1 > \}$ is IF closed set in $X$. Since $g$ is IF continuous, $g^{-1}(U)$ is IF closed set. Therefore $f$ is IF irresolute mapping.

Remark 3.14: Gz closed sets and IF closed sets are independent to each other for example.

Example 3.15: Let $X = \{ a,b \}$ and $\tau = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.3, 0.7 > \}$ is IF closed set but not IF closed set. Example 3.16: Let $X = \{ a, b \}$ and let $\tau = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.3, 0.7 > \}$ is IF closed set but not IF closed set.

Example 3.17: Let $X = \{ a,b \}$ and $\tau = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.3, 0.7 > \}$ is IF closed set but not IF closed set.

Example 3.18: Let $X = \{ a,b \}$ and $\tau = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.3, 0.7 > \}$ is IF closed set but not IF closed set.

Example 3.19: Let $X = \{ a,b \}$ and $\tau = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ be an intuitionistic fuzzy topology on $X$. Then the intuitionistic fuzzy set $A = \{ < a, 0.9, 0.1 >, < b, 0.8, 0.2 >, < c, 0.3, 0.7 > \}$ is IF closed set but not IF closed set.

Remark 3.20: From above examples and remarks we get following diagram of implication.

In this diagram

$A \rightarrow B$ means that $A$ implies $B$

$A \rightarrow B$ means that $B$ does not imply $A$

$A \rightarrow B$ means that $A$ and $B$ are independent to each other.

4. $\lambda$- irresolute mappings in intuitionistic fuzzy topological spaces.

In this section we introduce intuitionistic fuzzy $\lambda$- irresolute mapping and study some of its properties.

Definition 4.1: A mapping $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy $\lambda$- irresolute (IF- $\lambda$- irresolute) if $f^{-1}(V)$ is IF $\lambda$- closed set in $(X, \mathcal{T})$ for every IF $\lambda$- closed set $V$ of $(Y, \sigma)$.

Theorem 4.2: If a mapping $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ is called IF $\lambda$- irresolute mapping then it is IF $\lambda$- continuous mapping but not conversely.

Proof: Let $V$ be any closed set in $Y$, since every closed set is IF $\lambda$- closed set, $V$ is IF $\lambda$- closed set in $Y$. By assumption $f^{-1}(V)$ is IF $\lambda$- closed set in $X$. Thus $f$ is IF $\lambda$- continuous.

The converse of the above theorem need not be true as seen in the following example:

Example 4.3: Let $X = \{ a, b \}$, and intuitionistic fuzzy sets $U$ and $V$ are defined as follows.

$U = \{ < a, 0.5, 0.5 >, < b, 0.2, 0.7 > \}$

$V = \{ < a, 0.5, 0.5 >, < b, 0.6, 0.4 > \}$

be the intuitionistic fuzzy sets respectively and let $\mathcal{T} = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$ and $\sigma = \{ \emptyset, \{ a \}, \{ a,b \}, X \}$. Then $U$ and $V$ be intuitionistic fuzzy topologies on $X$ and $Y$ respectively. And let $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ be the identity mapping. Clearly $f$ is IF $\lambda$- continuous mapping. $f^{-1}(\{< a, 0.5, 0.5 >, < b, 0.4, 0.6 >\}) = \{< a, 0.5, 0.5 >, < b, 0.4, 0.6 >\}$ in not IF $\lambda$- closed set in $X$. Therefore $f$ is not IF $\lambda$- irresolute mapping.

Theorem 4.4: Let $X,Y$, and $Z$ be any topological space. For any IF $\lambda$- irresolute map $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ and IF $\lambda$- continuous map $g: (Y, \sigma) \rightarrow (Z, \upsilon)$ the composition $g \circ f: (X, \mathcal{T}) \rightarrow (Z, \upsilon)$ is IF $\lambda$- continuous.

Proof: Let $V$ be any closed set in $Z$, since $g$ is IF $\lambda$- continuous map, $g^{-1}(V)$ is IF $\lambda$- closed in $Y$. Since $f$ is IF irresolute map $f^{-1}(g^{-1}(V))$ is IF $\lambda$- closed in $X$. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ which is $\lambda$- closed in $X$. Thus $g \circ f$ is IF $\lambda$- continuous map.

Theorem 4.5: If $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ is IF open mapping and IF continuous then $f$ is IF $\lambda$- irresolute map.

Proof: Let $A$ be a $\lambda$- closed set in $X$. Let $f^{-1}(A)$ be open in $Y$, and $f(O)$ is open in $Y$. Thus we have $A \subseteq f(O)$. This implies $A \subseteq Cl_f(f(O))$. But $Cl_f(f(O)) \supseteq f(\text{Cl}_O)$ because $f$ is IF continuous. Therefore $A \subseteq f(\text{Cl}_O)$. Hence $f^{-1}(A)$ is IF $\lambda$- closed set in $X$. Therefore $f$ is IF $\lambda$- irresolute map.

Theorem 4.6: If $X$, $Y$, and $Z$ are topological spaces and $f: (X, \mathcal{T}) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \upsilon)$ are IF $\lambda$- irresolute and IF $\lambda$- continuous maps respectively then composition $g \circ f$ is IF $\lambda$- irresolute map.

Proof: Let $V$ be a IF $\lambda$- closed set in $Z$. Since $g$ is IF $\lambda$- irresolute map, $g^{-1}(V)$ is IF $\lambda$- closed set in $Y$. Also we are given that $f$ is IF $\lambda$- irresolute and therefore $f^{-1}(g^{-1}(V))$ is IF $\lambda$- closed set in $(X, \mathcal{T})$. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Hence $g \circ f$ is IF $\lambda$- irresolute map from $X$ to $Z$. Therefore $f$ is IF $\lambda$- irresolute map.
Theorem 4.8: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y.

Then the following conditions are equivalent if X and Y are IF λ-T_{1/2} spaces.

(i) f is an IFλ- irresolute mapping
(ii) \( f^{-1}(B) \) is an IFCS in Y for each IFCS in Y
(iii) \( \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \) for each IFS B of Y.

Proof: (i) ⇒ (ii): It can be proved by using complement and definition 4.1.

(ii) ⇒ (iii): Let B be any IFS in Y and B \( \subseteq \text{cl}(B) \). Then \( f^{-1}(B) \subseteq f^{-1}(\text{cl}(B)) \). Since \( \text{cl}(B) \)

is an IFCS in Y, \( \text{cl}(B) \) is an IFλ-CS in Y. Therefore \( f^{-1}(\text{cl}(B)) \) is an IFCS in X.

by hypothesis. Since X is IF λ-T_{1/2} space. \( f^{-1}(\text{cl}(B)) \) is an IFCS in X. Hence

\( \text{cl}(f^{-1}(B)) \subseteq \text{cl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B)) \). That is \( f^{-1}(B) \subseteq f^{-1}(\text{cl}(B)) \).

(iii) ⇒ (i): Let B be an IF λ-CS in Y. Since Y is an IF λ-T_{1/2} space, B is an IFCS in Y and

\( \text{cl}(B) = B \). Hence \( f^{-1}(B) = f^{-1}(\text{cl}(B)) \subseteq \text{cl}(f^{-1}(B)) \). But clearly \( f^{-1}(B) \supseteq \text{cl}(f^{-1}(B)) \).

Therefore \( \text{cl}(f^{-1}(B)) = f^{-1}(B) \). This implies \( f^{-1}(B) \) is an IFCS and hence it is an IFλ-CS in X. Thus f is an IFλ-

irresolute mapping.

Theorem 4.9: Let f : (X, τ) → (Y, σ) be an IFλ- irresolute mapping from an IFTS X into an IFTS Y. Then \( f^{-1}(B) \subseteq \lambda\text{-int}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))))) \) for every IFλ-OS B in Y, if X and Y are IF λ-T_{1/2} spaces.

Proof: Let B be an IFλ-OS in Y. Then by hypothesis \( f^{-1}(B) \) is an IFλ-OS in X. Since X is an IF λ-T_{1/2} space, \( f^{-1}(B) \) is an IFOS in X. Therefore \( \lambda\text{-int}(f^{-1}(B)) \subseteq \lambda\text{-int}(B) \). Since Y is an IFλ-T_{1/2} space, B is an IFλ-OS in Y and \( B \subseteq \text{cl}(\text{int}(\text{cl}(B))) \). Now \( f^{-1}(B) = \lambda\text{-int}(f^{-1}(B)) \) implies,

\( f^{-1}(B) \subseteq \lambda\text{-int}(\text{cl}(\text{int}(\text{cl}(B)))) \).

Theorem 4.10: If a mapping f : X → Y is intuitionistic fuzzy λ- irresolute mapping, then

\( f(\lambda\text{-cl}(B)) \subseteq \lambda\text{-cl}(f(B)) \) for every IFS B of X.

Proof: Let B be an IFS of X. Since \( \text{cl}(f(B)) \) is an IFλ-CS in Y , by our assumption

\( f^{-1}(\text{cl}(f(B))) \) is an IFλ-CS in X. Furthermore \( B \subseteq f^{-1}(\text{cl}(f(B))) \).

\( \lambda\text{-cl}(B) \subseteq f^{-1}(\text{cl}(f(B))) \) and consequently \( f(\lambda\text{-cl}(B)) \subseteq f(f^{-1}(\text{cl}(f(B)))) \) \( \subseteq \text{cl}(f(B)) \).

Theorem 4.11: If any union of IFλ-CS is an IFλ-CS, then a mapping f : X → Y from an IFTS X into an IFTS Y is intuitionistic fuzzy λ- irresolute if and only if for each IFP \( P(αβ) \) in X and IFλ-CS B in Y such that \( f(P(αβ)) \supseteq B \), there exists an IFλ-CS A in X such that \( P(αβ) \subseteq A \) and

\( f(A) \subseteq B \).

Proof: Let f be any intuitionistic fuzzy irresolute mapping, \( P(αβ) \) an IFP in X and B be any IFλ-CS in Y , such that f (P(αβ)) \( \supseteq B \). Then \( f^{-1}(B) = \lambda\text{-cl}(f^{-1}(B)) \). We take

A = \( \lambda\text{-cl}(f^{-1}(B)) \). Then A is an IFλ-CS in X, containing IFP \( P(αβ) \) and

\( f(A) = f(\lambda\text{-cl}(f^{-1}(B))) \subseteq f(f^{-1}(B)) \subseteq B \).

Conversely assume that B be any IFλ-CS in Y and IFP \( P(αβ) \) in X, such that

\( P(αβ) \subseteq f^{-1}(B) \). By assumption there exists IFλ-CS A in X such that \( P(αβ) \subseteq A \) and (f(A) \( \subseteq B \). Therefore \( P(αβ) \subseteq f^{-1}(B) \) and \( P(αβ) \subseteq A = \lambda\text{-cl}(A) \subseteq \lambda\text{-cl}(f^{-1}(B)) \). Since \( P(αβ) \) is an arbitrary IFP and \( f^{-1}(B) \) is union of all IFP contained in \( f^{-1}(B) \), \( f^{-1}(B) \) is an IFλ-CS in X, so f is an intuitionistic fuzzy λ-

irresolute mapping.

Corollary 4.12. A mapping f : X → Y from an IFTS X into an IFTS Y is intuitionistic fuzzy λ- irresolute if and only if f is a mapping from an IFλ-OS in X to an IFλ-CS in Y such that

\( f(P(αβ)) \supseteq B \). Such that \( f(P(αβ)) \subseteq B \). Therefore \( f(P(αβ)) \supseteq B \).

That is, there exists an IFλ-CS A in X such that \( P(αβ) \subseteq A \) and \( A \subseteq f^{-1}(B) \).

Proof: Follows from Theorem 4.11.

5. CONCLUSION

In this paper we have studied the relations between intuitionistic fuzzy λ-closed sets and the other intuitionistic fuzzy sets already exists. Also we studied the intuitionistic fuzzy λ- irresolute map and some of its properties.

6. REFERENCES


