Range Limited Quad-Histogram Equalization Method for Image Contrast Enhancement

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ABSTRACT
Histogram equalization is a very popular technique for image contrast enhancement. Histogram equalization defines the clarity of picture and sometime improves brightness. But sometime it is not very good for consumer electronics, because of preserving the original brightness. Range limited bi-histogram equalization RLBHE applied for consumer electronics. This paper proposes novel extension of RLBHE referred to as range limited quad-histogram equalization RLQHE. This method is applied as first it divides the input histogram into two sub-images by a threshold. Then again two sub-images divide in to two sub-images by using another threshold that minimizes intra-class variance. All these sub-images separate the object from background. Then, Range means it calculate the minimum absolute mean brightness error between actual image and equalized image. The result shows that proposed method is better than previous method for image contrast enhancement. Proposed method preserves original brightness as actual image. It is very well suited for consumer electronics product.

General Terms

Keywords
Image Contrast Enhancement, Quad Histogram Equalization, Brightness Preserving Enhancement; Range Limit.

1. INTRODUCTION
Global histogram equalization (GHE) is a one of the most commonly method that improves the contrast in an image, in order to stretch out the intensity range. Histogram equalization implies mapping one distribution to another distribution as the given histogram to a wider and more uniform distribution of intensity values. It is achieved by normalizing the intensity distribution using cumulative distribution function so that the result may have a uniform distribution of intensity [1].

GHE is rarely employed in consumer electronics such as TV. In spite of its high performance in enhancing contrast of a given image, Histogram equalization may change the original brightness of an input image, introduce some annoying artifacts and increase noise level [2].

During the last few decades, several useful approaches are applied to overcome this problem include GHE technique. These techniques are Brightness Preserving Bi-Histogram Equalization (BBHE) [2], Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE) [3] and Minimum Mean Brightness Error Bi-Histogram Equalization (MMEBHE) [4] and Range Limited Bi-Histogram equalization (RLBHE) [5]. Also RMSHE (Recursive Mean-Separate Histogram Equalization) [6] and RSHE (Recursive Sub-Image Histogram Equalization) [7] are recursive algorithms of BBHE and DSIHE, etc. BBHE decomposes the input image into two parts based on its mean value. Input image histogram divides into two parts which is equalized independently. This technique preserves the original brightness up to some extent. Another advance method of it is MMBEBHE which provide maximal brightness preservation based on threshold value, in this method calculating the minimum difference between input and output mean. This method is some better than the previous method. Give good contrast enhancement of image but variation of gray level of distribution in histogram equalization, some annoying artifact generated. Two recursive methods RMSHE and RSIHE also give the improved result as compare to the previous methods. Some annoying side effect reduced by these methods. Another extension of bi-histogram equalization is RLBHE. This method gives more improvement for image contrast enhancement and brightness improvement and avoided over enhancement. The Otsu’s method which is threshold selection method [8] applied to perform histogram thresholding. In this method limit the range and mean input brightness equal to the mean output brightness.

In this paper presents new histogram equalization method called Range Limited Quad-Histogram Equalization [9]. This method is extension of RLBHE. Range limited bi-histogram divide the input histogram into two sub-images but quad-histogram divide the input histogram into two sub-images and again these two sub-images into two sub-images by another two threshold by applying Otsu’s method which is threshold selection method. After this when combine the images, it gives much better result. Much better contrast enhancement images and brightness improved as compare to the previous method. Absolute mean brightness error (AMBE) is minimum as compare to the other method.

In what follows, GHE and bi-histogram equalization for digital image is reviewed with its mathematical formulation in Section 2 and 3, respectively. The RLBHE method and RLQHE method presented in Section 4 and 5. Section 6 lists a few experimental results to give the performance of RLQHE. Section 7 serves as the conclusion of this paper.

2. GLOBAL HISTOGRAM EQUALIZATION
Suppose that \( D = \{ D(i,j) \} \) define the digital image, Where \( D(i,j) \) denotes the gray level of pixels at (i,j) place. The total number of pixels of image is \( m \), and the image intensity is digitized into \( L \) levels that are \( \{ D_0, D_1, D_2, \ldots \ldots \ldots \ldots \ldots, D_{L-1} \} \). So \( v(i,j) \in \{ D_0, D_1, D_2, \ldots \ldots \ldots \ldots \ldots, D_{L-1} \} \). \( m_a \) denotes a total no. of pixels with gray level of \( D_k \) in the image. So the probability density of \( D_k \) will be
\[ p(D_k) = \frac{m_k}{m}, \quad k=0,1,2,3, \ldots, L-1 \]  
(1)

\[ p(D_k) \text{ and } D_k \text{ is defined as a probability density function (PDF), its cumulative distribution function is defined as} \]

\[ c(D_k) = \sum_{j=0}^{k-1} p(D_j) = \sum_{j=0}^{k-1} \frac{m_k}{m} \]
(2)

where \( k=0,1,2,\ldots,L-1 \) and \( c(D_{L-1})=1 \). Let us define a transform function \( f(d) \) based on the cumulative density function as

\[ f(d)=D_b+(D_{L-1}-D_b)c(d) \]
(3)

Output image of the GHE, \( G=\{G(i,j)\} \), which can be expressed as

\[ G= f(D) = \{f(D(i,j)\} \quad D(i,j) \in D \}
(4)

\( G \) follows a uniform distribution. Suppose that \( D \) is a continuous random variable, i.e., \( L=\infty \), then the output of GHE, \( G \) is also regarded as a continuous random variable and

\[ p(g) = \frac{1}{D_{L-1}-D_b} \]
(5)

It is so easy to show that mean brightness of output image

\[ E(G) = \frac{D_{L-1}}{D_b} \cdot \int_{D_b}^{D_{L-1}} g p(g) dg = \frac{D_{L-1}}{D_b} \cdot \int_{D_b}^{D_{L-1}} \frac{D_{L-1}-D_b}{D_{L-1}-D_b} dg = \frac{D_{L-1}}{2} \]
(6)

Where \( E(.) \) denotes statistical expectation. It shows that the output mean of the histogram equalization does not take the mean brightness of the original image into account. The input image is dark. This property is not very good for some application such as consumer electronics because it change the brightness of input image which is not very good.

### 3. BI-HISTOGRAM EQUALIZATION

Some bi-histogram equalization methods have been proposed to overcome the aforementioned problems, these all method divide the input image into two sub-parts. These two sub-part than equalize independently. The difference among these methods in this family is the criteria used to chose the threshold for separation denoted by \( D_T \). And \( D_T=D_bD_{L-1} \). The input image \( D \) can be divided into two sub parts \( D_L \) and \( D_B \) based on threshold

\[ D = D_L \cup D_B \]
(7)

Where

\[ D_L=\{D(i,j)\}D(i,j)\leq D_T \lor D(i,j)<D_T \}
(8)

And

\[ D_B=\{D(i,j)\}D(i,j)>D_T \lor D(i,j)<D_T \]
(9)

Now, respective PDF of the sub-images \( D_L \) and \( D_B \) as

\[ p_L(D_k) = \frac{m_k}{m_L}, \quad k=0,1, \ldots, T \]
(10)

And

\[ p_B(D_k) = \frac{m_k}{m_B}, \quad k=T+1,1 \ldots, L-1 \]
(11)

\( m_k \) Represent the numbers of \( D_k \) in \( D_L \) and \( D_B \) and \( m_L \) and \( m_B \) are the total number of samples in \( D_L \) and \( D_B \). Then the respective cumulative density functions for \( D_L \) and \( D_B \) are then defined as

\[ c_L(D_k) = \sum_{j=0}^k p_L(D_j) \]
(12)

And

\[ c_B(D_k) = \sum_{j=T+1}^k p_B(D_j) \]
(13)

Same as in GHE where cumulative density function is used as a transform function, Let us define the following transform functions exploiting the cumulative density functions

\[ f_L(D_k) = D_b+(D_T-D_b)c_L(D_k), \quad k=0,1, \ldots, T \]
(14)

And

\[ f_B(D_k) = D_T+(D_{L-1}-D_T)c_B(D_k), \quad k=T+1,1 \ldots, L-1 \]
(15)

BBHE [2] divides the input histogram into two parts which is based on threshold that is mean brightness of the input

\[ D_T = D_m = \sum_{j=0}^{L-1} D_j p(D_j) \]
(16)

Mean brightness of the output of the BBHE can be expressed as [2]

\[ E(G) = \frac{D_m}{2} \]
(17)

BBHE preserve the mean brightness of image up to some extent. To overcome the problem DSIHE [3] came. It give the better result as compare to the BBHE. It is same as the BBHE, except that it selects the \( D_B \) median value as gray level.

\[ \sum_{j=0}^{D_m} p(D_j) \approx \frac{1}{2} \]
(18)

The output image follows

\[ E(G) = \frac{D_m}{2} \]
(19)

MMBEBHE [4] performs separation based on threshold level, which calculate minimum difference between input mean and output mean. Only threshold is selected and other procedure are same.

### 4. RANGE LIMITED BI-HISTOGRAM EQUALIZATION

RLBHE performs basically three steps

1. Selecting a threshold.
2. Determine the upper and lower bound for histogram equalization.
3. Equalize each part.

#### 4.1 Selecting a Threshold for Histogram Separation

Otsu’s method [8] performs basically sum of variance based image thresholding. It searches threshold that minimize intra-class variance, calculated weighted sum of variances of two classes

\[ \sigma^2(D_T) = W_L(E(D_L) - E(D))^2 + W_B(E(D_B) - E(D))^2 \]
(20)
Where $E(D_L)$ and $E(D_U)$ stands for average brightness of the two sub-images thresholded by $D_I$. $E(D)$ is the mean brightness of the whole image. $W_l$ and $W_u$ stands for the fraction to define the numbers of two classes of the whole pixels.

$$W_L = \frac{m_L}{m} \quad (21)$$

Threshold calculated by otsu’s method as

$$D_o = \arg \max_{D_o} \{ \sigma^2(D_o),T=0,1,2,\ldots,L-1 \} \quad (22)$$

### 4.2 Determine The Upper and Lower Bounds for Histogram Equalization

Otsu’s method separate the object from the background. The mean brightness of output image of bi-histogram equalization as

$$E(G) = E(G|D) \leq D_o \{ p(D|D_o) = E(D|D_o) \}$$

$$= \frac{D_o + D_o}{2} (\sum_{i=0}^{D_o} p(D_i)) + \frac{D_o + D_o}{2} (\sum_{i=D_o+1}^{L} p(D_i))$$

$$= \frac{1}{2} [D_o + D_o] (\sum_{i=0}^{D_o} p(D_i) + (D_o + D_o - 1) (1 - \sum_{i=0}^{D_o} p(D_i))] \quad (23)$$

The output image should keep the mean brightness of original image as possible

$$E(G) \approx E(D) = \sum_{j=0}^{L} D_j p(D_j) \quad (24)$$

Then

$$\frac{1}{2} [(D_o + D_o)(\sum_{i=0}^{D_o} p(D_i)) + (D_o + D_o) + 1 + D_o - 1) (1 - \sum_{i=0}^{D_o} p(D_i))]$$

$$= D_m \quad (25)$$

By otsu’s method minimum Absolute Mean Brightness Error (AMBE) between equalize image and original image:

$$(D'_{L-1}, D'_o) = \arg \min_{D_{L-1}, D_o} \{|E(G) - E(D)|\}$$

$$= \arg \min_{D_{L-1}, D_o} \left\{ \frac{1}{2} [(D_o + D_o)(\sum_{i=0}^{D_o} p(D_i)) + (D_o + D_o - 1) \sum_{i=D_o+1}^{L} p(D_i)))] \right\}$$

$$* (1 - \sum_{i=0}^{D_o} p(D_i)) = D_m \mid \arg \min_{D_{L-1}, D_o} \left\{ \frac{1}{2} [(D_o + D_o)(\sum_{i=0}^{D_o} p(D_i)) + (D_o + D_o - 1) \sum_{i=D_o+1}^{L} p(D_i)))] \right\}$$

$$D'_{L-1}, D'_o) \text{ and } G \text{ denote the input and output image. Equation "25" can be simplified as}$$

$$(D'_{L-1}, D'_o) = \arg \min_{D_{L-1}, D_o} \left\{ (aD_o + (1-a) D'_{L-1} \cdot b)^2 \right\} \quad (27)$$

Where $a = \sum_{i=D_o}^{D_o} p(D_i)$, $b = 2D_m - D_o -(1 - \sum_{i=D_o}^{D_o} p(D_i))$. Then

$$\begin{cases} 0 \leq D_o \leq D \quad (28) \\
D_o < D'_{L-1} \leq D_{L-1} \end{cases}$$

Give the best brightness preservation.

### 4.3 Equalize Each Part

RLBHE [5] equalize sub-image independently. Final transform function

$$f_{k}(D_k) = D_k + (D_o - D_k)c_k(D_k), \quad k = O, 1, \ldots, O \quad (29)$$

And

$$f_k(D_k) = D_{k+1} + (D_k - D_{k+1})c_k(D_k), \quad k = O+1, 1, \ldots, L-1 \quad (30)$$

The recombine sub-images. The output of RLBHE, $G$, is finally expressed as

$$G = \{ G(i, j) \} = G_L \cup G_U = f_L(D_L) \cup f_U(D_U) \quad (31)$$

Where

$$G_L = \{ f_L(D_L) \} = \{ f(D(i, j)) \} \quad (32)$$

And

$$G_U = \{ f_U(D_U) \} = \{ f(D(i, j)) \} \quad (33)$$

### 5. RANGE LIMITED QUAD-HISTOGRAM EQUALIZATION

RLQHE performs basically three steps

1. Choosing a proper threshold for histogram separation. Than image is sub-divided into two sub-images and another two different threshold are choosing for histogram separation.
2. Determine the upper and lower bound for histogram equalization for three threshold.
3. Equalize each partition independently.

#### 5.1 Choosing a Proper Threshold for Histogram Separation. Than Image is Sub-Divided into Two Sub-Images and Another Two Different Threshold are Choosing for Histogram Separation

Histogram is dividing into two parts by selecting a threshold. Otsu’s method [8] is used to perform histogram based thresholding. As RLQHE [5] this algorithm assumes that image to be threshold contains two classes of pixels (e.g. foreground and background). Calculate intra-class variance. Calculate weighted sum of variances of the two classes:

$$\sigma^2(D_T) = W_L (E(D_L) - E(D))^2 + W_U (E(D_U) - E(D))^2 \quad (34)$$

Where $E(D_L)$ and $E(D_U)$ stands for average brightness of the two sub-images thresholded by $D_T$. $E(D)$ is the mean brightness of the whole image. $W_L$ and $W_U$ stands for the fraction to define the numbers of two classes of the whole pixels.

Thresholded image contain two classes of pixels. Then again image is sub-divided into two parts and calculates the weighted sum of variances of the two classes. For first sub-image:

$$\sigma^2(D_{T1}) = W_{L1} (E(D_{L1}) - E(D))^2 + W_{U1} (E(D_{U1}) - E(D))^2 \quad (35)$$

For second sub-image:

$$\sigma^2(D_{T2}) = W_{L2} (E(D_{L2}) - E(D))^2 + W_{U2} (E(D_{U2}) - E(D))^2 \quad (36)$$
5.2 Determine The Upper and Lower Bound for Histogram Equalization for Three Threshold

Otsu’s method separate the object from the background. The mean brightness of output image of bi-histogram equalization as

\[ E(G) = E(GD \leq D_0)p(D \leq D_0) + E(GD > D_0)p(D > D_0) \]

\[ = \frac{D_0 + D_1}{2} \left( \sum_{i=0}^{D_1} p(D_i) \right) + \frac{D_0 + D_{L-1}}{2} \left( \sum_{i=D_1+1}^{L-1} p(D_i) \right) \]

\[ = \frac{1}{2} [ (D_0+D_1) \sum_{i=0}^{D_1} p(D_i) + (D_0+1+D_{L-1}) \sum_{i=D_1+1}^{D_0} p(D_i) ] \]

(39)

The output image should keep the mean brightness of original image as possible

\[ E(G) \approx E(D) = \sum_{j=0}^{L-1} \frac{1}{2} D_j p(D_j) \]

(40)

Then

\[ \frac{1}{2} [ (D_0+D_0) \sum_{i=0}^{D_0} p(D_i) - (D_0+1+D_{L-1}) \sum_{i=D_0+1}^{D_0} p(D_i) ] \]

\[ = D_m \]

(41)

By otsu’s method minimum Absolute Mean Brightness Error (AMBE) between equalize image and original image:

\[ \left( D'_{L-1}, D'_0 \right) = \arg \min_{D'_{L-1}, D'_0} \left\{ E(G) - E(D) \right\} \]

\[ = \arg \min_{D'_{L-1}, D'_0} \left\{ \frac{1}{2} [ (D_0+D_0) \sum_{i=0}^{D_0} p(D_i) + (D_0+1+D_{L-1}) \sum_{i=D_0+1}^{D_0} p(D_i) ] \right\} \]

(42)

D and G denote the input and output image. Equation"39" can be simplified as

\[ (D'_{L-1}, D'_0) = \arg \min_{D'_{L-1}, D'_0} \left\{ (aD'_0 + (1-a) D'_{L-1} - b)^2 \right\} \]

(43)

5.3 Equalize Each Partition Independently

RLBHE [5] equalize sub-image independently. Final transform function

\[ f_k(D_k) = D'_0 + (D_0 - D'_0) c_k(D_k), \quad k=0,1,\ldots,O \]

(45)
Fig 4: Two Sub Images of Second Sub Image of Original F16 image and there Histograms with respect to Third Threshold Level

And

\[ f_O(D_k) = D_{O+1} + (D_{L-1} - D_{O+1}) c_O(D_k), \ k = 0, 1, \ldots, L-1 \] (46)

The recombine sub-images. The output of RLQHE, \( G \), is finally expressed as

\[ G = \{ G(i,j) \} = G_U \cup G_O = f_L(D_L) \cup f_U(D_U) \] (47)

Where

\[ G_L = f_L(D) = \{ f(D(i,j)) : D(i,j) \in D_L \} \] (48)

And

\[ G_U = f_U(D) = \{ f(D(i,j)) : D(i,j) \in D_U \} \] (49)

This method is also applied to two sub-images.

For first sub-image equalization is:

\[ G = \{ G(i,j) \} = G_{L1} \cup G_{U1} = f_L(D_{L1}) \cup f_U(D_{U1}) \] (50)

Where

\[ G_{L1} = f_L(D) = \{ f(D(i,j)) : D(i,j) \in D_{L1} \} \] (51)

\[ G_{U1} = f_U(D) = \{ f(D(i,j)) : D(i,j) \in D_{U1} \} \] (52)

For Second sub-image equalization is:

\[ G = \{ G(i,j) \} = G_{L2} \cup G_{U2} = f_L(D_{L2}) \cup f_U(D_{U2}) \] (53)

Where

\[ G_{L2} = f_L(D) = \{ f(D(i,j)) : D(i,j) \in D_{L2} \} \] (54)

And

\[ G_{U2} = f_U(D) = \{ f(D(i,j)) : D(i,j) \in D_{U2} \} \] (55)
Fig 8: (a) Original image of Aircraft. (b) Result of RLQHE. (c) Result of RLBHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBE

Fig 9: (a) Original image of F16. (b) Result of RLQHE. (c) Result of RLBHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE

Fig 10: (a) Original image of Plane. (b) Result of RLQHE. (c) Result of RLBHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE

Fig 11: (a) Original image of Tank. (b) Result of RLQHE. (c) Result of RLBHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE
RLQHE method divides the image into two sub-images and chooses three threshold for these sub-images. When image is divided into quad form by RLQHE than it gives better result for image contrast enhancement. When recombine the images, It gives more better result.

Figure.1 shows the test image of F16 that is original image of F16 and its histogram. Figure.2 shows the two sub-image of original F16 image and its histogram with respect to first threshold level. Figure.3 shows the two sub-images of first sub-image of original image F16 and its histogram with respect to second threshold level.

Figure.4 shows the sub-images of second sub-image of original F16 image and its histogram with respect to third threshold level. Figure.5 shows the RLBHE[5] response the first sub-image of original F16 image and its histogram with respect to second threshold level. Figure.6 RLBHE[5] response the second sub-image of original F16 image and its histogram with respect to third threshold level.

Figure.7 shows the RLQHE response the original image and its histogram. RLQHE gives good result as compared to previous method. Give good contrast enhancement and more clarity of picture.

6. RESULT AND DISCUSSION

In this paper, many bi-histogram methods are discussing as GHE, BBHE, DSIHE, MMBEBHE and RLBHE. The proposed method is RLQHE. Table 1 lists the AMBE with proposed method result.

The first test image in Figure.8 GHE, BBHE and DSIHE over enhance the trail of the aircraft. MMBEBHE enhances the background’s contrast good, but changes the aircraft body. RLBHE gives the better result. RLQHE gives the best result than BBHE and DSIHE.

The second test image is F16 in Figure.9. The output of BBHE, MMBEBHE and RLBHE is very similar while the result of GHE and DSIHE shows changes in brightness and decrease of contrast.

Figure.10 shows the image of plane. The result of RLQHE gives the better result as compare to other method. The result of GHE, BBHE and DSIHE are too dark when compared to the original image. MMBEBHE gives good result up to some extent, but the brightness is not kept well. RLBHE preserve the brightness of image and RLQHE gives natural enhancement and give more better result.

Figure.11 shows the image of tank. Resulting images of GHE, BBHE, DSIHE and MMBEBHE have mean brightness is much brighter compared to the original image. RLBHE has preserved the brightness and natural enhancement and RLQHE give more better result.

7. CONCLUSION

In this paper many histogram method discussed. These methods give good result up to some extent. In this paper we proposed a novel extension of RLBHE which is RLQHE Range Limited Quad-Histogram Equalization using Otsu’s[8] method. The proposed method gives better brightness and contrast enhancement and natural enhancement of image. It is better for consumer electronics and real time processing. RLQHE can also be implemented with other thresholding method for betterment of the image contrast.

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9. REFERENCES


Table 1. The Result of AMBE for GHE, BBHE, DSIHE, MMBEBHE, RLBHE and RLQHE

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Image Name</th>
<th>GHE</th>
<th>BBHE</th>
<th>DSIHE</th>
<th>MMBEBHE</th>
<th>RLBHE</th>
<th>RLQHE</th>
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<tr>
<td>F16</td>
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<td>1.463</td>
<td>23.75</td>
<td>0.060</td>
<td>0.862</td>
<td>6.857</td>
<td>0</td>
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<td>Plane</td>
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<td>1.013</td>
<td>18.29</td>
<td>0.023</td>
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<td>0.050</td>
<td>1</td>
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<tr>
<td>Tank</td>
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<td>16.71</td>
<td>29.36</td>
<td>2.820</td>
<td>0.872</td>
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