Non Coherent Block Coded Modulation using Linear Components Codes

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ABSTRACT
This paper derives minimum non coherent distances of block-coded TAPSK (twisted amplitude and phase shift keying) and 16QAM (quadrature-amplitude modulation), both using linear component codes. According to the derived distances, non coherent block-coded TAPSK (NBC-TAPSK) and non coherent block-coded 16QAM (NBC-16QAM) are proposed. If the block length is very small, NBC-16TAPSK performs best among all non coherent schemes and NBC-16QAM performs worse due to its small minimum non coherent distance. However, if the block is not short, NBC-16QAM has the best error performance because the code words with small non coherent distances are rare. Here it also changes the value of r and see the performance of BER and also see the effect of Rayleigh channel on BER.

Index Terms
Non coherent detection, BCH Codes Block coded modulation, multilevel coding.

1. INTRODUCTION
The Additive White Gaussian Noise (AWGN) channel which introduces an unknown carrier phase rotation has been investigated in many works, for example, [1]-[8]. This channel offers a useful abstraction of the flat fading channel, when the effects of the phase rotation need to best studied independently of the amplitude variations. A simple model that is commonly used is one where the unknown carrier phase is constant over a block of N symbols and independent from block to block, [1], [2]. This model is correct for frequency hopping systems. For this non coherent channel with large N, pilot symbols used for the carrier phase estimation combined with codes designed for coherent decoding perform well. However, for small N, block codes designed for non coherent decoding outperform these training-based non coherent codes. The minimum non coherent distances of codes are obtained by brute-force searching for all codeword-pairs.For the transmitted baseband codeword **x** =($x1,x2, \cdots, xN$), the received baseband block y = ($y1$, $y2, \cdots, yN$) is given by y = x exp{$j\theta$} + n .signal point in the signal constellation of 8PSK, is labeled by (a, b, c) where a, b, and $c \in \{0, 1\}$. Let ($a1$, $b1$, $c1$), ($a2$, $b2$, $c2$), $\cdots$, (aN, bN, cN) be a block of transmitted signals. If ca = ($a1$, $a2, \cdots, aN$), cb = ($b1$, $b2, \cdots, bN$) and cc = ($c1$, $c2, \cdots$, cN) are code words of binary block codes Ca, Cb and Cc, are also called components codes. the minimum non coherent Hamming distance of Ci is defined by $dncH,i$ = min{di,min,$N - di$,max} where d,min and di,max denote the minimum and maximum values of Hamming distance between any two code words corresponding to different data bits in C.

2. NONCOHERENT BLOCK MODULATION USING LINEAR COMPONENT CODES
2.1 TAPSK
For TAPSK with labeling in Fig. 1, the bit in level a decides Symbol energy. The radiuses of the inner and outer circles are denoted by r_0 and r_1, respectively. The values of r_1 and r_0 $(r_0 \le 1 \le r_1)$ satisfy r=2 when a=0 has the same probability as a =1, $r_0^2 + r_1^2 = 2$. With the proof given in Appendix A, it has the following theorem Define $f(d)$ by (d)=

$$\frac{r_1^2 - r_0^2}{2} d - \sqrt{(r_0^2(N-d) + r_0 r_1 \cos\Phi)^2 + (r_0 r_1 d \sin\Phi)^2}$$

Block coded generalized-8TAPSK C whose component codes are all linear, the minimum squared non coherent distance is $d_{nc}^2 = \min\{d_{nc,a}^2, d_{nc,b}^2, d_{nc,c}^2\}$,where

$d_{nc,a}^2 = \min\{f(d_{a,\min}), f(d_{a,\max})\}$, $d_{nc,b}^2 = r_0^2(N - \sqrt{(N - d_{ncH,b})^2}$) $d_{nc,c}^2 = 2r_0^2 d_{ncH,c}$ For block-coded generalized-16TAPSK, by a similar derivation $d_{nc}^2 = \min\{d_{nc,a}^2, d_{nc,b}^2, d_{nc,c}^2, d_{nc,d}^2\}$ Where,

$$d_{nc,a}^2 = \min\{f(d_{a,\min)}, f(d_{a,\max})\},$$

$$d_{nc,b}^2 = r_0^2(N - \sqrt{(N - \frac{2-\sqrt{2}}{2} d_{ncH,b})^2 + \frac{d_{ncH,b}^2}{2}}),$$

$$d_{nc,c}^2 = r_0^2(N - \sqrt{(N - d_{ncH,c})^2 + d^2_{ncH,c}}) \text{ and}$$

$d_{nc,d}^2 = 2r_0^2 d_{ncH,d}$,for NBC-8TAPSK. Table I compares NBC-8TAPSK with NBC-8PSK in terms of d for N =15, 31, 63, and $N \rightarrow 8$. In this paper, only (15,11,1) code, (31,26, 1) code and (63,57,1) BCH codes are used as component codes. The values of (da,min,db,min,dc,min).are shown in the column of "code", and the values of r which maximize the same rate and N, NBC-8TAPSK always has larger d^2nc than NBC-8PSK. Figure 2 presents the results for N =4. For the pilot optimized 16QAM, the amplitude of the pilot signal is 1.225. NBC-16QAM has better BER than 16 QAM (H) and 16 QAM (L), but they all do not decrease exponentially.

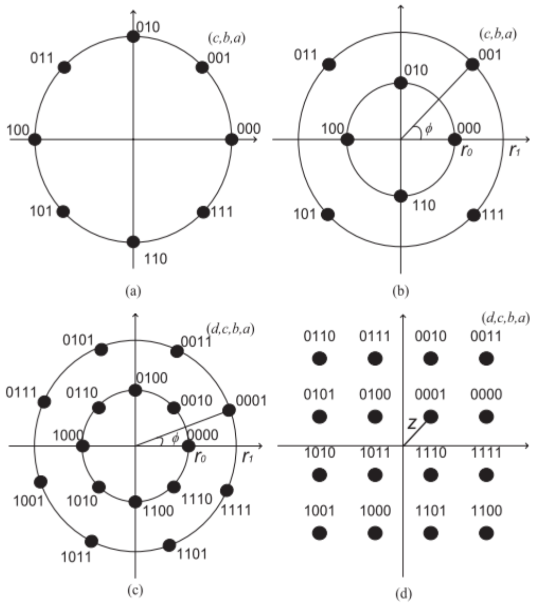
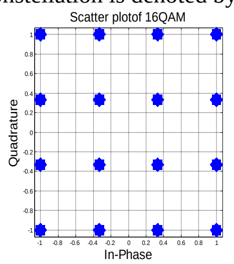


Fig. 1. Constellations with bit labeling for (a) 8PSK (b) 8TAPSK ($\phi = \pi/4$) (c) 16TAPSK ($\phi = \pi/8$) (d) 16QAM

When r =1, i.e. TAPSK becomes MPSK, we have (N)=0and $f(d)=f(N - d)\forall d$. Consequently, d of block-coded MPSK is equal to (d). Therefore, for Block coded MPSK, Ca should be a binary block code with large d. It proposed NBC-MPSK in [5] by setting $d_{a,\max}, d_{ncH,a} = N - d_{a,\min}$ such that $dncH,a= da$,min at the price of sacrificing one data bit. But as r increases, (N) also increases. For block-coded 8TAPSK where r is large enough, (N)= $(r_1 - r_0)^2 N/2$ can be larger than $f(da,\min)$. If r>1.61238, (N) is always larger than (da,min) for any value of da,min (d= N/2).In such case, since $d^2_{nc,a} = f(da,\min)$, Ca,min could be a normal code with large da,min and thus the one-bit loss is unnecessary.

2.2 16QAM

The distance between the smallest-energy point and the origin in the 16QAM constellation is denoted by z.



From this diagram it can calculate minimum non coherent distance $d^2_{nc\min}$. If it defines $d_{\min H} = 4$, then

$$d_{0\min} = (d_{\min H}/\delta_0^2), d_{1,\min} = (d_{\min H}/\delta_1^2),$$

$$d_{2,\min} = (d_{\min H}/\delta_2^2) \text{ and } d_{3,\min} = (d_{\min H}/\delta_3^2)$$

For block coded 16QAM C whose component codes are all linear, the minimum squared non coherent distance

$$d^2_{nc} = \min\{d^2_{nc,a}, d^2_{nc,b}, d^2_{nc,c}, d^2_{nc,d}\}$$

Table I. compares NBC-8TAPSK with NBC-8PSK in terms of d^2_{nc} for N =15, 31, 63, and $N \rightarrow 8$.

Spectral efficiency	code	d^2_{nc}		
		N=15	N=31	N=63
4.34	8PSK	0.234	0.254	0.267
2.24	8TAPSK(H)	0.342	0.350	0.355
2.43	8TAPSK(L)	0.344	0.352	0.357
3.23	16QAM	0.420	0.432	0.437
2.56	16TAPSK	0.530	0.543	0.547

In this paper, only (15,11,1) code, (31,27,1) code and (63,57,1) BCH CODES are used as component codes. The values of (da,min,db,min dc,min)For the same rate and N, NBC-8TAPSK always has larger dnc than NBC-8PSK.

Table II. Comparison Of Theoretical Best Values And Simulation Best Values Of r For Nbc-16tapsk.

Spectral efficienc y	N=15		N=31		N=63	
	Theo .	Simu .	The o	Sim u	The o	sim u
2.23	0.50	0.51	0.45	0.47	0.62	0.67
3.24	0.45	0.47	0.56	0.59	0.57	0.60
3.67	0.44	0.43	0.54	0.52	0.65	0.63
4.34	0.32	0.35	0.45	0.47	0.56	0.50

For NBC-16TAPSK, Table II compares the best values of r for simulations with the theoretical best values of r that maximize dnc. The values of (da,min,db,min,dc,min) are shown in the column of “data rate" In the multistage decoding, a decoding error in level a probably causes error propagation, so slightly larger r which results in better BER in level a would have the best overall BER. Let N_a and N_b denote the numbers of the nearest-neighbor code words for C_a and C_b respectively, shown in Table II also. It finds that if is less than or approximately equal to 1, the best r for simulations is close to (slightly larger than) the best r for dnc.But if N is not small, the BER in level a is increased due to the large number of the nearest-neighbor code words, so the best r for simulations is larger than the best r for d_{nc}. NBC-16TAPSK is better than NBC16QAM at high SNRs which agree with the minimum non coherent distance analysis. For NBC-16QAM, the gap between non coherent decoding and ideal coherent decoding is quite wide.

3. SIMULATION RESULTS & DISCUSSIONS

At high SNRs, the pilot-optimized 16QAM outperforms NBC-16QAM, and NBC-16TAPSK is the best among all non coherent schemes. The results for N =15 are shown in Fig. 3 in which the amplitude of the pilot signal is 1.673. It finds that the average number of code words with small non coherent distances is too tiny to affect the curves above BER of 10^{-6} for all non coherent 16QAM schemes.

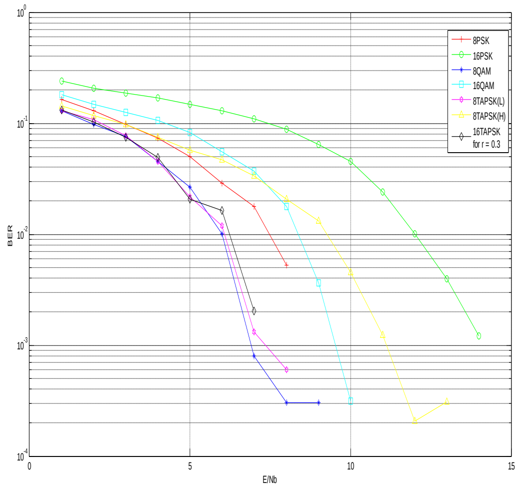


Fig.2 (BER V*s* E/N*b* a t(r=o.3))

At the receivers, the channel-quantization decoding algorithm in [6, Sec. III] is used. This algorithm uses the estimate of θ from the family T={0,2π/MQ, $\cdots$,2$\pi(Q$-1)/MQ},M=4for NBC-8TAPSK and NBC-16QAM, M =8 for NBC-8PSK and NBC-16TAPSK. In all simulations, it set Q =6. Note that the labeling in Fig.1(a) and Fig. 1(b) is Ungerboeck labeling, but if C'$b$and C'c are uncoded bits (db,min= dc,min=1), the labeling of bits b and c should be Gray labeling of QPSK for the minimization of bit error rate (BER).The labeling in Fig. 1(c) For NBCTAPSK and nonlinear NBC-TAPSK, it looks for the value of r that needs the lowest SNR at the BER of 10^{-6} by simulation results, and use it in simulations. In Fig. 2 and Fig. 3, it considers non coherent block codes using sixteen signal points with data rate (4N - 4)/N bits/symbol, including NBC-16TAPSK and NBC-16QAM whose (da,min,db,min, dc,min,dd,min) is (2, 1, 1, 1), and the differentially-encoded16QAM scheme in [9] denoted by 16QAM(H). It modifies the scheme in [9] by choosing the low energy code words instead of the high-energy code words, denoted by 16QAM (L), as suggested by [7]. The results of ideal coherent decoding for NBC-16TAPSK and NBC 16QAM are explained in [7] is also compared. Figure 2 presents the results for N =31. For the pilot optimized 16QAM, the amplitude of the pilot signal is 1.225. NBC-16QAM has better BER than 16QAM (L), but they all do not decrease exponentially because the average number of code words with small non coherent distances is little, but not little enough. For ideal coherent decoding, NBC-16TAPSK is worse than NBC-16QAM. But for non coherent decoding, NBC-16TAPSK is better than NBC16QAM at high SNRs which agree with the minimum non coherent distance analysis. For NBC-16QAM, the gap between non coherent decoding and ideal coherent decoding is quite wide given as references. But here it takes fixed minimum hamming distance, then find $\delta_a^2, \delta_b^2, \delta_c^2$, and then minimum required $d^2{}_{\min}$ .for evaluating system performance; it computes BER versus E/Nb graph for the AWGN channel or Rayleigh channel. For encoding it uses the BCH encoder, then transmitted the signals by this encoding, at the receiver it uses same type of decoder and see the error which place we have to correct.

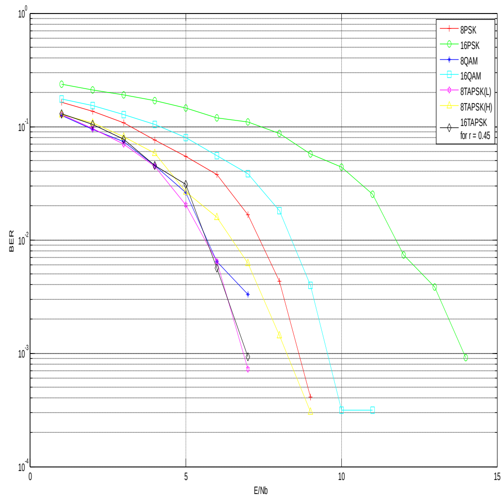


Fig.3 (BER V*s* E/N*b* a t(r=o.45))

At high SNRs, the pilot-optimized 16QAM outperforms NBC-16QAM, and NBC-16TAPSK is the best among all non coherent schemes. The results for N =15 are shown in Fig. 3 in which the amplitude of the pilot signal is 1.673. It finds that the average number of code words with small non coherent distances is too tiny to affect the curves above BER of 10^{-6} for all non coherent 16QAM schemes. NBC16QAM outperforms NBC-16TAPSK and the pilot-optimized 16QAM, and its gap between non coherent decoding and ideal coherent decoding is less than 1dB.

Various non coherent block codes using eight or sixteen signal points with data rate (3N -3)/N bits/symbol for N =16 are compared in Fig. 4. NBC-16TAPSK and NBC-16QAM both use (da,min,db,min,dc,min,dd,min)=(8, 4, 1, 1),and NBC-8TAPSK using $C(H)$ (denoted by NBC-8TAPSK(H) and NBC-8TAPSK (denoted by NBC-8TAPSK(L)) and both use (da,min,db,min,dc,min))=(1, 1, 1). NBC-8TAPSK using (O) has almost the same BER as NBC-8TAPSK and thus is not shown in the figure-2. The used values of r are 1.94, 1.95 and 1.6 for NBC 8TAPSK (H), NBC-8TAPSK (L) and NBC-8TAPSK, respectively. It finds that NBC-8PSK is the worst, and NBC-8TAPSK has better BER than NBC 8TAPSK (L) and NBC-8TAPSK (H). At high SNRs, NBC16TAPSK outperforms NBC-8TAPSK. This is reasonable since its dnc, 0.6277, is larger than $d$nc of NBC-8TAPSK, 0.6030. After all, NBC-16QAM whose dnc is only 0.1649 is the best. It provides about 1.6dB gain over NBC-16TAPSK at a BER of 10^{-6}. Quite different from NBC-MPSK and NBC-TAPSK, the average number of nearest neighbors of NBC-16QAM is very small. It is complicated to compute the average number of nearest neighbors of NBC-16QAM, so it takes an example to illustrate this point as follows. Suppose that the transmitted has component codeword in level a ca= 0. Consider another component codeword ca. Help of scatter plot shown in above figure-4.then it computes BER for 16 QAM , For N=31, the minimum non coherent distance of energy constraint 16-MAPSK is larger than that of energy constraint 16-QAM. Therefore, it is reasonable that the performance of energy constraint 16-MAPSK is better than energy constraint 16-QAM.

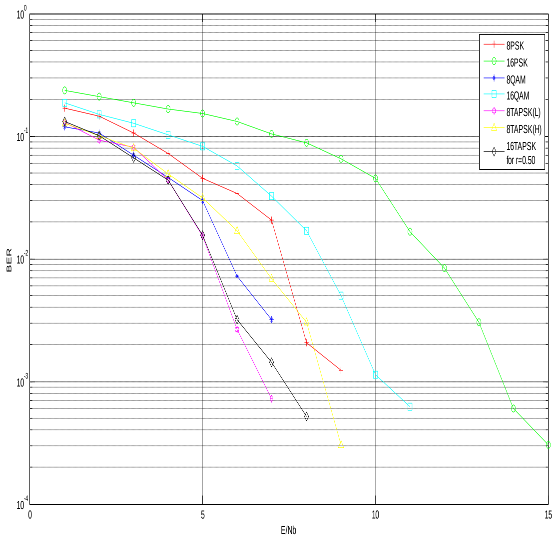


Fig 4. (BER V*s* E/N*b* a t(r=o.5))

Quite different from NBC-MPSK and NBC-TAPSK, the average number of nearest neighbors of NBC-16QAM is very small. It is complicated to compute the average number of nearest neighbors of NBC-16QAM, so it takes an example to illustrate this point as follows. Suppose that the transmitted codeword, denoted by x, has a component codeword in level a ca= 0. Consider another component codeword cin level a and the Hamming distance between ca and c'a is dmin, Assume that dnc= dnc, . For this case, it computes the number of nearest neighbors caused by C'a for NBC-16TAPSK and NBC-16QAM as follow.

4. CONCLUSION

In this paper, the minimum non-coherent distances of block coded TAPSK and 16QAM using linear component codes are derived. The minimum non-coherent distance of block-coded QAM with more signal points can be derived similarly. It finds that the minimum non coherent distance of block-coded MPSK derived in [5] is a special case of the derived minimum non coherent distance of block-coded TAPSK. According to the derived distances, it proposes NBC-TAPSK and NBC16QAM. The comparison of minimum non coherent distances shows the superiority of NBC-TAPSK over NBC-MPSK at high data rates. It compares various non-coherent block codes based on the simulation results. If the block is very short, NBC-16QAM has worse error performance due to its small minimum non coherent distance, and NBC-16TAPSK has the best error performance. But if the block length is not small, NBC-16QAM has the best error performance because the code words with small non coherent distances become rare.

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