**Strong Split Geodetic Number of a Graph**

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**ABSTRACT**

A set $S \subseteq V(G)$ is a strong split geodetic set of $G$, if $S$ is a geodetic set and $(V - S)$ is totally disconnected. The strong split geodetic number of a graph $G$, is denoted by $ss(G)$, is the minimum cardinality of a strong split geodetic set of $G$. In this paper we investigate many bounds on strong split geodetic number in terms of cardinality of a strong split geodetic set and covering number of $G$, further the relationship between strong split geodetic number and split geodetic number.

**Keywords:**  
Cartesian product, Distance, Edge covering number, Split geodetic number, Vertex covering number.

**1. INTRODUCTION**

In this paper we follow the notations of [1]. As usual $n = |V|$ and $m = |E|$ denote the number of vertices and edges of a graph $G$ respectively. The graphs considered here have at least one component which is not complete or at least two non trivial components. The distance $d(u, v)$ between two vertices $u$ and $v$ in a connected graph $G$ is the length of a shortest $u - v$ path in $G$. It is well known that this distance is a metric on the vertex set $V(G)$. For a vertex $v$ of $G$, the eccentricity $e(v)$ is the distance between $v$ and a vertex farthest from $v$. The minimum eccentricity among the vertices of $G$ is radius, rad $G$, and the maximum eccentricity is the diameter, diam $G$. A $u - v$ path of length $d(u, v)$ is called a $u - v$ geodesic. We define $I[u, v]$ to the set (interval) of all vertices lying on some $u - v$ geodesic of $G$ and for a nonempty subset $S$ of $V(G)$, $I[S] = \bigcup_{u \in S} I[u, v]$.

A set $S$ of vertices of $G$ is called a geodetic set in $G$ if $I[S] = V(G)$, and a geodetic set of minimum cardinality is a minimum geodetic set. The cardinality of a minimum geodetic set in $G$ is called the geodetic number of $G$, and we denote it by $g(G)$.

Split geodetic number of a graph was studied by in [4]. A geodetic set $S$ of a graph $G = (V, E)$ is a split geodetic set if the induced subgraph $(V - S)$ is disconnected. The split geodetic number $ss(G)$ of $G$ is the minimum cardinality of a split geodetic set. Now we define strong split geodetic number of a graph. A set $S'$ of vertices of $G = (V, E)$ is called the strong split geodetic set if the induced subgraph $(V - S')$ is totally disconnected and a strong split geodetic set of minimum cardinality is the strong split geodetic number of $G$ and is denoted by $ss(G)$.

A vertex $v$ is an extreme vertex in a graph $G$, if the subgraph induced by its neighbors is complete. A vertex cover in a graph $G$ is a set of vertices that covers all edges of $G$. The minimum number of vertices in a vertex cover of $G$ is the vertex covering number $\alpha(G)$ of $G$. An edge cover of a graph $G$ without isolated vertices is a set of edges of $G$ that covers all the vertices of $G$. The edge covering number $\beta(G)$ of a graph $G$ is the minimum cardinality of an edge cover of $G$.

For any undefined term in this paper, see [1] and [2].

**2. PRELIMINARY NOTES**

We need the following results to prove further results.

**Theorem 2.1.** [3] Every geodetic set of a graph contains its extreme vertices.

**Theorem 2.2.** [3] For any path $P_n$, with $n$ vertices, $g(P_n) = 2$.

**Theorem 2.3.** [3] For integers $r, s \geq 2$, $g(K_{r,s}) = \min\{r, s, 4\}$.

**Theorem 2.4.** [3] Let $G$ be a connected graph of order at least 3. If $G$ contains a minimum geodetic set $S$ with a vertex $x$ such that every vertex of $G$ lies on some $x - w$ geodesic in $G$ for some $w \in S$, then $g(G) = g(G \times K_2)$.

**Theorem 2.5.** [2] For any graph $G$, $\alpha_0 + \beta_0 = \alpha_1 + \beta_1$.

**Proposition 2.6.** For any graph $G$, $g_s(G) \leq g_s(G)$.

**Proposition 2.7.** For any tree $T$ of order $n$ and number of cut vertices $c$, then the number of end edges is $n - c$. 

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3. MAIN RESULTS

**Theorem 3.1.** Let $T$ be a tree that has at least three internal vertices. If $T$ has $k$ end-vertices, then $g_{ss}(T) = k + \left\lceil \frac{n-k+1}{2} \right\rceil$.

Proof. Let $F = \{v_1, v_2, ..., v_k\}$ be the set of all end vertices in $T$, $|F| = k$. Consider $S = F \cup H$, where $H \subseteq V(T) - F$, such that $H$ contains a vertex of maximum degree and a minimum set of alternating vertices in $V - F$, $|H| = \left\lceil \frac{n-k+1}{2} \right\rceil$. Now $S$ be the minimal set of vertices which covers all the vertices in $T$. Clearly it follows that, $|S| = |F \cup H| = k + \left\lceil \frac{n-k+1}{2} \right\rceil$. Therefore $g_{ss}(T) = k + \left\lceil \frac{n-k+1}{2} \right\rceil$.

**Corollary 3.2.** For any path $P_n$, $n \geq 5$, $g_{ss}(P_n) = 2 + \left\lceil \frac{n-2}{2} \right\rceil$.

Proof. Proof follows from the above theorem.

**Theorem 3.3.** For cycle $C_n$ of order $n > 3$

$g_{ss}(C_n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$

Proof. Let $n > 3$, we have the following cases.

Case 1: Let $n$ be even.

Consider $\{v_1, v_2, ..., v_n\}$ be a cycle with $n$ vertices where $n$ is even, let $S = \{v_1, v_2, ..., v_n\}$ be the set of alternating vertices which covers all the vertices of $C_n$ and for any $v_i \in V - S$, $deg_{C_n} v_i = 0$. Clearly $S$ forms minimal strong split geodetic set of $C_n$, it follows that $|S| = \frac{n}{2}$. Therefore $g_{ss}(C_n) = \frac{n}{2}$.

Case 2: Let $n$ be odd.

Consider $\{v_1, v_2, ..., v_n\}$ be a cycle with $n$ vertices where $n$ is odd, let $S = \{v_1, v_2, ..., v_{n-1}, v_{n-2}\}$ which covers all the vertices of $C_n$ and for any $v_i \in V - S$, $deg_{C_n} v_i = 0$. Clearly $S$ forms minimal strong split geodetic set of $C_n$, it follows that $|S| = \frac{n+1}{2}$. Therefore $g_{ss}(C_n) = \frac{n+1}{2}$.

**Corollary 3.4.** For any cycle $C_n$ of order $n > 3$, $g_{ss}(C_n) = \alpha_0(C_n)$.

Proof. We have the following cases.

Case 1: Let $n$ be even.

Let $n > 3$ be the number of vertices which is even and $\alpha_0$ is the vertex covering number of $C_n$. We have by Case 1 of Theorem 3.3, $g_{ss}(C_n) = \frac{n}{2}$. Also for even cycle, vertex covering number is $\alpha_0(C_n) = \frac{n}{2}$. Hence $g_{ss}(C_n) = \alpha_0(C_n)$.

Case 2: Let $n$ be odd.

Let $n > 3$ be the number of vertices which is odd and $\alpha_0$ is the vertex covering number of $C_n$. We have by Case 2 of Theorem 3.3, $g_{ss}(C_n) = \frac{n+1}{2}$. Also for odd cycle, vertex covering number is $\alpha_0(C_n) = \frac{n+1}{2}$. Hence $g_{ss}(C_n) = \alpha_0(C_n)$.

**Theorem 3.5.** For the wheel $W_n = K_1 + C_{n-1}$ ($n \geq 6$),

$g_{ss}(W_n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$

Proof. Let $W_n = K_1 + C_{n-1}$ ($n \geq 6$) and let $V(W_n) = \{x, u_1, u_2, ..., u_{n-1}\}$, where $deg(x) = n - 1 > 3$ and $deg(u_i) = 3$ for each $i \in \{1, 2, ..., n-1\}$. We have the following cases

Case 1. Let $n$ be even. Consider geodesic $P : \{u_1, u_2, u_4\}$, $Q : \{u_3, u_4, u_5\}$.

It is clear that the vertices $u_2, u_4, ..., u_{n-2}$ lies on the geodesics $P, Q$. Also $S = \{u_1, u_3, u_5, ..., u_{n-2}, u_{n-1}, x\}$ is a minimal strong split geodetic set such that $V - S$ is totally disconnected and it has $\frac{n}{2} + 1$ vertices. Hence $g_{ss}(W_n) = \frac{n+2}{2}$.

Case 2. Let $n$ be odd. Consider geodesic $P : \{u_1, u_2, u_3\}$, $Q : \{u_4, u_5, u_6\}$.

It is clear that the vertices $u_2, u_4, ..., u_{n-2}$ lies on the geodesic $P, Q$. Also $S = \{u_1, u_3, u_5, ..., u_{n-2}, u_{n-1}, x\}$ is a minimal strong split geodetic set such that $V - S$ is totally disconnected and it has $\frac{n-1}{2} + 1$ vertices. Hence $g_{ss}(W_n) = \frac{n+1}{2}$.

**Corollary 3.6.** For the wheel $W_n = K_1 + C_{n-1}$ ($n \geq 6$),

$g_{ss}(W_n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$

Proof. Let $W_n = K_1 + C_{n-1}$ ($n \geq 6$) and let $V(W_n) = \{x, u_1, u_2, ..., u_{n-1}\}$, where $deg(x) = n - 1 > 3$ and $deg(u_i) = 3$ for each $i \in \{1, 2, ..., n-1\}$. Maximum degree($\Delta$) of $W_n$ is $n - 1$ and minimum degree($\delta$) of $W_n$ is 3.

We have the following cases

Case 1: Let $n$ be even. We have from Case 1 of Theorem 3.5

$g_{ss}(W_n) = \frac{n+1}{2}$. Hence $g_{ss}(W_n) = \alpha_0(C_n)$.

Case 2: Let $n$ be odd. We have from Case 2 of Theorem 3.5

$g_{ss}(W_n) = \frac{n}{2}$. Hence $g_{ss}(W_n) = \alpha_0(C_n)$.

**Theorem 3.7.** Let $G$ be a connected graph of order $n$ and diameter $d$. Then $g_{ss}(G) \leq n - d + 2$, except for tree.

Proof. Let $u$ and $v$ be vertices of $G$ for which $d(u, v) = d$ and let $S = \{v_1, v_2, ..., v_d\}$. Then $I[S] = V(G), V - (S \cup \{v_1, v_2, ..., v_d\})$ is totally disconnected and thus $g_{ss}(G) \leq |S| + 1 = n - d + 2$.

**Theorem 3.8.** For any tree $T$ with at least three internal vertices and order $n$, diameter $d$. Then $g_{ss}(G) \leq n - d + k$, where $k$ be the number of end vertices.

Proof. Let $u$ and $v$ be vertices of $G$ for which $d(u, v) = d$ and let $S = \{v_1, v_2, ..., v_d\}$. Then $I[S] = V(G), V - (S \cup \{v_1, v_2, ..., v_d\})$ is totally disconnected and thus $g_{ss}(G) \leq |S| + 1 = n - d + k$. 

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THEOREM 3.9. For any integers $r, s \geq 2$ $g_{ss}(K_{r,s}) = \min\{r,s\}$.

Proof. Let $G = K_{r,s}$, such that $U = \{u_1, u_2, ..., u_r\}$, $W = \{w_1, w_2, ..., w_s\}$ are the partite sets of $G$, where $r \leq s$ and also $V = U \cup W$.

Consider $S = U$, for every $w_k$, $1 \leq k \leq s$ lies on the $u_i - u_j$ geodesic for $1 \leq i \neq j \leq r$. Since $V - S$ is totally disconnected, we have $S$ is a strong split geodetic set of $G$.

Let $X = \{u_1, u_2, ..., u_{r-1}\}$ be any set of vertices such that $|X| < |S|$, then $X$ is not a geodetic set of $G$, since $u_r \notin I[X]$. It is clear that $S$ is a minimum strong split geodetic set of $G$. Hence $g_{ss}(K_{r,s}) = |S| = r$.

THEOREM 3.10. For any connected graph $G$ of order $n$, $g_s(G) + g_{ss}(G) < 2n$.

Proof. Suppose $S = \{v_1, v_2, ..., v_n\} \subseteq V(G)$ be the set of vertices which covers all the vertices in $G$ and $V - S$ is disconnected. Then $S$ is a minimal split geodetic set of $G$. Further if the subgraph $G - S$ contains the set of vertices $v_i$, $1 \leq i \leq n$, such that $deg(v_i) = 0$. Then $S$ is an strong split geodetic set of $G$.

Otherwise, $S' = S_1 \cup I$, where $S_1 \subseteq S$ and $I \subseteq V(G) - S$ is the minimum set of alternate vertices, $S'$ forms a minimum strong split geodetic set of $G$. Since $V - S'$ contains isolated vertices, it follows that $|S| + |S'| < 2n$. Therefore, $g_s(G) + g_{ss}(G) < 2n$.

The following corollaries are immediate consequence of above Theorem and Theorem 2.5.

COROLLARY 3.11. For any connected graph $G$ of order $n$, $g_s(G) + g_{ss}(G) < 2(o_0(G) + \beta_0(G))$.

COROLLARY 3.12. For any connected graph $G$ of order $n$, $g_s(G) + g_{ss}(G) < 2(o_1(G) + \beta_1(G))$.

4. ADDING AN END EDGE

For an edge $e = (u, v)$ of a graph $G$ with $deg(u) = 1$ and $deg(v) > 1$, we call $e$ an end-edge and $u$ an end-vertex.

THEOREM 4.1. $G'$ be the graph obtained by adding an end edge $(u, v)$ to a cycle $C_n = G$ of order $n > 3$, with $u \in G$ and $v \notin G$. Then

$$g_{ss}(G') = \begin{cases} \frac{n+2}{2} & \text{for even cycle} \\ \frac{n+1}{2} & \text{for odd cycle}. \end{cases}$$

Proof. Let $\{u_1, u_2, ..., u_n, u_1\}$ be a cycle with $n$ vertices. Let $G'$ be the graph obtained from $G = C_n$ by adding an end-edge $(u, v)$ such that $u \in G$ and $v \notin G$.

We have the following cases.

Case 1: Let $G$ be an even cycle.

Let $S = \{v, u_i\} \subseteq V(G')$, where $v \notin G$ is an end vertex of $G'$ and $u_i$ is an antipodal vertex of $v$. Consider $S' = S \cup H$, where $H \subseteq V(G') - S$ is a minimum set of non-adjacent vertices, $|H| = \frac{n}{2} - 1$. Now $S'$ be the minimal set of vertices which covers all the vertices of $G'$. Clearly for any $u_i \in G' - S', deg(u_i) = 0$.

by the above argument it follows that $S'$ is a minimal strong split geodetic set of $G'$. Clearly $|S'| = |S \cup H| = 2 + \frac{n-2}{2} = \frac{n+2}{2}$.

Therefore $g_{ss}(G') = \frac{n+2}{2}$.

Case 2: Let $G$ be an odd cycle.

(a) When $n = 5$

Let $S = \{v, a, b\}$ be a geodetic set, where $v \notin G$, is an end-vertex of $G'$ and $a, b \in G$, such that $2d(u, a) = d(u, b)$ and $d(a, b) = 2$.

Thus $I[S] = V(G')$ and $V - S$ is an induced subgraph which has two components. Let $S' = S \cup H$ where $H \subseteq V - S$ such that $H$ contains minimum alternate vertices from both the components having $\frac{n-3}{2}$ vertices. Clearly $S'$ forms the minimal strong split geodetic set of $G'$, since $V - S'$ forms an independent set. Clearly $|S'| = |S \cup H| = 3 + \frac{n-3}{2} = \frac{n+3}{2}$. Therefore $g_{ss}(G') = \frac{n+3}{2}$.

(b) When $n > 5$

Let $S = \{v, a, b\}$ be a geodetic set where $v \notin G$ is an end-vertex of $G'$ and $a, b \in G$, such that $d(u, a) = d(u, b)$ and $d(a, b) is the diameter of $G$. Thus $I[S] = V(G')$ and $V - S$ is an induced subgraph which has two components. Let $S' = S \cup H$ where $H \subseteq V - S$ such that $H$ contains minimum alternate vertices from both the components having $\frac{n-3}{2}$ vertices. Clearly $S'$ forms the minimal strong split geodetic set of $G'$, since $V - S'$ forms an independent set. Clearly $|S'| = |S \cup H| = 3 + \frac{n-3}{2} = \frac{n+3}{2}$. Therefore $g_{ss}(G') = \frac{n+3}{2}$.

THEOREM 4.2. Let $G'$ be the graph obtained by adding end edge $(u_i, v_i)$, $i = 1, 2, ..., n$, to each vertex of $G = C_n$ of order $n > 3$ such that $u_i \in G$, $v_i \notin G$. Then

$$g_{ss}(G') = \begin{cases} k + \frac{n}{2} & \text{for even cycle} \\ k + \frac{n+1}{2} & \text{for odd cycle}. \end{cases}$$

Proof. Let $G = C_n = \{u_1, u_2, ..., u_n, u_1\}$ be a cycle with $n$ vertices. Let $G'$ be the graph obtained by adding an end-edge $(u_i, v_i)$, $i = 1, 2, ..., n = k$ to each vertex of $G$ such that $u_i \in G$, $v_i \notin G$.

Case 1: Let $G$ be an even cycle.

Let $F = \{u_i, v_1, ..., v_k\}$ be the $k$ number of end-vertices of $G'$ and $H \subseteq V(G') - F$ is an even cycle. Let $S = F \cup H_1$, where $H_1 \subseteq H$ such that $H_1 \notin E(H)$. Now $S$ be the minimal set of vertices which covers all the vertices in $G'$. Clearly for any $u_i \in G'$, $deg(u_i) = 0$.

Then by the above argument $S$ is the minimal strong split geodetic set of $G'$, it follows that $|S'| = |F \cup H_1| = k + \frac{n}{2}$. Therefore $g_{ss}(G') = k + \frac{n}{2}$.

Case 2: Let $G$ be odd cycle.

Let $F = \{u_1, v_2, ..., v_k\}$ be the $k$ number of end-vertices of $G'$ and $H \subseteq V(G') - F$ is an odd cycle. Let $S = F \cup \{u_1, u_n\} \cup H_1$, where $H_1 \subseteq H$ such that $H_1 \notin E(H)$. Now $S$ be the minimal set of vertices which covers all the vertices in $G'$. Clearly for any $u_i \in G'$, $deg(u_i) = 0$.

Then by the above argument $S$ is the minimal strong split geodetic set of $G'$, it follows that $|S'| = |F \cup \{u_1, u_n\} \cup H_1| = k + 2 + \frac{n-3}{2}$. Therefore $g_{ss}(G') = k + \frac{n+1}{2}$. 

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5. CARTESIAN PRODUCT

The cartesian product of the graphs $H_1$ and $H_2$, written as $H_1 \times H_2$, is the graph with vertex set $V(H_1) \times V(H_2)$, two vertices $u_1, u_2$ and $v_1, v_2$ being adjacent in $H_1 \times H_2$ if and only if either $u_1 = v_1$ and $(u_2, v_2) \in E(H_2)$, or $u_2 = v_2$ and $(u_1, v_1) \in E(H_1)$.

THEOREM 5.1. For any path $P_n$ of order $n$, $g_{ss}(K_2 \times P_n) = n$.

Proof. Consider $G = P_n$. Let $K_2 \times P_n$ be graph formed from two copies $G_1$ and $G_2$ of $G$. Let $V = \{u_1, u_2, \ldots, u_n\}$ be the vertices of $G_1$, $W = \{v_1, v_2, \ldots, v_n\}$ be the vertices of $G_2$ and $U = V \cup W$.

Case 1. Let $n$ be even.

Consider $S = H_1 \cup H_2$, where $H_1 = \{v_1, v_2, \ldots, v_n\} \subseteq V$ having $\frac{n}{2}$ vertices, $H_2 = \{u_1, u_2, \ldots, u_n\} \subseteq W$ having $\frac{n}{2}$ vertices. Now $S$ be the minimal set of vertices which covers all the vertices in $K_2 \times P_n$. Such that set of vertices of a subgraph $U - S$ is isolated, then by the above argument $S$ is a minimal strong split geodetic set of $K_2 \times P_n$. Clearly it follows that, $|S| = |H_1 \cup H_2| = \frac{n}{2} + \frac{n}{2} = n$. Therefore $g_{ss}(K_2 \times P_n) = n$.

Case 2. Let $n$ be odd.

Consider $S = H_1 \cup H_2$, where $H_1 = \{v_2, v_4, \ldots, v_{n-1}\} \subseteq V$ having $\frac{n+1}{2}$ vertices, $H_2 = \{u_1, u_3, \ldots, u_n\} \subseteq W$ having $\frac{n+1}{2}$ vertices. Now $S$ be the minimal set of vertices which covers all the vertices in $K_2 \times P_n$. Such that set of vertices of a subgraph $U - S$ is isolated, then by the above argument $S$ is a minimal strong split geodetic set of $K_2 \times P_n$. Clearly it follows that, $|S| = |H_1 \cup H_2| = \frac{n+1}{2} + \frac{n+1}{2} = n$. Therefore $g_{ss}(K_2 \times P_n) = n$.

The following Corollaries are immediate consequence of above
Theorem and Theorem 2.5.

COROLLARY 5.2. For any path $P_n$ of order $n$, $g_{ss}(K_2 \times P_n) = \alpha_0 + \beta_0$.

COROLLARY 5.3. For any path $P_n$ of order $n$, $g_{ss}(K_2 \times P_n) = \alpha_1 + \beta_1$.

THEOREM 5.4. For any complete graph of order $n$, $g_{ss}(K_2 \times K_n) = 2n - 2$.

Proof. Let $G_1$ and $G_2$ be disjoint copies of $G = K_n$, $n \geq 2$. Let $V = \{v_1, v_2, \ldots, v_n\}$ and $W = \{w_1, w_2, \ldots, w_n\}$ be the vertex set of $G_1$ and $G_2$ respectively and let $v_i, w_i \in E(K_2 \times K_n)$ for $i \in \{1, 2, \ldots, n\}$. Let $S$ be the minimum geodetic set of $K_2 \times K_n$ by Theorem 2.4 $g(K_2 \times K_n) = g(K_n) = n$. Consider $S = H \cup H'$, where $H \subseteq U - S$ having $n - 2$ vertices, since $U - S$ has two components which are complete graphs. Now $S'$ be the minimal set of vertices which covers all the vertices in $K_2 \times K_n$, such that set of vertices of subgraph $U - S'$ is isolated, then by the above argument $S'$ is a minimal strong split geodetic set of $K_2 \times K_n$. Clearly it follows that $|S'| = |S \cup H'| = n + n - 2 = 2n - 2$.

OBSERVATION 5.5. For any complete graph of order $n$, $g(K_2 \times K_n) = g(K_n)$.

THEOREM 5.6. For any complete graph of order $n$, $g_{ss}(K_2 \times K_n) = 3n - 3$.

Proof. Let $G_1$ and $G_2$ be disjoint copies of $G = K_n$, $n \geq 2$. Let $X = \{x_1, x_2, \ldots, x_n\}$, $Y = \{y_1, y_2, \ldots, y_n\}$ and $Z = \{z_1, z_2, \ldots, z_n\}$ be the vertex set of $G_1$, $G_2$ and $G_3$ respectively. Let $S$ be the minimum geodetic set of $K_3 \times K_n$ by Observation 5.5 $g(K_3 \times K_n) = g(K_n) = n$. Consider $S' = S \cup H$, where $H \subseteq V - S$ having $2n - 3$ vertices. Now $S'$ be the minimal set of vertices which covers all the vertices in $K_2 \times K_n$, such that set of vertices of subgraph $V - S'$ are isolated, then by the above argument $S'$ is a minimal strong split geodetic set of $K_3 \times K_n$. Clearly it follows that $|S'| = |S \cup H| = n + 2n - 3 = 3n - 3$.

THEOREM 5.7. $G'$ be the graph obtained by adding an end edge $(u, v)$ to a cycle $C_n = G$ of order $n > 3$, with $u \in G$ and $v \notin G$. Then $g_{ss}(K_2 \times G') = n + 2$.

Proof. Let $\{u_1, u_2, \ldots, u_n, u_1\}$ be a cycle with $n$ vertices. Let $G'$ be the graph obtained from $G = C_n$ by adding an end-edge $(u, v)$ such that $u \in G$ and $v \notin G$.

We have the following cases.

Case 1: Let $G$ be an even cycle.

Let $S$ be the minimum geodetic set of $K_2 \times G'$, by Theorem 2.4 $g(K_2 \times G') = g(G) = 2$. Consider $S = S \cup H$, where $H \subseteq V - S$ having $n$ vertices. Now $S'$ be the minimal set of vertices which covers all the vertices in $K_2 \times G'$, such that set of vertices of subgraph $V - S'$ are totally disconnected. Then by the above argument $S'$ is a minimal strong split geodetic set of $K_2 \times G'$. Clearly it follows that $|S'| = |S \cup H'| = 2 + n$.

Case 2: Let $G$ be an odd cycle.

Let $S$ be the minimum geodetic set of $K_2 \times G'$, by Theorem 2.4 $g(K_2 \times G') = g(G') = 3$. Consider $S = S \cup H$, where $H \subseteq V - S$ having $n - 1$ vertices. Now $S'$ be the minimal set of vertices which covers all the vertices in $K_2 \times G'$, such that set of vertices of subgraph $V - S'$ are totally disconnected. Then by the above argument $S'$ is a minimal strong split geodetic set of $K_2 \times G'$. Clearly it follows that $|S'| = |S \cup H'| = 3 + n - 1 = n + 2$.

6. CONCLUSION

In this paper we establish many bounds on strong split geodetic number in terms of elements of $G$ and covering number of $G$, further the relationship between strong split geodetic number and split geodetic number.

7. REFERENCES