\(M_X G\zeta^*\)-Interior and \(M_X G\zeta^*\)-Closure in \(M\)-Structures

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ABSTRACT
In this paper, we introduce \(M_X G\zeta^*\)-Interior, \(M_X G\zeta^*\)-Closure and some of its basic properties.

Keywords
\(M_X G\zeta^*\)-open, \(M_X G\zeta^*\)-closed, \(M_X \text{Int}_{G\zeta^*}(A), M_X \text{Cl}_{G\zeta^*}(A)\).

1. INTRODUCTION
N.Levine [14] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Balachandran, Sundaram and Maki [5], Bhattacharyya and Lahiri[6], Arockiarani [3], Dunham [9], Gnanambal [10], Malghan [11], Palaniappan and Rao [21], Park [22], Arya and Gupta [4] and Devi [8] have worked on generalized closed sets, their definitions and characterizations of separation axioms by using the concept of minimal structures. V.Popa and T.Noiri [24] obtained the important results compatible by the general topology case.

In 2000, V.Popa and T.Noiri [23] introduced the notion of minimal structure. Also they introduce the notion of \(m_X\)-open sets and \(m_X\)-closed sets and characterized those sets using \(m_X\)-closure and \(m_X\)-interior respectively. Further they introduced \(m\)-continuous functions and studied some of its basic properties. V.Popa and T.Noiri [24] obtained the definitions and characterizations of separation axioms by using the concept of minimal structures. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [1], [2], [7], [15], [16], [19] and [20].

Already Kokilavani V [12] et al. introduce \(M_X G\zeta^*\)-closed set. In this paper, the notion of \(M_X G\zeta^*\)-interior is defined and some of its basic properties are studied. Also we introduce the concept of \(M_X G\zeta^*\)-closure in topological spaces using the notions of \(M_X G\zeta^*\)-closed sets, and we obtain some related results.

2. PRELIMINARIES
In this paper, we introduce the notion of \(M_X G\zeta^*\)-interior is defined and some of its basic properties are studied. Also we introduce the concept of \(M_X G\zeta^*\)-closure in topological spaces using the notions of \(M_X G\zeta^*\)-closed sets, and we obtain some related results.

**DEFINITION 2.1.** [17] Let \(X\) be a nonempty set and let \(m_X \subseteq P(X)\) where \(P(X)\) denote the power set of \(X\) where \(m_X\) is an \(M\)-structure (or a minimal structure) on \(X\), if \(\varphi\) and \(x\) belong to \(m_X\).

The members of the minimal structure \(m_X\) are called \(m_X\)-open sets, and the pair \((X, m_X)\) is called an \(m\)-space. The complement of \(m_X\)-open set is said to be \(m_X\)-closed.

**DEFINITION 2.2.** [17] Let \(X\) be a nonempty set and \(m_X\) an \(M\)-structure on \(X\). For a subset \(A\) of \(X\), \(m_X\)-closure of \(A\) and \(m_X\)-interior of \(A\) are defined as follows:
\[
m_X\text{-cl}(A) = \bigcap \{F : A \subseteq F, X - F \in m_X\} \\
m_X\text{-int}(A) = \bigcup \{U : U \subseteq A, U \in m_X\}
\]

**LEMMA 2.3.** [17] Let \(X\) be a nonempty set and \(m_X\) an \(M\)-structure on \(X\). For subsets \(A\) and \(B\) of \(X\), the following properties hold:
(a) \(m_X\text{-cl}(X - A) = X - m_X\text{-int}(A)\) and \(m_X\text{-int}(X - A) = X - m_X\text{-cl}(A)\).
(b) \(X \in m_X\text{-cl}(A)\) if \(A \in m_X\text{-cl}(X - A)\).
(c) \(m_X\text{-cl}(\varphi) = \varphi, m_X\text{-cl}(X) = X, m_X\text{-int}(\varphi) = \varphi, m_X\text{-int}(X) = X\).
(d) If \(A \subseteq B\) then \(m_X\text{-cl}(A) \subseteq m_X\text{-cl}(B)\) and \(m_X\text{-int}(A) \subseteq m_X\text{-int}(B)\).
(e) \(A \subseteq m_X\text{-cl}(A)\) and \(m_X\text{-int}(A) \subseteq A\).
(f) \(m_X\text{-cl}(m_X\text{-cl}(A)) = m_X\text{-cl}(A)\) and \(m_X\text{-int}(m_X\text{-int}(A)) = m_X\text{-int}(A)\).
(g) \(m_X\text{-int}(A \cap B) = (m_X\text{-int}(A)) \cap (m_X\text{-int}(B))\) and \(m_X\text{-cl}(A \cup B) = (m_X\text{-cl}(A)) \cup (m_X\text{-cl}(B))\).
(h) \(m_X\text{-cl}(A \cup B) \subseteq (m_X\text{-cl}(A)) \cup (m_X\text{-cl}(B))\).

**LEMMA 2.4.** [16] Let \((X, m_X)\) be an \(m\)-space and \(A\) be a subset of \(X\). Then \(x \in m_X\text{-cl}(A)\) if and only if \(U \cap A \neq \varphi\) for every \(U \in m_X\) containing \(x\).

**DEFINITION 2.5.** [25] Let \(X\) be a nonempty set and \(m_X\) an \(M\)-structure on \(X\). For a subset \(A\) of \(X\), \(m_X\)-closure of \(A\) and \(m_X\)-interior of \(A\) are defined as follows:
\[
amcl(A) = \bigcap \{F : A \subseteq F, X - F \in m_X\text{-closed}\} \\
amint(A) = \bigcup \{U : U \subseteq A, U \in m_X\text{-open}\}
\]

**DEFINITION 2.6.** A subset \(A\) of an \(m\)-space \((X, m_X)\) is called as
(i) \(m_X\)-\(\alpha\)-closed set if \(amcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(m_X\)-open in \(X\).
(ii) \(m_X\)-\(\beta\)-open set if \(amcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(m_X\)-\(\alpha\)-open in \(X\).
(iii) \(M_X G\zeta^*\)-closed set if \(amcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(m_X\)-\(\beta\)-open in \(X\).

The complement of \(M_X G\zeta^*\)-closed set is called as \(M_X G\zeta^*\)-open set.

**DEFINITION 2.7.** [13] A subset \(S\) is said to be an \(M_X G\zeta^*\)-neighbourhood of a point \(x\) of \(X\) if there exists a \(M_X G\zeta^*\)-open set \(U\) such that \(x \in U \subseteq S\).

**DEFINITION 2.8.** [23] A minimal structure \(m_X\) on a nonempty set \(X\) is said to have the property \(\mathcal{B}\) if the union of any family of subsets belonging to \(m_X\) belongs to \(m_X\).
REMARK 2.9. A minimal structure \( m_X \) with the property \( B \) coincides with a generalized topology on the sense of Lugojan.

LEMMA 2.10. [2] Let \( X \) be a nonempty set and \( m_X \) an \( \mathcal{M} \)-structure on \( X \) satisfying the property ‘\( B \)’. For a subset \( A \) of \( X \), the following property hold:

(a) \( A \in m_X \) iff \( m_X - \text{int}(A) = A \).
(b) \( A \in m_X \) iff \( m_X - \text{cl}(A) = A \).
(c) \( m_X - \text{int}(A) \in m_X \) and \( m_X - \text{cl}(A) \in m_X \).

3. \( M_X G^\ast \)-INTERIOR AND \( M_X G^\ast \)-CLOSURE

We introduce the following definition:

DEFINITION 3.1. Let \( X \) be a nonempty set and \( m_X \) an \( \mathcal{M} \)-structure on \( X \). For a subset \( A \) of \( X \), the following properties hold:

(a) \( A \in m_X \) iff \( m_X - \text{int}(A) = A \).
(b) \( A \in m_X \) iff \( m_X - \text{cl}(A) = A \).
(c) \( m_X - \text{int}(A) \in m_X \) and \( m_X - \text{cl}(A) \in m_X \).

3. \( M_X G^\ast \)-INTERIOR AND \( M_X G^\ast \)-CLOSURE

We introduce the following definition:

DEFINITION 3.2. Let \( X \) be a subset of \( X \). A point \( x \in X \) is said to be \( M_X G^\ast \)-interior point of \( A \) if \( A \) is a \( M_X G^\ast \)-nbhd of \( x \).

LEMMA 3.3. If \( A \) is a subset of \( X \), then \( m_X - \text{int}(A) = U \in \mathcal{M} \) iff \( m_X - \text{cl}(A) = U \).

PROOF. Let \( A \) be a subset of \( X \). Then \( m_X - \text{int}(A) = U \) iff \( m_X - \text{cl}(A) = U \).

EXAMPLE 3.6. Let \( X = \{a, b, c\} \) with \( m_X \)-open set \( A = \{a, b, c\} \). Then \( M_X G^\ast \)-open set \( A \) is not a \( M_X G^\ast \)-open set in \( X \).

THEOREM 3.4. Let \( A \) and \( B \) be subsets of \( X \). Then \( m_X - \text{int}(A) = U \in \mathcal{M} \) iff \( m_X - \text{cl}(A) = U \).

PROOF. We know that \( A \) and \( B \) are subsets of \( X \). Then \( m_X - \text{int}(A) = U \in \mathcal{M} \) iff \( m_X - \text{cl}(A) = U \).

EXAMPLE 3.7. Let \( X = \{a, b, c\} \) with \( m_X \)-open set \( A = \{a, b, c\} \). Then \( M_X G^\ast \)-open set \( A \) is not a \( M_X G^\ast \)-open set in \( X \).

THEOREM 3.8. If \( A \) and \( B \) are subsets of \( X \), then \( M_X - \text{int}(A) \cup M_X - \text{int}(B) = M_X - \text{int}(A \cup B) \).

PROOF. We know that \( A \) and \( B \) are subsets of \( X \). Then \( M_X - \text{int}(A) \cup M_X - \text{int}(B) = m_X - \text{int}(A \cup B) \).

THEOREM 3.9. If \( A \) is a subset of \( X \), then \( m_X - \text{int}(A) \subseteq M_X - \text{int}(A) \).

PROOF. Let \( A \) be a subset of \( X \). Then \( x \in m_X - \text{int}(A) \Rightarrow x \in U \in \mathcal{M} \) iff \( m_X - \text{cl}(A) = U \).

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Thus $x \in m_x\operatorname{-int}(A) \Rightarrow x \notin \mathcal{M}_x\operatorname{Int}_{G^c}(A)$. Hence $m_x\operatorname{-int}(A) \subset \mathcal{M}_x\operatorname{Int}_{G^c}(A)$.

**Remark 3.10.** Containment relation in the above Theorem 3.9 may be proper as seen from the following example.

**Example 3.11.** Let $X = (a, b, c)$ with $m_x\operatorname{-open}$ set $= \{\phi, X, (a), (a, b), (a, c), (b, c), (b, c), \}$. Then $m_{G^c}G^c\operatorname{-O}(X) = \{\phi, X, (a), (a, b), (a, c), \}$. Let $A = (a, c)$. Now $\mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A) = (a, c)$ and $m_x\operatorname{-int}(A) = \{a, c\}$. It follows that $m_x\operatorname{-int}(A) \subset \mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A)$ and $m_x\operatorname{-int}(A) \notin \mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A)$.

**Theorem 3.12.** If $A$ is a subset of $X$, then $\operatorname{amint}(A) \subset \mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A)$ where $\operatorname{amint}(A)$ is given by $\operatorname{amint}(A) = \bigcup \{G : G \text{ is an am-open, } G \subset A\}$.

**Proof.** Let $A$ be a subset of a space $X$. Let $x \in \operatorname{amint}(A) \Rightarrow x \in \bigcup \{G : G \text{ is an am-open, } G \subset A\}$.

$\Rightarrow$ there exists a $G$ such that $x \in G \subset A$.

$\Rightarrow$ there exist an $m_{G^c}G^c\operatorname{-open}$ set $G$ such that $x \in G \subset A$.

Every $m_{G^c}G^c\operatorname{-open}$ set is a $m_{G^c}G^c\operatorname{-closed}$ set in $X$. $\Rightarrow x \in \bigcup \{G : G \in X, G \text{ is } m_{G^c}G^c\text{-open}, G \subset A\}$. $\Rightarrow x \in m_{G^c}\operatorname{ag\operatorname{-int}}(A)$. Thus $x \in m_{G^c}\operatorname{Int}_{G^c}(A) \Rightarrow x \in m_{G^c}\operatorname{ag\operatorname{-int}}(A)$.

**Remark 3.13.** Containment relation in the above Theorem 3.12 may be proper as seen from the following example.

**Example 3.14.** Let $X = (a, b, c)$ with $m_x\operatorname{-open}$ set $= \{\phi, X, (a), (a, b), (a, c), (b, c), \}$. Then $m_{G^c}G^c\operatorname{-O}(X) = \{\phi, X, (a), (a, b), (b, c), \}$. Let $A = (a, b)$. Now $m_{G^c}\operatorname{Int}_{G^c}(A) = \phi$ and $m_{G^c}\operatorname{ag\operatorname{-int}}(A) = \{a, b\}$. It follows that $m_{G^c}\operatorname{Int}_{G^c}(A) \subset m_{G^c}\operatorname{ag\operatorname{-int}}(A)$ and $m_{G^c}\operatorname{ag\operatorname{-int}}(A) \notin \mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A)$.

**Definition 3.17.** Let $A$ be a subset of a space $X$. We define the $m_{G^c}G^c\operatorname{-closure}$ of $A$ to be the intersection of all $m_{G^c}G^c\operatorname{-closed}$ sets containing $A$. In symbols, $m_{G^c}\operatorname{Cl}_{G^c}(A) = \{F : A \subset F, \mathcal{M}_{G^c}\operatorname{Cl}_{G^c}(F) \}$

**Theorem 3.18.** Let $A$ and $B$ be subsets of a space $X$.

(i) $m_{G^c}\operatorname{Cl}_{G^c}(X) = X$ and $m_{G^c}\operatorname{Cl}_{G^c}(\phi) = \phi$.

(ii) $A \subset m_{G^c}\operatorname{Cl}_{G^c}(A)$.

(iii) If $B$ is any $m_{G^c}G^c\operatorname{-closed}$ set containing $A$, then $m_{G^c}\operatorname{Cl}_{G^c}(A) \subset B$.

(iv) If $A \subset B$ then $m_{G^c}\operatorname{Cl}_{G^c}(A) \subset m_{G^c}\operatorname{Cl}_{G^c}(B)$.

**Proof.**

(i) By the definition of $m_{G^c}G^c\operatorname{-closure}$, $X$ is the only $m_{G^c}G^c\operatorname{-closed}$ set containing $X$. Therefore $m_{G^c}\operatorname{Cl}_{G^c}(X) = \bigcap \{F : F \supseteq X, \mathcal{M}_{G^c}\operatorname{Cl}_{G^c}(F) \}$.

(ii) By the definition of $m_{G^c}G^c\operatorname{-closed}$ sets containing $\phi$, it is obvious that $\phi = m_{G^c}\operatorname{Cl}_{G^c}(\phi)$.

(iii) Let $B$ be any $m_{G^c}G^c\operatorname{-closed}$ set containing $A$. Since $m_{G^c}\operatorname{Cl}_{G^c}(A)$ is contained in every $m_{G^c}G^c\operatorname{-closed}$ set containing $A$, $m_{G^c}\operatorname{Cl}_{G^c}(A)$ is contained in every $m_{G^c}G^c\operatorname{-closed}$ set containing $A$. Hence in particular $m_{G^c}\operatorname{Cl}_{G^c}(A) \subset B$.

(iv) Let $A$ and $B$ be subsets of $X$ such that $A \subset B$. By the definition of $m_{G^c}G^c\operatorname{-closure}$, $m_{G^c}\operatorname{Cl}_{G^c}(B) = \bigcap \{F : F \supseteq B, \mathcal{M}_{G^c}\operatorname{Cl}_{G^c}(F) \}$.

**Theorem 3.19.** Let $A$ be a subset of space $X$. $m_{G^c}\operatorname{Cl}_{G^c}(A)$ is $m_{G^c}G^c\operatorname{-closed}$ set contained $A$. In symbols, $m_{G^c}\operatorname{Cl}_{G^c}(A)$ is $m_{G^c}G^c\operatorname{-closed}$ set contained in $A$. From Theorem 3.18. (iii) $m_{G^c}\operatorname{Cl}_{G^c}(A) \subset A$. Hence $m_{G^c}\operatorname{Cl}_{G^c}(A) = A$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.20.** Let $X = (a, b, c, d)$ with $m_x\operatorname{-open}$ set $= \{\phi, X, (a), (a, b), (a, c), (a, b), (a, c)\}$. Then $m_{G^c}G^c\operatorname{-O}(X) = \{\phi, X, (a), (a, b), (a, c), \}$. Let $A = (a, b, c)$ and $m_{G^c}\operatorname{ag\operatorname{-int}}(A) = \{a, b, c\}$. It follows that $m_{G^c}\operatorname{ag\operatorname{-int}}(A) \subset m_{G^c}\operatorname{Int}_{G^c}(A)$ and $m_{G^c}\operatorname{ag\operatorname{-int}}(A) \notin \mathcal{M}_{G^c}\operatorname{Int}_{G^c}(A)$.

**Theorem 3.21.** Let $A$ and $B$ be subsets of $X$, then $m_{G^c}\operatorname{Cl}_{G^c}(A \cap B) \subset m_{G^c}\operatorname{Cl}_{G^c}(A) \cap m_{G^c}\operatorname{Cl}_{G^c}(B)$.
PROOF. Let $A$ and $B$ be subsets of $X$. Clearly $A \cap B \subseteq A$ and $A \cap B \subseteq B$. We have, Theorem 3.18.

$(iv). \mathcal{M}_X Cl_G^*(A \cap B) \subseteq \mathcal{M}_X Cl_G^*(A)$ and $\mathcal{M}_X Cl_G^*(A \cap B) \subseteq \mathcal{M}_X Cl_G^*(A \cap B)$. This implies that $\mathcal{M}_X Cl_G^*(A \cap B) \subseteq \mathcal{M}_X Cl_G^*(A) \cap \mathcal{M}_X Cl_G^*(B)$.

THEOREM 3.22. If $A$ and $B$ are subsets of a space $X$, then $\mathcal{M}_X Cl_G^*(A \cup B) = \mathcal{M}_X Cl_G^*(A) \cup \mathcal{M}_X Cl_G^*(B)$.

PROOF. Let $A$ and $B$ be subsets of $X$. We know that $A \subseteq A \cup B$ and $A \subseteq B$. Hence $\mathcal{M}_X Cl_G^*(A) \subseteq \mathcal{M}_X Cl_G^*(A \cup B)$. Let $x \in \mathcal{M}_X Cl_G^*(A)$ and suppose $x \notin \mathcal{M}_X Cl_G^*(A \cup B)$. Then there exists $\mathcal{M}_X G^*_c$-open set $A_1$ and $B_1$ such that $x \notin A_1 \cup B_1$. We have $A \subseteq A_1 \cup B_1$ and $A_1 \cup B_1$ is $\mathcal{M}_X G^*_c$-closed set by the theorem 4.1 in [11] such that $x \notin A_1 \cup B_1$. Thus $x \notin \mathcal{M}_X Cl_G^*(A \cup B)$ which is a contradiction to $x \in \mathcal{M}_X Cl_G^*(A \cup B)$. Hence $\mathcal{M}_X Cl_G^*(A \cup B) \subseteq \mathcal{M}_X Cl_G^*(A)$.

THEOREM 3.23. For an $x \in X$, $x \in \mathcal{M}_X Cl_G^*(A)$ if and only if $V \cap A \neq \emptyset$ for every $\mathcal{M}_X G^*_c$-open sets $V$ containing $x$.

PROOF. Let $x \in X$ and $x \in \mathcal{M}_X Cl_G^*(A)$. To prove $V \cap A \neq \emptyset$ for every $\mathcal{M}_X G^*_c$-open sets $V$ containing $x$. We know that result by contradiction. Suppose there exists a $\mathcal{M}_X G^*_c$-open set $V$ containing $x$ such that $V \cap A = \emptyset$. Then $A \subseteq X - V$ and $X - V$ is $\mathcal{M}_X G^*_c$-closed set. We have $\mathcal{M}_X Cl_G^*(A) \subseteq X - V$. This shows that $x \notin \mathcal{M}_X Cl_G^*(A)$, which is contradiction. Hence $V \cap A \neq \emptyset$ for every $\mathcal{M}_X G^*_c$-open set $V$ containing $x$.

Conversely, let $V \cap A \neq \emptyset$ for every $\mathcal{M}_X G^*_c$-open set $V$ containing $x$. To prove $x \in \mathcal{M}_X Cl_G^*(A)$ we know that result by contradiction. Suppose $x \notin \mathcal{M}_X Cl_G^*(A)$. Then there exists a $\mathcal{M}_X G^*_c$-closed subset $A_1$ containing $A$ such that $x \notin F$. Then $x \notin F$. Hence $x \notin \mathcal{M}_X Cl_G^*(A)$, which is a contradiction. Hence $x \in \mathcal{M}_X Cl_G^*(A)$.

THEOREM 3.24. If $A$ is a subset of a space $X$, then $\mathcal{M}_X Cl_G^*(A) \subseteq m_x cl(A)$.

PROOF. Let $A$ be a subset of a space $X$. By the definition of closure, $cl(A) = \cap \{F \subseteq X : A \subseteq F \subseteq C(X)\}$. If $A \subseteq F \subseteq C(X)$, then $A \subseteq F \subseteq \mathcal{M}_X G^*_c(X)$, because every $m_x$-closed set is $\mathcal{M}_X G^*_c$-closed. Therefore $\mathcal{M}_X Cl_G^*(A) \subseteq cl(A)$.

REMARK 3.25. Containment relation in the above Theorem 3.24 may be proper as seen from following example.

EXAMPLE 3.26. Let $X = \{a, b, c, d\}$ with $m_x$-open set $\{\varnothing, X, \{a, b\}, \{c, d\}\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}, \{c, d\}\}$. Let $A = \{a, b, d\}$. Note that $\mathcal{M}_X Cl_G^*((a, b, d)) = \{a, b, d\}$ and $m_x cl((a, b, d)) = X$. It follows that $\mathcal{M}_X Cl_G^*(A) \subseteq m_x cl(A)$ and $\mathcal{M}_X Cl_G^*(A) \neq m_x cl(A)$.

THEOREM 3.27. If $A$ is a subset of a space $X$, then $\mathcal{M}_X Cl_G^*(A) \subseteq amc l(A)$, where $amc l(A)$ is given by $amc l(A) = \cap \{F \subseteq X : A \subseteq F \subseteq \mathcal{M}_X G^*_c(X)\}$. This implies that $\mathcal{M}_X Cl_G^*(A) \subseteq amc l(A)$.

PROOF. Let $A$ be a subset of $X$. By definition of $amc l(A)$ and $\mathcal{M}_X Cl_G^*(A)$, we know that $amc l(A) \subseteq \mathcal{M}_X G^*_c(X)$ and $\mathcal{M}_X Cl_G^*(A) \subseteq \mathcal{M}_X G^*_c(X)$. We have $amc l(A) \subseteq \mathcal{M}_X Cl_G^*(A)$.

REMARK 3.28. Containment relation in the above Theorem 3.27 may be proper as seen from following example.

EXAMPLE 3.29. Let $X = \{a, b, c\}$ with $m_x$-open set $\{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $amc l(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $A = \{a, b\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}\}$.

THEOREM 3.30. If $A$ is a subset of a space $X$, then $m_x a g cl(A) \subseteq \mathcal{M}_X Cl_G^*(A)$ and $m_x a g cl(A) = amc l(A)$.

PROOF. Let $A$ be a subset of $X$. By the definition of $amc l(A)$ and $\mathcal{M}_X Cl_G^*(A)$, we know that $amc l(A) \subseteq \mathcal{M}_X Cl_G^*(A)$ and $\mathcal{M}_X Cl_G^*(A) \subseteq amc l(A)$.

REMARK 3.31. Containment relation in the above Theorem 3.30 may be proper as seen from following example.

EXAMPLE 3.32. Let $X = \{a, b, c\}$ with $m_x$-open set $\{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $amc l(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $A = \{a, b\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}\}$.

THEOREM 3.31. Let $A$ be any subset of $X$. Then $\mathcal{M}_X Cl_G^*(A)$ is $\mathcal{M}_X G^*_c$-open.

PROOF. Let $x \notin \mathcal{M}_X Cl_G^*(A)$. Then $x \notin \mathcal{M}_X Int_G^*(A)$ and $\mathcal{M}_X Cl_G^*(A) = \mathcal{M}_X Cl_G^*(A)$.

REMARK 3.32. Containment relation in the above Theorem 3.31 may be proper as seen from following example.

EXAMPLE 3.33. Let $X = \{a, b, c\}$ with $m_x$-open set $\{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $amc l(A) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $A = \{a, b\}$. Then $\mathcal{M}_X Cl_G^*(A) = \{\varnothing, X, \{a, b\}\}$.
let \( x \in \mathcal{M}_xGL(A^c) \). Then by Theorem 3.23., every \( \mathcal{M}_xGL(A^c) \)-open set \( U \) containing \( x \) such that \( U \cap A^c \neq \emptyset \). That is every \( \mathcal{M}_xGL(A^c) \)-open set \( U \) containing \( x \) such that \( U \subseteq A \).
This implies by Definition of \( \mathcal{M}_xGL(A^c) \)-interior of \( A, x \notin \mathcal{M}_xGL(A^c) \). That is \( x \in (\mathcal{M}_xGL(A)^c)^c \) and \( \mathcal{M}_xGL(A^c) \subseteq (\mathcal{M}_xGL(A)^c)^c \). Thus \( (\mathcal{M}_xGL(A^c)^c)^c = \mathcal{M}_xGL(A^c)^c \).

(ii) Follows by taking complements in (i).

(iii) Follows by replacing \( A \) by \( A^c \) in (i).

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